Weak Soft Separation Axioms and Weak Soft (1,2)*-$\tilde{D}$-Separation Axioms in Soft Bitopological Spaces

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Abstract

In this article we introduce and study new types of soft sets in soft bitopological spaces, namely, soft (1,2)*-difference sets and soft (1,2)*-b-difference sets by using the notion of soft $\tilde{\tau}_1\tilde{\tau}_2$-open sets and soft (1,2)*-b-open sets respectively. Furthermore we use these soft sets to define and study new types of soft separation axioms, namely, soft (1,2)*-$\tilde{D}_i$-spaces and soft (1,2)*-b-$\tilde{D}_i$-spaces for $i = 0,1,2$ which are weaker than soft (1,2)*-$\tilde{T}_i$-spaces and soft (1,2)*-b-$\tilde{T}_i$-spaces for $i = 0,1,2$ respectively. The basic properties and characteristics each of soft (1,2)*-b-$\tilde{T}_i$-spaces, soft (1,2)*-$\tilde{D}_i$-spaces and soft (1,2)*-b-$\tilde{D}_i$-spaces for $i = 0,1,2$ also have been studied.

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Introduction

Soft set theory was firstly introduced by Molodtsov D. [1] as a new mathematical tool for dealing with uncertainty while modeling problems in economics, medical sciences, computer science, engineering physics and social sciences. Senel G. and Çagman N. [5] defined the theory of soft bitopological spaces over an initial universe with a fixed set of parameters. Revathi N. and Bageerathi K. [4] introduce and study soft (1,2)*-b-open sets in soft bitopological spaces $(X, \tilde{\tau}_1, \tilde{\tau}_2, E)$ as a generalization of soft $\tilde{\tau}_1\tilde{\tau}_2$-open sets. The purpose of this paper is to introduce new kinds of soft spaces, namely, soft (1,2)*-$\tilde{D}_i$-spaces and soft (1,2)*-b-$\tilde{D}_i$-spaces for $i = 0,1,2$ by using the concepts of soft (1,2)*-difference sets and soft (1,2)*-b-difference sets. These soft spaces which are weaker than soft (1,2)*-$\tilde{T}_i$-spaces and soft (1,2)*-b-$\tilde{T}_i$-spaces for $i = 0,1,2$ respectively. The characteristics and basic properties each of soft (1,2)*-b-$\tilde{T}_i$-spaces, soft (1,2)*-$\tilde{D}_i$-spaces and soft (1,2)*-b-$\tilde{D}_i$-spaces for $i = 0,1,2$ also have been studied.

1. Preliminaries

In this paper, X refers to an initial universe, P(X) is the power set of X, E is the set of parameters. Now, we recall the following definitions and propositions.

Definition (1.1)[1]:

A soft set over X is a pair (A, U), where A is a function given by $A: U \to P(X)$ and U is a non-empty subset of E.

Definition (1.2)[3]:

If (A, U) is a soft set over X. Then $\tilde{a} = (e, \{a\})$ is called a soft point of (A, U), if $e \in U$ and $a \in A(e)$, and is denoted by $\tilde{a} \tilde{\in} (A, U)$.

Definition (1.3)[6]:

If $\tilde{\tau}$ is a family of soft sets over X. Then $\tilde{\tau}$ is called a soft topology on X if $\tilde{\tau}$ satisfies the following:

i) $\tilde{X}, \tilde{\phi}$ belong to $\tilde{\tau}$.

ii) If $(A_1, E), (A_2, E) \tilde{\in} \tilde{\tau}$, then $(A_1, E) \tilde{\cap} (A_2, E) \tilde{\in} \tilde{\tau}$.

(iii) If $(A_\alpha, E) \tilde{\in} \tilde{\tau}$, $\forall \alpha \in \Lambda$, then $\bigcup_{\alpha \in \Lambda} (A_\alpha, E) \tilde{\in} \tilde{\tau}$.

The triple $(X, \tilde{\tau}, E)$ is called a soft topological space over X. The members of $\tilde{\tau}$ are called soft open soft subsets of $\tilde{X}$.

Definition (1.4)[5]:

Let $X \neq \phi$, and let $\tilde{\tau}_1$ and $\tilde{\tau}_2$ be soft topologies over X. Then $(X, \tilde{\tau}_1, \tilde{\tau}_2, E)$ is called a soft bitopological space over X.

Definition (1.5)[5]

A soft subset (U, E) of a soft bitopological space $(X, \tilde{\tau}_1, \tilde{\tau}_2, E)$ is called soft $\tilde{\tau}_1\tilde{\tau}_2$-open if $(U, E) = (V_1, E) \tilde{\bigcup} (V_2, E)$ where $(V_1, E) \tilde{\in} \tilde{\tau}_1$ and $(V_2, E) \tilde{\in} \tilde{\tau}_2$. The complement of a soft $\tilde{\tau}_1\tilde{\tau}_2$-open set is defined to be soft $\tilde{\tau}_1\tilde{\tau}_2$-closed.

The family of all soft $\tilde{\tau}_1\tilde{\tau}_2$-open sets need not form a soft topology as shown by the following example:

***Example (1.6)** :*

Let $X = \{a, b, c\}$ and $E = \{e_1, e_2\}$, and let $\tilde{\tau}_1 = \{\tilde{X}, \tilde{\phi}, (A_1, E)\}$ and $\tilde{\tau}_2 = \{\tilde{X}, \tilde{\phi}, (A_2, E)\}$ be two soft topologies over X, where $(A_1, E) = \{(e_1, \{X\}), (e_2, \{a, b\})\}$ and $(A_2, E) = \{(e_1, \{X\}), (e_2, \{a, c\})\}$. The soft sets in $\{\tilde{X}, \tilde{\phi}, (A_1, E), (A_2, E)\}$ are soft $\tilde{\tau}_1\tilde{\tau}_2$-open set in $\tilde{X}$. Since $(A_1, E) \tilde{\bigcap} (A_2, E) = \{(e_1, \{X\}), (e_2, \{a\})\} = (A, E)$, but (A, E) is not soft $\tilde{\tau}_1\tilde{\tau}_2$-open set in $\tilde{X}$. Thus $(X, \tilde{\tau}_1, \tilde{\tau}_2, E)$ is not soft topology over X.

