

Design of an Einzel Lens Using Non-Classical Variation Technique

Fatin A. J. Al-Moudarris, Oday A. Hussein and Maher M. Abd Ali

Department of Physics, College of Science, Al-Nahrain University, Baghdad-Iraq.

E-mail: udayalobaidy@yahoo.com.

Abstract

A computational investigation has been carried out in the field of charged-particle optics. The work is concerned with the design of einzel (unipotential) lens by using non-classical variation method under zero magnification condition.

The potential field distribution of the einzel lens has been represented by the following suggested function:

$U(z) = \exp[-C_1(z-5)^2] + C_2$ where C_1 and C_2 are constants, z is the optical axis and $U(z)$ is the axial potential distribution. The paraxial ray equation has been solved for finding the short beam trajectory of the charged particles traversing the lens.

The axial potential distributions and its first and second derivatives, the optical properties such as the focal length and spherical and chromatic aberrations are determined by using non-classical variation method. The electrode shape of the einzel lens has been determined in the two dimensions.

The aberrations of the electrostatic lens from our results depend on the beam trajectory of the charged particles, where the aberration is small when the beam trajectory is shorter.

1. Introduction

The most commonly used einzel (unipotential) lenses have three electrodes. The distinctive feature of einzel lenses is that they have the same constant potential U_1 at both the object and the image side, the central electrode is at a different potential U_2 , therefore, they are used when only focusing is required but the beam energy must be retained.

Einzel lenses are symmetrical with respect to the center of the lens for both of its foci. Hence, they are frequently called symmetrical lenses [1]. Einzel lenses are finding increasing applications in many areas of science and technology, because of their versatility and the rapid development of modern instrumentation [2,3,4,5,6]. It is possible, to destroy the symmetry by applying different voltages to the two outer electrodes and still have a practicable lens. The focusing action of the system remains essentially the same as in the symmetrical case, but it is not in general used owing to the obvious advantages of the latter [7].

1. Axial Potential Distribution of an Einzel Lens

It is aimed in the present work to find a more simple expression that would describe the axial potential distribution of an einzel lens with acceptable aberrations.

The following expression is suggested to represent the potential distribution along the optical axis of an einzel lens:

$$U(z) = \exp[-C_1(z-5)^2] + C_2 \dots\dots\dots(1)$$

where C_1 and C_2 are constant, and this equation is chosen because the path of the beam trajectory is shorter between the starting and ending points. And C_1 is representing the change in the length of outer electrodes and is C_2 the change in the ratio of voltage.

The axial field distribution given in equation (1) for an einzel lens and its first and second derivatives is shown in Fig.(1). This field has been used for determining the trajectory and the aberration coefficients of the lens. Fig.(1) shows the axial field distribution of an einzel lens whose central electrode is at higher voltage than the two outside electrodes. Since the potential distribution $U(z)$ is constant at the boundaries, then its first derivative $U'(z)$ is zero. This indicates that there is no electric field outside the lens i.e. there is a field-free region away from the lens terminals where the trajectory of the charged particles beam is a straight line due to the absence of any force acting on it. The second derivative of the potential; $U''(z)$ has two inflection points hence the lens has three electrodes.

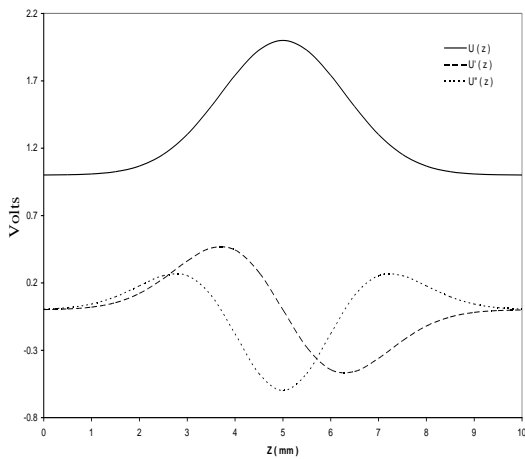


Fig.(1) The axial potential distribution $U(z)$ and its first and second derivative $U'(z)$ and $U''(z)$ of an einzel lens.