***Definition (1.7)[5]**:*

If $(X, \tilde{\tau}_1, \tilde{\tau}_2, E)$ is a soft bitopological space and $(A, E) \tilde{\subseteq} \tilde{X}$. Then:

i) $\tilde{\tau}_1\tilde{\tau}_2 cl(A, E) = \tilde{\bigcap}\{(F, E) : (F, E)$ is soft $\tilde{\tau}_1\tilde{\tau}_2$-closed set in $\tilde{X}$ and $(A, E) \tilde{\subseteq} (F, E)\}$ is called the soft $\tilde{\tau}_1\tilde{\tau}_2$-closure of (A, E).

ii) $\tilde{\tau}_1\tilde{\tau}_2 \operatorname{int}(A, E) = \tilde{\bigcup}\{(U, E) : (U, E)$ is soft $\tilde{\tau}_1\tilde{\tau}_2$-open set in $\tilde{X}$ and $(U, E) \tilde{\subseteq} (A, E)\}$ is called the soft $\tilde{\tau}_1\tilde{\tau}_2$-interior of (A, E)

Proposition (1.8)[2] :

If $(X, \tilde{\tau}_1, \tilde{\tau}_2, E)$ is a soft bitopological space and $(A_1, E), (A_2, E) \tilde{\subseteq} \tilde{X}$. Then:

i) $\tilde{\tau}_1\tilde{\tau}_2 \operatorname{int}(A_1, E) \tilde{\subseteq} (A_1, E)$ and $(A_1, E) \tilde{\subseteq} \tilde{\tau}_1\tilde{\tau}_2 cl(A_1, E)$.

ii) The union of soft $\tilde{\tau}_1\tilde{\tau}_2$-open sets in $\tilde{X}$ is soft $\tilde{\tau}_1\tilde{\tau}_2$-open.

iii) The intersection of soft $\tilde{\tau}_1\tilde{\tau}_2$-closed sets in $\tilde{X}$ is soft $\tilde{\tau}_1\tilde{\tau}_2$-closed.

iv) $\tilde{\tau}_1\tilde{\tau}_2 \operatorname{int}(A_1, E)$ is soft $\tilde{\tau}_1\tilde{\tau}_2$-open set in $\tilde{X}$ and $\tilde{\tau}_1\tilde{\tau}_2 cl(A_1, E)$ is soft $\tilde{\tau}_1\tilde{\tau}_2$-closed set in $\tilde{X}$.

v) (A_1, E) is soft $\tilde{\tau}_1\tilde{\tau}_2$-open set in $\tilde{X}$ iff $(A_1, E) = \tilde{\tau}_1\tilde{\tau}_2 \operatorname{int}(A_1, E)$.

vi) (A_1, E) is soft $\tilde{\tau}_1\tilde{\tau}_2$-closed set in $\tilde{X}$ iff $(A_1, E) = \tilde{\tau}_1\tilde{\tau}_2 cl(A_1, E)$.

vii) $\tilde{\tau}_1\tilde{\tau}_2 cl(\tilde{\tau}_1\tilde{\tau}_2 cl(A_1, E)) = \tilde{\tau}_1\tilde{\tau}_2 cl(A_1, E)$ and $\tilde{\tau}_1\tilde{\tau}_2 \operatorname{int}(\tilde{\tau}_1\tilde{\tau}_2 \operatorname{int}(A_1, E)) = \tilde{\tau}_1\tilde{\tau}_2 \operatorname{int}(A_1, E)$.

viii) $(\tilde{\tau}_1\tilde{\tau}_2 \operatorname{int}(A, E))^c = \tilde{\tau}_1\tilde{\tau}_2 cl((A, E)^c)$ and $\tilde{\tau}_1\tilde{\tau}_2 \operatorname{int}((A, E)^c) = (\tilde{\tau}_1\tilde{\tau}_2 cl(A, E))^c$.

ix) If $(A_1, E) \tilde{\subseteq} (A_2, E)$, then $\tilde{\tau}_1\tilde{\tau}_2 cl(A_1, E) \tilde{\subseteq} \tilde{\tau}_1\tilde{\tau}_2 cl(A_2, E)$.

x) If $(A_1, E) \tilde{\subseteq} (A_2, E)$, then $\tilde{\tau}_1\tilde{\tau}_2 \operatorname{int}(A_1, E) \tilde{\subseteq} \tilde{\tau}_1\tilde{\tau}_2 \operatorname{int}(A_2, E)$.

Definition (1.9)[4] :

A soft subset (A, E) of a soft bitopological space $(X, \tilde{\tau}_1, \tilde{\tau}_2, E)$ is called soft (1,2)*-b-open if $(A, E) \tilde{\subseteq} \tilde{\tau}_1\tilde{\tau}_2 \operatorname{int}(\tilde{\tau}_1\tilde{\tau}_2 cl(A, E)) \tilde{\bigcup} \tilde{\tau}_1\tilde{\tau}_2 cl(\tilde{\tau}_1\tilde{\tau}_2 \operatorname{int}(A, E))$.

The complement of a soft (1,2)*-b-open set is defined to be soft (1,2)*-b-closed. The family of all soft (1,2)*-b-open (resp. soft (1,2)*-b-closed) subsets of $(X, \tilde{\tau}_1, \tilde{\tau}_2, E)$ is denoted by (1,2)*-b-O($\tilde{X}$) (resp. (1,2)*-b-C($\tilde{X}$)).

Every soft $\tilde{\tau}_1\tilde{\tau}_2$-open set is soft (1,2)*-b-open, but the converse is not true in general. We see that by the following example:

Example (1.10):

Let $X = \{a, b, c\}$ and $E = \{e_1, e_2\}$, and let $\tilde{\tau}_1 = \{\tilde{X}, \tilde{\phi}, (A_1, E)\}$ and $\tilde{\tau}_2 = \{\tilde{X}, \tilde{\phi}, (A_2, E)\}$ be two soft topologies over X, where $(A_1, E) = \{(e_1, \{X\}), (e_2, \{a\})\}$ and $(A_2, E) = \{(e_1, \{X\}), (e_2, \{a, b\})\}$. The soft sets in $\{\tilde{X}, \tilde{\phi}, (A_1, E), (A_2, E)\}$ are soft $\tilde{\tau}_1\tilde{\tau}_2$- open. Thus $(A, E) = \{(e_1, \{X\}), (e_2, \{a, c\})\}$ is a soft

(1,2)*-b-open set in $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$, but is not soft $\tilde{\tau}_1\tilde{\tau}_2$-open.

Definition(1.11)[4] :

Let (A,E) be a soft subset of a soft bitopological space $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$. Then:

i) The soft (1,2)*-b-closure of (A,E), denoted by (1,2)*-bcl(A,E) is the intersection of all soft (1,2)*-b-closed sets in $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$ which contains (A,E).

ii) The soft (1,2)*-b-interior of (A,E), denoted by (1,2)*-bint(A,E) is the union of all soft (1,2)*-b-open sets in $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$ which are contained in (A,E).