Fig.(2) shows the axial potential field distribution $U(z)$ at various values of the constant C_1 and at constant value for $C_2 = 1$. It is seen that the change in C_1 don't influent to the peak of the curve but only in the width of the curve, that means decreasing C_1 will increase the separation distances between the central electrode and the outer electrodes. Therefore, studying the optical properties with changing C_1 means the effect of the separation distance on the optical properties.

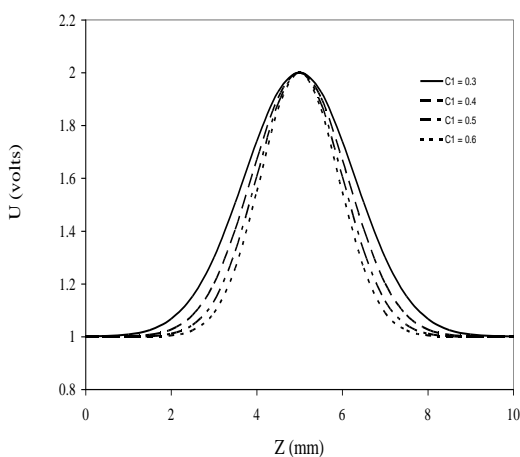


Fig.(2) The axial potential $U(z)$ as a function of z at various values of C_1 and constant value of $C_2=1$.

Fig.(3) shows the axial potential field $U(z)$ as a function of the optical axis z for different values of C_2 and C_1 constant ($C_1=1$). In Fig.(3) the peaks of the curves increase with increasing C_2 with same behaviors for all the curves. Therefore, changing C_2 will change the voltage ratio U_i/U_o where U_i the voltage of the central electrode and U_o is the voltage of the outer electrode.

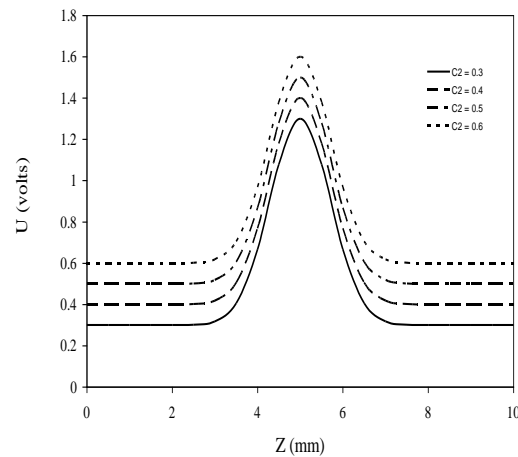


Fig.(3) The axial potential $U(Z)$ as a function of Z at various values of C_2 and constant value of $C_1=1$.

From the values of the axial potential distribution and its first and second derivatives the electrodes profile has been obtained as shown in Fig.(4). The radial and the axial dimensions of the electrodes r and z respectively have been normalized in terms of the total lens length L which has been taken in the present work to be equal to 10 mm .

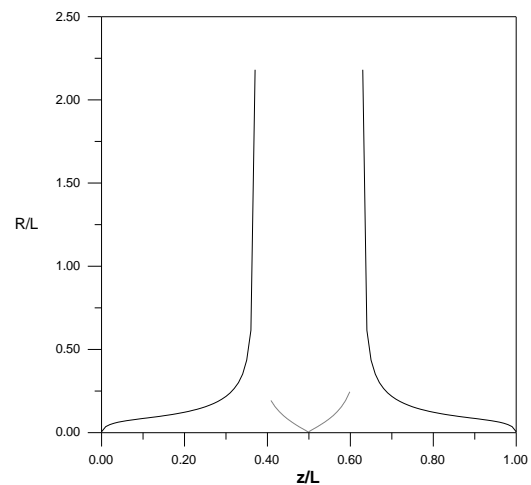


Fig.(4) The three-electrodes of einzel lens which is corresponding to the axial distribution of constant $C_1 = 0.3$, $C_2 = 0.07$ and $L = 10$.

3. The Trajectory of Charged – particles Beam

The trajectory of charged – particles beam through an axially symmetric electrostatic lens field, in terms of the axial potential U and its first and second derivatives $U'(z)$ and $U''(z)$ respectively, is given by the following equation [8]:

$$\frac{d^2 r}{dz^2} + \frac{U'(z)}{2 \cdot U(z)} \cdot \frac{dr}{dz} + \frac{U''(z)}{4 \cdot U(z)} \cdot r = 0 \quad \dots\dots(2)$$

where r is the radial displacement of the beam from the optical axis z , and the primes denote a derivative with respect to z .

By using the non-classical variation method the trajectory of the beam is given by:

$$r(z) = 1 + A_0 z^2 + A_1 z^3 + A_2 z^4 \quad \dots\dots\dots(3)$$

where A_0, A_1, A_2 are constant.

The electron beam trajectory along the electrostatic einzel lens field under zero magnification condition is shown in Fig.(5), the computation is made for $C_1 = 0.3$ and $C_2 = 1$.

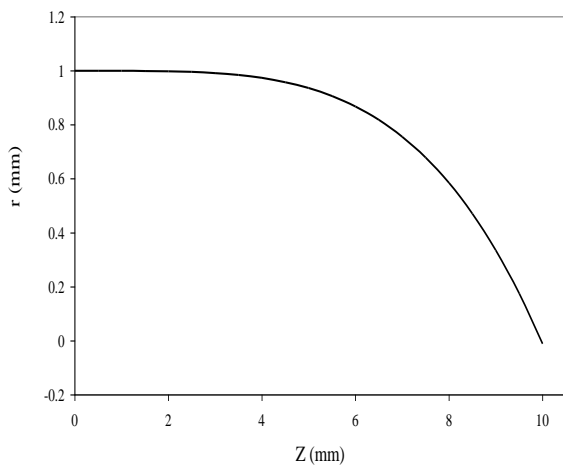


Fig.(5) The trajectory of the charged particles traversing the field of the lens operated under zero magnification condition at $C_1 = 0.3$ and $C_2 = 1$.

4. Spherical Aberration

The spherical aberration coefficient C_{si} at the image plane has been computed with the aid of the following formula [9] and [10]:

$$C_{Si} = \frac{(U_i)^{-1/2}}{16 \cdot (r_i')^4} \int_{z_0}^{z_i} \left[\frac{5}{4} \cdot \left(\frac{U''(z)}{U(z)} \right)^2 + \frac{5}{24} \cdot \left(\frac{U'(z)}{U(z)} \right)^4 + \frac{14}{3} \cdot \left(\frac{U'(z)}{U(z)} \right)^3 \cdot \left(\frac{r'}{r} \right) - \frac{3}{2} \cdot \left(\frac{U'(z)}{U(z)} \right)^2 \cdot \left(\frac{r''}{r} \right) \right] \cdot \sqrt{U(z)} \cdot r^4 dz \quad \dots\dots\dots(4)$$

where $U = U(z)$ is the axial potential, the axial potential, the primes denote derivative with respect to z , and $U_i = U(z_i)$ is the potential at the image where $z = z_i$.

Fig.(6) shows the image-side relative spherical aberration coefficient C_s / f_i of the electrostatic einzel lens operated under zero magnification condition as a function of the voltage ratio U_i / U_0 at various values of C_1 . The trajectories which are shown in Fig.(5) have been used for computing the relative spherical aberration coefficients as a function of U_i / U_0 at the value of $C_1 = 0.3, 0.4, 0.5$ and 0.6 . The relative spherical aberration coefficients decreases as the ratio U_i / U_0 increases up to limited values of U_i / U_0 , where all the curves have the minimum values.

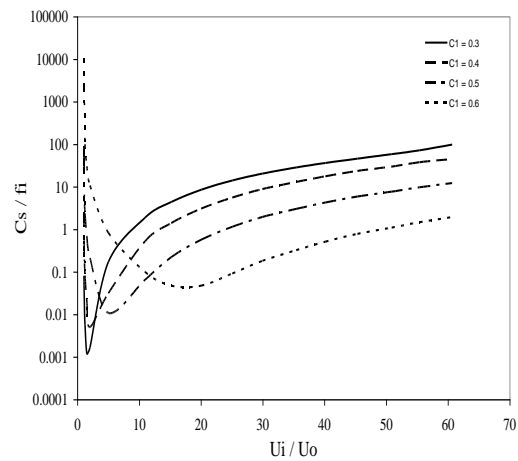


Fig.(6) The relative spherical aberration coefficient C_s / f_i as a function of U_i / U_0 at various C_1 .