Proposition (1.12) :

If $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$ is a soft bitopological space and $(A_1,E),(A_2,E)\tilde{\subseteq}\tilde{X}$. Then:

i) $\tilde{\tau}_1\tilde{\tau}_2\,\mathrm{int}(A_1,E)\tilde{\subseteq}(1,2)^*\text{-}b\,\mathrm{int}(A_1,E)\tilde{\subseteq}(A_1,E)$.

ii) $(A_1,E)\tilde{\subseteq}(1,2)^*\text{-}\mathrm{bcl}(A_1,E)\tilde{\subseteq}\tilde{\tau}_1\tilde{\tau}_2\mathrm{cl}(A_1,E)$.

iii) The union of soft (1,2)*-b-open sets in $\tilde{X}$ is soft (1,2)*-b-open. [4].

iv) The intersection of soft (1,2)*-b-closed sets in $\tilde{X}$ is soft (1,2)*-b-closed. [4].

v) $(1,2)^*\text{-}b\,\mathrm{int}(A_1,E)$ is soft (1,2)*-b-open and $(1,2)^*\text{-}\mathrm{bcl}(A_1,E)$ is soft (1,2)*-b-closed.

vi) (A_1,E) is soft (1,2)*-b-open in $\tilde{X}$ iff $(1,2)^*\text{-}b\,\mathrm{int}(A_1,E)=(A_1,E)$. [4].

vii) (A_1,E) is soft (1,2)*-b-closed in $\tilde{X}$ iff $(1,2)^*\text{-}\mathrm{bcl}(A_1,E)=(A_1,E)$. [4].

viii) If $(A_1,E)\tilde{\subseteq}(A_2,E)$, then $(1,2)^*\text{-}b\,\mathrm{int}(A_1,E)\tilde{\subseteq}(1,2)^*\text{-}b\,\mathrm{int}(A_2,E)$.

ix) If $(A_1,E)\tilde{\subseteq}(A_2,E)$, then $(1,2)^*\text{-}\mathrm{bcl}(A_1,E)\tilde{\subseteq}(1,2)^*\text{-}\mathrm{bcl}(A_2,E)$.

x) $\tilde{x}\tilde{\in}(1,2)^*\text{-}\mathrm{bcl}(A,E)$ iff for every soft (1,2)*-b-open set (V,E) containing $\tilde{x}$, $(V,E)\tilde{\cap}(A,E)\neq\tilde{\phi}$.

2. Weak Soft Separation Axioms

In this section we define and study new types of soft separation axioms and weak soft separation axioms in soft bitopological spaces, namley, soft (1,2)*-$\tilde{T}_i$-spaces and soft (1,2)*-b-$\tilde{T}_i$-spaces for $i=0,1,2$. The characteristics and the relations among these soft spaces also have been studied.

Definitions (2.1) :

A soft bitopological space $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$ is called a soft (1,2)*-$\tilde{T}_0$-space (resp. soft (1,2)*-b-$\tilde{T}_0$-space) if for any two distinct soft points $\tilde{x}$ and $\tilde{y}$ of $\tilde{X}$, there exists a soft $\tilde{\tau}_1\tilde{\tau}_2$-open set (resp. soft (1,2)*-b-open set) of $\tilde{X}$ containing one of the soft points but not the other.

Remark (2.2) :

Every soft (1,2)*-$\tilde{T}_0$-space is soft (1,2)*-b-$\tilde{T}_0$-space, but the converse is not true in general. As we see by the following example:

Example (2.3) :

Let $X=\{a,b\}$ and $E=\{e_1,e_2\}$ and let $\tilde{\tau}_1=\{\tilde{X},\tilde{\phi},(A_1,E)\}$ and $\tilde{\tau}_2=\{\tilde{X},\tilde{\phi},(A_2,E)\}$ be soft topologies over X, where $(A_1,E)=\{(e_1,\{a\}),(e_2,\{a\})\}$ and $(A_2,E)=\{(e_1,\{b\}),(e_2,\{b\})\}$. The soft sets in $\{\tilde{X},\tilde{\phi},(A_1,E),(A_2,E)\}$ are soft $\tilde{\tau}_1\tilde{\tau}_2$-open. Thus $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$ is a soft (1,2)*-b-$\tilde{T}_0$-space, but is not soft (1,2)*-$\tilde{T}_0$-space, since $\tilde{x}=(e_1,\{a\})\neq\tilde{y}=(e_2,\{a\})$, but there exists no soft $\tilde{\tau}_1\tilde{\tau}_2$-open set containing $\tilde{x}$, but not containing $\tilde{y}$.

Now, we proceed to prove that every soft bitopological space is soft (1,2)*-b-$\tilde{T}_0$-space.

Proposition (2.4) :

Let $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$ be a soft bitopological space. If for some $\tilde{x}\tilde{\in}\tilde{X}$, $\{\tilde{x}\}$ is soft (1,2)*-b-open, then $\tilde{x}\tilde{\notin}(1,2)^*\text{-}\mathrm{bcl}(\{\tilde{y}\})$ for all $\tilde{y}\neq\tilde{x}$.

Proof:

If $\{\tilde{x}\}$ is soft (1,2)*-b-open for some $\tilde{x}\tilde{\in}\tilde{X}$, then $\{\tilde{x}\}^c$ is soft (1,2)*-b-closed and $\tilde{y}\tilde{\in}\{\tilde{x}\}^c$ for all $\tilde{y}\neq\tilde{x}$. Hence $\{\tilde{y}\}\tilde{\subseteq}\{\tilde{x}\}^c$ and $(1,2)^*\text{-}\mathrm{bcl}(\{\tilde{y}\})\tilde{\subseteq}\{\tilde{x}\}^c$ for all $\tilde{y}\neq\tilde{x}$. If $\tilde{x}\tilde{\in}(1,2)^*\text{-}\mathrm{bcl}(\{\tilde{y}\})$ for some $\tilde{y}\neq\tilde{x}$, then $\tilde{x}\tilde{\in}\{\tilde{x}\}^c$ which is not true. Therefore, $\tilde{x}\tilde{\notin}(1,2)^*\text{-}\mathrm{bcl}(\{\tilde{y}\})$ for all $\tilde{y}\neq\tilde{x}$.

Theorem (2.5) :

In a soft bitopological space $(X, \tilde{\tau}_1, \tilde{\tau}_2, E)$, distinct soft points have distinct soft (1,2)*-b-closures.

Proof:

Let $\tilde{x}, \tilde{y} \tilde{\in} \tilde{X}$ such that $\tilde{x} \neq \tilde{y}$. Take $(A, E) = \{\tilde{x}\}^c$. Then $\tilde{\tau}_1\tilde{\tau}_2 cl(A, E) = (A, E)$ or $\tilde{\tau}_1\tilde{\tau}_2 cl(A, E) = \tilde{X}$.

Case (a)

If $\tilde{\tau}_1\tilde{\tau}_2 cl(A, E) = (A, E)$, then (A, E) is soft $\tilde{\tau}_1\tilde{\tau}_2$-closed and hence soft (1,2)*-b-closed. Then $(A, E)^c = \{\tilde{x}\}$ is soft (1,2)*-b-open, not containing $\tilde{y}$. Therefore by proposition (2.4), $\tilde{x} \tilde{\notin} (1,2)^*\text{-}bcl(\{\tilde{y}\})$. But $\tilde{x} \tilde{\in} (1,2)^*\text{-}bcl(\{\tilde{x}\})$ which implies that $(1,2)^*\text{-}bcl(\{\tilde{x}\})$ and $(1,2)^*\text{-}bcl(\{\tilde{y}\})$ are distinct.

Case (b)

If $\tilde{\tau}_1\tilde{\tau}_2 cl(A, E) = \tilde{X}$, then (A, E) is soft (1,2)*-b-open and hence $\{\tilde{x}\}$ is soft (1,2)*-b-closed which shows that $(1,2)^*\text{-}bcl(\{\tilde{x}\}) = \{\tilde{x}\}$ which is not equal to $(1,2)^*\text{-}bcl(\{\tilde{y}\})$.

Theorem (2.6) :

Every soft bitopological space $(X, \tilde{\tau}_1, \tilde{\tau}_2, E)$ is a soft (1,2)*-b-$\tilde{T}_0$-space.

Proof :

Let $\tilde{x}, \tilde{y} \tilde{\in} \tilde{X}$, $\tilde{x} \neq \tilde{y}$. Then by theorem (2.5), $(1,2)^*\text{-}bcl(\{\tilde{x}\})$ is not equal to $(1,2)^*\text{-}bcl(\{\tilde{y}\})$. Then there exists $\tilde{z} \tilde{\in} \tilde{X}$ such that $\tilde{z} \tilde{\in} (1,2)^*\text{-}bcl(\{\tilde{x}\})$, but $\tilde{z} \tilde{\notin} (1,2)^*\text{-}bcl(\{\tilde{y}\})$ or $\tilde{z} \tilde{\in} (1,2)^*\text{-}bcl(\{\tilde{y}\})$, but $\tilde{z} \tilde{\notin} (1,2)^*\text{-}bcl(\{\tilde{x}\})$. Without loss of generality, let $\tilde{z} \tilde{\in} (1,2)^*\text{-}bcl(\{\tilde{x}\})$, but $\tilde{z} \tilde{\notin} (1,2)^*\text{-}bcl(\{\tilde{y}\})$. If $\tilde{x} \tilde{\in} (1,2)^*\text{-}bcl(\{\tilde{y}\})$, then $(1,2)^*\text{-}bcl(\{\tilde{x}\})$ is contained in $(1,2)^*\text{-}bcl(\{\tilde{y}\})$ and therefore, $\tilde{z} \tilde{\in} (1,2)^*\text{-}bcl(\{\tilde{y}\})$. which is a contradiction. Thus we get $\tilde{x} \tilde{\notin} (1,2)^*\text{-}bcl(\{\tilde{y}\})$. This implies that $\tilde{x} \tilde{\in} ((1,2)^*\text{-}bcl(\{\tilde{y}\}))^c$. Therefore, $(X, \tilde{\tau}_1, \tilde{\tau}_2, E)$ is a soft (1,2)*-b-$\tilde{T}_0$-space.

Now, we introduce the definition of soft (1,2)*-$\tilde{T}_1$-spaces and soft (1,2)*-b-$\tilde{T}_1$-spaces also we study the characteristics and the relations between these soft spaces and the previous soft spaces.

Definition (2.7) :

A soft bitopological space $(X, \tilde{\tau}_1, \tilde{\tau}_2, E)$ is called a soft (1,2)*-$\tilde{T}_1$-space (resp. soft (1,2)*-b-$\tilde{T}_1$-space) if for any two distinct soft points $\tilde{x}$ and $\tilde{y}$ of $\tilde{X}$, there exists a soft $\tilde{\tau}_1\tilde{\tau}_2$-open set (resp. soft (1,2)*-b-open set) of $\tilde{X}$ containing $\tilde{x}$ but not $\tilde{y}$ and a soft $\tilde{\tau}_1\tilde{\tau}_2$ -open set (resp. soft (1,2)*-b -open set) of $\tilde{X}$ containing $\tilde{y}$ but not $\tilde{x}$.

Remark (2.8) :

Every soft (1,2)*-b-$\tilde{T}_1$-space is a soft (1,2)*-b-$\tilde{T}_0$-space, but the converse is not true in general. We see that by the following example:

Example (2.9) :

Let $X = \{a, b\}$ and $E = \{e_1, e_2\}$ and let $\tilde{\tau}_1 = \{\tilde{X}, \tilde{\phi}, (A_1, E)\}$ and $\tilde{\tau}_2 = \{\tilde{X}, \tilde{\phi}, (A_2, E)\}$ be soft topologies over X, where $(A_1, E) = \{(e_1, \{a\}), (e_2, \{a\})\}$ and $(A_2, E) = \{(e_1, \{a\}), (e_2, \{\phi\})\}$. The soft sets in $\{\tilde{X}, \tilde{\phi}, (A_1, E), (A_2, E)\}$ are soft $\tilde{\tau}_1\tilde{\tau}_2$-open. Thus $(X, \tilde{\tau}_1, \tilde{\tau}_2, E)$ is a soft (1,2)*-b-$\tilde{T}_0$-space, but is not soft (1,2)*-b-$\tilde{T}_1$-space.

Remark (2.10) :

Every soft (1,2)*-$\tilde{T}_1$-space is a soft (1,2)*-b-$\tilde{T}_1$-space, but the converse is not true in general. We see that by the following example:

Example (2.11) :

Let $X = \{a, b\}$ and $E = \{e_1, e_2\}$ and let $\tilde{\tau}_1 = \{\tilde{X}, \tilde{\phi}, (A_1, E)\}$ and $\tilde{\tau}_2 = \{\tilde{X}, \tilde{\phi}, (A_2, E)\}$ be soft topologies over X, where $(A_1, E) = \{(e_1, \{a\}), (e_2, \{a\})\}$ and $(A_2, E) = \{(e_1, \{b\}), (e_2, \{b\})\}$. The soft sets in $\{\tilde{X}, \tilde{\phi}, (A_1, E), (A_2, E)\}$ are soft $\tilde{\tau}_1\tilde{\tau}_2$-open. Thus $(X, \tilde{\tau}_1, \tilde{\tau}_2, E)$ is a soft (1,2)*-b-$\tilde{T}_1$-space, but is not soft (1,2)*-$\tilde{T}_1$-space.

Remark (2.12) :

Every soft (1,2)*-$\tilde{T}_1$-space is a soft (1,2)*-$\tilde{T}_0$-space, but the converse is not true in general. We see that by the following example:

Example (2.13) :

Let $X=\{a,b,c\}$ and $E=\{e\}$ and let $\tilde{\tau}_1=\{\tilde{X},\tilde{\phi},(A_1,E)\}$ and $\tilde{\tau}_2=\{\tilde{X},\tilde{\phi},(A_2,E)\}$ be soft topologies over X, where $(A_1,E)=\{(e,\{a\})\}$ and $(A_2,E)=\{(e,\{b\})\}$.

The soft sets in $\{\tilde{X},\tilde{\phi},(A_1,E),(A_2,E),(A_3,E)\}$ are soft $\tilde{\tau}_1\tilde{\tau}_2$-open sets, where $(A_3,E)=\{(e,\{a,b\})\}$. Thus $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$ is a soft (1,2)*-$\tilde{T}_0$-space, but is not soft (1,2)*-$\tilde{T}_1$-space.