It is seen that C_s / f_i has a minimum value at $U_i / U_0 = 1.5$ which refer to the value of $C_1 = 0.3$. The calculation shows that the $C_1 = 0.6$ gives the optimum value of the relative spherical aberration coefficient for $U_i / U_0 > 20$ and the value of this coefficient in this region still less than unity up to $U_i / U_0 = 50$.

5. Chromatic aberration

The chromatic aberration coefficient C_{ci} at the image plane has been computed with the aid of the following formula [8]:

$$C_{ci} = \frac{(U_i)^{1/2}}{(r_i')^2} \int_{z_0}^{z_i} \left[\frac{U'(z)}{2 \cdot U(z)} \cdot r \cdot r' + \frac{U''(z)}{4 \cdot U(z)} \cdot r' \right] \cdot U(z)^{-1/2} \cdot dz \quad \dots\dots\dots(5)$$

Within the trajectories which are shown in Fig.(5), the image-side relative chromatic aberration coefficient C_{ci} / f_i has been

computed as a function of the voltage ratio U_i/U_o at various values of C_1 . Fig.(7) shows that C_{ci}/f_i increases with increasing U_i/U_o . Low values of C_{ci}/f_i are achieved at high values of C_1 .

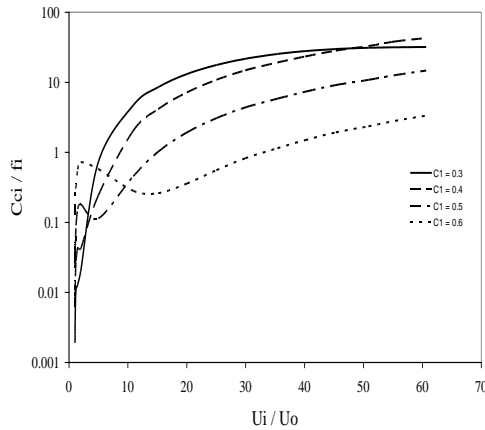


Fig.(7) The relative chromatic aberration coefficient C_{ci}/f_i as a function of U_i/U_o at various value C_1 .

6. Conclusion

It appears from the present investigation that it is possible to design various types of electrostatic lenses with small aberrations operated under different potential ratios.

It has been found that it is possible to design an einzel lens with small aberration and with minimum path of trajectory by used the non-classical variation method which can be used to solve the paraxial ray equation of charged particles beam.

From the results it has been found that the aberration coefficient decreases as the C_1 decreasing, where the best result occur at the C_1 is equal 0.3.

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الخلاصة

اجرى بحث حاسوبي في مجال بصريات الجسيمات المشحونة. يهتم البحث بتصميم عدسة احادية الجهد باستخدام طريقة التباير غير التقليدي تحت ظروف تكبير صفري. ان توزيع مجال الجهد لعدسة احادية الجهد تم تمثيله بالدالة المقترحة التالية: $U(z) = \exp[-C_1(z-5)^2]$ حيث $C_1 + C_2$ و C_2 ثوابت عددية، z هو المحور البصري و $U(z)$ هو توزيع الجهد المحوري. تم حل معادلة الاشعة المحورية للمجال المقترح عن طريق ايجاد اقصر مسار للجسيمات المشحونة المارة في العدسة. ومن توزيع الجهد المحوري ومشتقاته الاولى والثانية تم حساب الخواص البصرية كالبعد البؤري والزيغين الكروي واللوني. كذلك تم ايجاد شكل اقطاب عدسة كهروسكونية ببعدين. ويعتمد الزيوغ في العدسات على مسار الجسيمات المشحونة حيث يكون الزيوغ قليل عندما يكون مسار الجسيمات قصير وهذا وجدناه في هذه الدراسة وذلك باستخدام طريقة التباير غير التقليدي.