Theorem (2.14) :

In a soft bitopological space $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$ the following statements are equivalent:

(i) $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$ is a soft (1,2)*-b-$\tilde{T}_1$-space.

(ii) For each $\tilde{x}\tilde{\in}\tilde{X}$, $\{\tilde{x}\}$ is a soft (1,2)*-b-closed set in $\tilde{X}$.

(iii) Each soft subset of $\tilde{X}$ is the intersection of all soft (1,2)*-b-open sets containing it.

(iv) The intersection of all soft (1,2)*-b-open sets containing the soft point $\tilde{x}\tilde{\in}\tilde{X}$ is $\{\tilde{x}\}$.

Proof:

$(i)\Rightarrow(ii)$. Let $\tilde{x}\tilde{\in}\tilde{X}$. To prove that $\{\tilde{x}\}$ is soft (1,2)*-b-closed in $\tilde{X}$. Let $\tilde{y}\tilde{\notin}\{\tilde{x}\}$ $\Rightarrow \tilde{x}\neq\tilde{y}$. Since $\tilde{X}$ is a soft (1,2)*-b-$\tilde{T}_1$-space, then there is a soft (1,2)*-b-open set (U,E) in $\tilde{X}$ such that $\tilde{y}\tilde{\in}(U,E)$ and $\tilde{x}\tilde{\notin}(U,E)$ $\Rightarrow$ $\{\tilde{x}\}\tilde{\cap}(U,E)=\tilde{\phi}$ $\Rightarrow$ $\{\tilde{x}\}\tilde{\subseteq}(U,E)^c$ $\Rightarrow$ $(1,2)^*\text{-}bcl(\{\tilde{x}\})\tilde{\subseteq}(1,2)^*\text{-}bcl((U,E)^c)=(U,E)^c$. Since $\tilde{y}\tilde{\notin}(U,E)^c$ $\Rightarrow$ $\tilde{y}\tilde{\notin}(1,2)^*\text{-}bcl(\{\tilde{x}\})$ $\Rightarrow$ $(1,2)^*\text{-}bcl(\{\tilde{x}\})=\{\tilde{x}\}$. Therefore $\{\tilde{x}\}$ is a soft (1,2)*-b-closed set in $\tilde{X}$.

$(ii)\Rightarrow(iii)$. Let $(A,E)\tilde{\subset}\tilde{X}$ and $\tilde{y}\tilde{\notin}(A,E)$. Then $(A,E)\tilde{\subset}\{\tilde{y}\}^c$ and $\{\tilde{y}\}^c$ is soft (1,2)*-b-open in $\tilde{X}$ and $(A,E)=\tilde{\bigcap}\{\{\tilde{y}\}^c:\tilde{y}\tilde{\in}(A,E)^c\}$ which is the intersection of all soft (1,2)*-b-open sets containing (A,E).

$(iii)\Rightarrow(iv)$. Obvious.

$(iv)\Rightarrow(i)$. Let $\tilde{x},\tilde{y}\tilde{\in}\tilde{X}$, $\tilde{x}\neq\tilde{y}$. By our assumption, there exist at least a soft (1,2)*-b-open set containing $\tilde{x}$ but not $\tilde{y}$ and also a soft (1,2)*-b-open set containing $\tilde{y}$ but not $\tilde{x}$. Therefore, $\tilde{X}$ is a soft (1,2)*-b-$\tilde{T}_1$-space.

Now, we introduce the definition of soft (1,2)*-$\tilde{T}_2$-spaces and soft (1,2)*-b-$\tilde{T}_2$-spaces also we study the characteristics and the relations between these soft spaces and the previous soft spaces.

Definition (2.15) :

A soft bitopological space $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$ is called a soft (1,2)*-$\tilde{T}_2$-space (resp. soft (1,2)*-b-$\tilde{T}_2$-space) if for any two distinct soft points $\tilde{x}$ and $\tilde{y}$ of $\tilde{X}$, there are two soft $\tilde{\tau}_1\tilde{\tau}_2$-open sets (resp. soft (1,2)*-b-open sets) (U,E) and (V,E) of $\tilde{X}$ such that $\tilde{x}\tilde{\in}(U,E)$, $\tilde{y}\tilde{\in}(V,E)$ and $(U,E)\tilde{\cap}(V,E)=\tilde{\phi}$.

Remark (2.16) :

Every soft (1,2)*-b-$\tilde{T}_2$-space is a soft (1,2)*-b-$\tilde{T}_1$-space, but the converse is not true in general. We see that by the following example:

Example (2.17) :

Let $X=\{a,b,c\}$ and $E=\{e\}$ and let $\tilde{\tau}_1=\{\tilde{X},\tilde{\phi},(A_1,E)\}$ and $\tilde{\tau}_2=\{\tilde{X},\tilde{\phi},(A_2,E)\}$ be soft topologies over X, where $(A_1,E)=\{(e,\{a,b\})\}$ and $(A_2,E)=\{(e,\{b,c\})\}$. The soft sets in $\{\tilde{X},\tilde{\phi},(A_1,E),(A_2,E)\}$ are soft $\tilde{\tau}_1\tilde{\tau}_2$-open sets. Thus $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$ is a soft (1,2)*-b-$\tilde{T}_1$-space, but is not soft (1,2)*-b-$\tilde{T}_2$-space.

Remark (2.18) :

Every soft (1,2)*-$\tilde{T}_2$-space is a soft (1,2)*-b-$\tilde{T}_2$-space, but the converse is not true in general. In example (2.11), $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$ is a soft (1,2)*-b-$\tilde{T}_2$-space, but is not soft (1,2)*-$\tilde{T}_2$-space.

Remark (2.19) :

Every soft (1,2)*-$\tilde{T}_2$-space is a soft (1,2)*-$\tilde{T}_1$-space, but the converse is not true in general. We see that by the following example:

Example (2.20) :

Let $X = N$ and $E = \{e_1, e_2\}$ and let $\tilde{\tau}_1 = \{(U,E) \tilde{\subseteq} \tilde{X} : (U,E)^c$ is finite$\} \tilde{\bigcup} \{\tilde{\phi}\}$ and $\tilde{\tau}_2 = \{\tilde{X}, \tilde{\phi}\}$ be soft topologies over X. Thus $(X, \tilde{\tau}_1, \tilde{\tau}_2, E)$ is a soft (1,2)*-$\tilde{T}_1$-space, but is not soft (1,2)*-$\tilde{T}_2$-space.

<u>Definition (2.21) :</u>

A soft subset (A,E) of a soft bitopological space $(X, \tilde{\tau}_1, \tilde{\tau}_2, E)$ is called a soft (1,2)*-neighborhood (resp. soft (1,2)*-b-neighborhood) of a soft point $\tilde{x}$ in $\tilde{X}$ if there exists a soft $\tilde{\tau}_1\tilde{\tau}_2$-open (resp. soft (1,2)*-b-open) set (U,E) in $\tilde{X}$ such that $\tilde{x} \tilde{\in} (U,E) \tilde{\subseteq} (A,E)$.

<u>Theorem (2.22) :</u>

For a soft bitopological space $(X, \tilde{\tau}_1, \tilde{\tau}_2, E)$ the following statements are equivalent:

(i) $(X, \tilde{\tau}_1, \tilde{\tau}_2, E)$ is a soft (1,2)*-b-$\tilde{T}_2$-space.

(ii) If $\tilde{x} \tilde{\in} \tilde{X}$, then for each $\tilde{y} \neq \tilde{x}$, there is a soft (1,2)*-b-neighborhood (N,E) of $\tilde{x}$ such that $\tilde{y} \tilde{\notin}$ (1,2)*-$\text{bcl}(N,E)$.

(iii) For each $\tilde{x} \tilde{\in} \tilde{X}$, $\tilde{\bigcap}\{$(1,2)*-$\text{bcl}(N,E)$: (N,E) is a soft (1,2)*-b-neighborhood of $\tilde{x}\} = \{\tilde{x}\}$

Proof:

$(i) \Rightarrow (ii)$. Let $\tilde{x} \tilde{\in} \tilde{X}$. If $\tilde{y} \tilde{\in} \tilde{X}$ such that $\tilde{y} \neq \tilde{x}$, there exist disjoint soft (1,2)*-b-open sets $(U,E),(V,E)$ such that $\tilde{x} \tilde{\in} (U,E)$ and $\tilde{y} \tilde{\in} (V,E)$. Then $\tilde{x} \tilde{\in} (U,E) \tilde{\subseteq} (V,E)^c$ which implies that $(V,E)^c$ is a soft (1,2)*-b-neighborhood of $\tilde{x}$. Also $(V,E)^c$ is soft (1,2)*-b-closed and $\tilde{y} \tilde{\notin} (V,E)^c$. Let $(N,E) = (V,E)^c$. Then $\tilde{y} \tilde{\notin}$ (1,2)*-$\text{bcl}(N,E)$.

$(ii) \Rightarrow (iii)$. Obvious.

$(iii) \Rightarrow (i)$. Let $\tilde{x}, \tilde{y} \tilde{\in} \tilde{X}$, $\tilde{x} \neq \tilde{y}$. By hypothesis, there is at least a soft (1,2)*-b-neighborhood (N,E) of $\tilde{x}$ such that $\tilde{y} \tilde{\notin}$ (1,2)*-$\text{bcl}(N,E)$. We have $\tilde{x} \tilde{\notin}$ ((1,2)*-$\text{bcl}(N,E))^c$ which is soft (1,2)*-b-open. Since (N,E) is a soft (1,2)*-b-neighborhood of $\tilde{x}$, then there exists a soft (1,2)*-b-open set (U,E) such that $\tilde{x} \tilde{\in} (U,E) \tilde{\subseteq} (N,E)$ and $(U,E) \tilde{\bigcap}$ ((1,2)*-$\text{bcl}(N,E))^c = \tilde{\phi}$. Hence $(X, \tilde{\tau}_1, \tilde{\tau}_2, E)$ is a soft (1,2)*-b-$\tilde{T}_2$-space.

<u>Definition (2.23):</u>

A soft function $f : (X, \tilde{\tau}_1, \tilde{\tau}_2, E) \to (Y, \tilde{\sigma}_1, \tilde{\sigma}_2, E)$ is called soft (1,2)*-continuous (resp. soft (1,2)*-b-continuous) if $f^{-1}((U,E))$ is soft $\tilde{\tau}_1\tilde{\tau}_2$-open (resp. soft (1,2)*-b-open) set in $\tilde{X}$ for each soft $\tilde{\sigma}_1\tilde{\sigma}_2$-open set (U,E) in $\tilde{Y}$.

<u>Definition (2.24):</u>

A soft function $f : (X, \tilde{\tau}_1, \tilde{\tau}_2, E) \to (Y, \tilde{\sigma}_1, \tilde{\sigma}_2, E)$ is called soft (1,2)*-b-irresolute if $f^{-1}((U,E))$ is soft (1,2)*-b-open set in $\tilde{X}$ for each soft (1,2)*-b-open set (U,E) in $\tilde{Y}$.

<u>Theorem (2.25) :</u>

Let $f : (X, \tilde{\tau}_1, \tilde{\tau}_2, E) \to (Y, \tilde{\sigma}_1, \tilde{\sigma}_2, E)$ be a soft (1,2)*-b-continuous injective function. If $\tilde{Y}$ is a soft (1,2)*-$\tilde{T}_i$-space, then $\tilde{X}$ is a soft (1,2)*-b-$\tilde{T}_i$-space, $i = 0,1,2$.

Proof:

Suppose that $\tilde{Y}$ is a soft (1,2)*-$\tilde{T}_2$-space. Let $\tilde{x}, \tilde{y} \tilde{\in} \tilde{X}$ such that $\tilde{x} \neq \tilde{y}$. Since f is injective and $\tilde{Y}$ is a soft (1,2)*-$\tilde{T}_2$-space, then there exists disjoint soft $\tilde{\tau}_1\tilde{\tau}_2$-open sets (A_1,E) and (A_2,E) of $\tilde{Y}$ such that $f(\tilde{x}) \tilde{\in} (A_1,E)$ and $f(\tilde{y}) \tilde{\in} (A_2,E)$. By definition (2.23), $f^{-1}((A_1,E))$ and $f^{-1}((A_2,E))$ are soft (1,2)*-b-open sets in $\tilde{X}$ Such that $\tilde{x} \tilde{\in} f^{-1}((A_1,E)), \tilde{y} \tilde{\in} f^{-1}((A_2,E))$ and $f^{-1}((A_1,E)) \tilde{\bigcap} f^{-1}((A_2,E)) = \tilde{\phi}$. Hence $\tilde{X}$ is a soft (1,2)*- b-$\tilde{T}_2$-space. Similarly, we can prove $\tilde{X}$ is a soft (1,2)*-b-$\tilde{T}_i$-space when $i = 0,1$.

Theorem (2.26) :

Let $f:(X,\tilde{\tau}_1,\tilde{\tau}_2,E)\to(Y,\tilde{\sigma}_1,\tilde{\sigma}_2,E)$ be a soft (1,2)*-continuous injective function. If $\tilde{Y}$ is a soft (1,2)*-$\tilde{T}_i$-space, then so is $\tilde{X}$, $i=0,1,2$.

Theorem (2.27):

Let $f:(X,\tilde{\tau}_1,\tilde{\tau}_2,E)\to(Y,\tilde{\sigma}_1,\tilde{\sigma}_2,E)$ be a soft (1,2)*-b-irresolute injective function. If $\tilde{Y}$ is a soft (1,2)*-b-$\tilde{T}_i$-space, then so is $\tilde{X}$, $i=0,1,2$.

***Proof* :**

Suppose that $\tilde{Y}$ is a soft (1,2)*-b-$\tilde{T}_2$-space. Let $\tilde{x},\tilde{y}\tilde{\in}\tilde{X}$ such that $\tilde{x}\neq\tilde{y}$. Since f is injective and $\tilde{Y}$ is a soft (1,2)*-b-$\tilde{T}_2$-space, then there exists disjoint soft (1,2)*-b-open sets (A_1,E) and (A_2,E) of $\tilde{Y}$ such that $f(\tilde{x})\tilde{\in}(A_1,E)$ and $f(\tilde{y})\tilde{\in}(A_2,E)$. By definition (2.24), $f^{-1}((A_1,E))$ and $f^{-1}((A_2,E))$ are soft (1,2)*-b-open sets in $\tilde{X}$ such that $\tilde{x}\tilde{\in}f^{-1}((A_1,E))$, $\tilde{y}\tilde{\in}f^{-1}((A_2,E))$ and $f^{-1}((A_1,E))\tilde{\bigcap}f^{-1}((A_2,E))=\tilde{\phi}$. Hence $\tilde{X}$ is a soft (1,2)*-b-$\tilde{T}_2$-space.

3. Weak Soft (1,2)*-$\tilde{D}$-Separation Axioms

Now, we introduce and study new concepts, namely, soft (1,2)*-$\tilde{D}$-sets and soft (1,2)*-$\tilde{D}_b$-sets by using the notion of soft $\tilde{\tau}_1\tilde{\tau}_2$-open sets and soft (1,2)*-b-open sets respectively. Furthermore we use these soft sets to define and study new types of soft separation axioms, namely, soft (1,2)*-$\tilde{D}_i$-spaces and soft (1,2)*-b-$\tilde{D}_i$-spaces for $i=0,1,2$. Moreover we investigate the relation between the soft (1,2)*-$\tilde{T}_i$-spaces and each of soft (1,2)*-b-$\tilde{T}_i$-spaces, soft (1,2)*-$\tilde{D}_i$-spaces and soft (1,2)*-b-$\tilde{D}_i$-spaces for $i=0,1,2$. The characteristics of these soft spaces also have been studied.

Definition (3.1) :

A soft subset (A,E) of a soft bitopological space $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$ is called:

i) A soft (1,2)*-difference set (briefly soft (1,2)*-$\tilde{D}$-set) if there are two soft $\tilde{\tau}_1\tilde{\tau}_2$-open sets (U,E) and (V,E) in $\tilde{X}$ such that $(U,E)\neq\tilde{X}$ and $(A,E)=(U,E)\setminus(V,E)$.

ii) A soft (1,2)*-b-difference set (briefly soft (1,2)*-$\tilde{D}_b$-set) if there are two soft (1,2)*-b-open sets (U,E) and (V,E) in $\tilde{X}$ such that $(U,E)\neq\tilde{X}$ and $(A,E)=(U,E)\setminus(V,E)$

Remarks (3.2) :

i) In definition (3.1), if $(U,E)\neq\tilde{X}$ and $(V,E)=\tilde{\phi}$, then every proper soft $\tilde{\tau}_1\tilde{\tau}_2$-open (resp. soft (1,2)*-b-open) soft subset of $\tilde{X}$ is a soft (1,2)*-$\tilde{D}$-set (resp. soft (1,2)*-$\tilde{D}_b$-set).

ii) In any soft bitopological space $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$ any soft (1,2)*-$\tilde{D}$-set is soft (1,2)*-$\tilde{D}_b$-set, but the converse is not true in general. In example (1.10), $(A,E)=\{(e_1,\{X\}),(e_2,\{a,c\})\}$ is a soft (1,2)*-$\tilde{D}_b$-set, but is not soft (1,2)*-$\tilde{D}$-set.

Now, we define new types of soft separation axioms in soft bitopological spaces, namely, soft (1,2)*-$\tilde{D}_i$-spaces and soft (1,2)*-b-$\tilde{D}_i$-spaces for $i=0,1,2$.

Definitions (3.3) :

A soft bitopological space $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$ is called:

(i) A soft (1,2)*-$\tilde{D}_0$-space (resp. soft (1,2)*-b -$\tilde{D}_0$-space) if for any two distinct soft points $\tilde{x}$ and $\tilde{y}$ of $\tilde{X}$, there exists a soft (1,2)*-$\tilde{D}$ - set (resp. soft (1,2)*-$\tilde{D}_b$-set) of $\tilde{X}$ containing one of the soft points but not the other.

(ii) A soft (1,2)*-$\tilde{D}_1$-space (resp. soft (1,2)*-b-$\tilde{D}_1$-space) if for any two distinct soft points $\tilde{x}$ and $\tilde{y}$ of $\tilde{X}$, there exists a soft (1,2)*-$\tilde{D}$-set (resp. soft (1,2)*-$\tilde{D}_b$-set) of $\tilde{X}$ containing $\tilde{x}$ but not $\tilde{y}$ and a soft (1,2)*-$\tilde{D}$-set (resp. soft (1,2)*-$\tilde{D}_b$-set) of $\tilde{X}$ containing $\tilde{y}$ but not $\tilde{x}$.

(iii) A soft (1,2)*-$\tilde{D}_2$-space (resp. soft (1,2)*-b-$\tilde{D}_2$-space) if for any two distinct soft points $\tilde{x}$ and $\tilde{y}$ of $\tilde{X}$, there are two soft (1,2)*-$\tilde{D}$-sets (resp. soft (1,2)*-$\tilde{D}_b$-sets), (U,E) and (V,E) of $\tilde{X}$ such that $\tilde{x}\tilde{\in}(U,E), \tilde{y}\tilde{\in}(V,E)$ and $(U,E)\tilde{\bigcap}(V,E)=\tilde{\phi}$.

Theorem (3.4):

(i) Every soft (1,2)*-$\tilde{T}_i$-space (resp. soft (1,2)*-b-$\tilde{T}_i$-space) is soft (1,2)*-$\tilde{D}_i$-space (resp. soft (1,2)*-b-$\tilde{D}_i$-space), $i=0,1,2$.

(ii) Every soft (1,2)*-$\tilde{D}_i$-space (resp. soft (1,2)*-b-$\tilde{D}_i$-space) is soft (1,2)*-$\tilde{D}_{i-1}$-space (resp. soft (1,2)*-b-$\tilde{D}_{i-1}$-space), $i=1,2$.

iii) Every soft (1,2)*-$\tilde{D}_i$-space is soft (1,2)*-b-$\tilde{D}_i$-space, $i=0,1,2$.

Proof:

(i) Follows from Remark (3.2).

(ii) It is obvious.

(iii) It is obvious.

Remark (3.5):

The converse of theorem (3.4) , no. (i) may not be true in general. We see that by the following example:

***Example (3.6)** :*

Let $X=\{a,b,c\}$ and $E=\{e\}$ and let $\tilde{\tau}_1=\{\tilde{X},\tilde{\phi},(A_1,E),(A_2,E)\}$ and $\tilde{\tau}_2=\{\tilde{X},\tilde{\phi},(A_3,E)\}$ be soft topologies over X, where $(A_1,E)=\{(e,\{a\})\}$, $(A_2,E)=\{(e,\{a,b\})\}$ and $(A_3,E)=\{(e,\{a,c\})\}$.

The soft sets in $\{\tilde{X},\tilde{\phi},(A_1,E), (A_2,E), (A_3,E)\}$ are soft $\tilde{\tau}_1\tilde{\tau}_2$-open sets. Thus $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$ is a soft (1,2)*-$\tilde{D}_i$-space (resp. soft (1,2)*-b-$\tilde{D}_i$-space), but is not soft (1,2)*-$\tilde{T}_i$-space (resp. soft (1,2)*-b-$\tilde{T}_i$-space), $i=1,2$.

Remark (3.7) :

The converse of theorem (3.4), no. (ii) may not be true in general. We see that by the following example:

Example (3.8) :

Let $X=\{a,b\}$ and $E=\{e\}$ and let $\tilde{\tau}_1=\{\tilde{X},\tilde{\phi},(A,E)\}$ and $\tilde{\tau}_2=\{\tilde{X},\tilde{\phi}\}$ be soft topologies over X, where $(A,E)=\{(e,\{a\})\}$. The soft sets in $\{\tilde{X},\tilde{\phi},(A,E)\}$ are soft $\tilde{\tau}_1\tilde{\tau}_2$-open sets. Thus $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$ is a soft (1,2)*-$\tilde{D}_0$-space (resp. soft (1,2)*-b-$\tilde{D}_0$-space), but is not soft (1,2)*-$\tilde{D}_1$-space (resp. soft (1,2)*-b-$\tilde{D}_1$-space).

Remark (3.9) :

The converse of theorem (3.4), no. (iii) may not be true in general. In example (2.11), $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$ is a soft (1,2)*-b-$\tilde{D}_i$-space, but is not soft (1,2)*-$\tilde{D}_i$-space, $i=0,1,2$.

***Theorem (3.10)** :*

A soft bitopological space $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$ is a soft (1,2)*-b-$\tilde{D}_0$-space (resp. soft (1,2)*-$\tilde{D}_0$-space) if and only if it is a soft (1,2)*-b-$\tilde{T}_0$-space (resp. soft (1,2)*-$\tilde{T}_0$- space).

Proof:

Sufficiency, follows from theorem (3.4), no. (i). Necessity, let $\tilde{x},\tilde{y}\tilde{\in}\tilde{X}$ such that $\tilde{x}\neq\tilde{y}$ Since $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$ is a soft (1,2)*-b-$\tilde{D}_0$-space, then there exists a soft (1,2)*-$\tilde{D}_b$- set (U,E) such that $\tilde{x}\tilde{\in}(U,E)$, $\tilde{y}\tilde{\notin}(U,E)$. Let $(U,E)=(U_1,E)\setminus(U_2,E)$, where $(U_1,E)\neq\tilde{X}$ and $(U_1,E),(U_2,E)$ are soft (1,2)*-b-open sets in $\tilde{X}$. By $\tilde{y}\tilde{\notin}(U,E)$ we have two cases:

(i) $\tilde{y}\tilde{\notin}(U_1,E)$

(ii) $\tilde{y}\tilde{\in}(U_1,E)$ and $\tilde{y}\tilde{\in}(U_2,E)$.

In case (i) $\tilde{y}\tilde{\notin}(U_1,E)$ and $\tilde{x}\tilde{\in}(U,E)=(U_1,E)\setminus(U_2,E) \Rightarrow \tilde{x}\tilde{\in}(U_1,E)$ and $\tilde{y}\tilde{\notin}(U_1,E)$.

In case (ii) $\tilde{y}\tilde{\in}(U_1,E)$ and $\tilde{y}\tilde{\in}(U_2,E)$, but $\tilde{x}\tilde{\in}(U_1,E)\setminus(U_2,E) \Rightarrow \tilde{x}\tilde{\notin}(U_2,E) \Rightarrow$

$\tilde{y}\tilde{\in}(U_2,E)$ and $\tilde{x}\tilde{\notin}(U_2,E)$. Thus in both cases, we obtain that $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$ is a soft (1,2)*-b-$\tilde{T}_0$-space. Similarly, we can prove that $\tilde{X}$ is a soft (1,2)*-$\tilde{D}_0$-space if and only if it is a soft (1,2)*-$\tilde{T}_0$-space.

Theorem (3.11) :

A soft bitopological space $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$ is a soft (1,2)*-b-$\tilde{D}_1$-space (resp. soft (1,2)*-$\tilde{D}_1$-space) if and only if it is a soft (1,2)*-b-$\tilde{D}_2$-space (resp. soft (1,2)*-$\tilde{D}_2$-space).

Proof:

Sufficiency. Follows from theorem (3.4), no. (ii). Necessity. Let $\tilde{x},\tilde{y}\tilde{\in}\tilde{X}$ such that $\tilde{x}\neq\tilde{y}$. Since $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$ is a soft (1,2)*-b-$\tilde{D}_1$-space, then there exists soft (1,2)*-$\tilde{D}_b$-sets (U,E) and (V,E) in $\tilde{X}$ such that $\tilde{x}\tilde{\in}(U,E)$, $\tilde{y}\tilde{\notin}(U,E)$ and $\tilde{y}\tilde{\in}(V,E)$, $\tilde{x}\tilde{\notin}(V,E)$. Let $(U,E)=(U_1,E)\setminus(U_2,E)$ and $(V,E)=(U_3,E)\setminus(U_4,E)$, where $(U_1,E),(U_2,E),(U_3,E),(U_4,E)$ are soft (1,2)*-b-open sets in $\tilde{X}$ and $(U_1,E)\neq\tilde{X}$, $(U_3,E)\neq\tilde{X}$. By $\tilde{x}\tilde{\notin}(V,E)$ we have two cases: **(i)** $\tilde{x}\tilde{\notin}(U_3,E)$. **(ii)** $\tilde{x}\tilde{\in}(U_3,E)$ and $\tilde{x}\tilde{\in}(U_4,E)$.

In case (i): $\tilde{x}\tilde{\notin}(U_3,E)$. By $\tilde{y}\tilde{\notin}(U,E)$ we have two subcases:

(a) $\tilde{y}\tilde{\in}(U_1,E)$ and $\tilde{y}\tilde{\in}(U_2,E)$.

(b) $\tilde{y}\tilde{\notin}(U_1,E)$.

Subcase (a): $\tilde{y}\tilde{\in}(U_1,E)$ and $\tilde{y}\tilde{\in}(U_2,E)$. We have $\tilde{x}\tilde{\in}(U_1,E)\setminus(U_2,E)$, $\tilde{y}\tilde{\in}(U_2,E)$ and $((U_1,E)\setminus(U_2,E))\tilde{\cap}(U_2,E)=\tilde{\phi}$. Observe that $(U_2,E)\neq\tilde{X}$ since $(U,E)\neq\tilde{\phi}$, thus by Remarks (3.2),(i), (U_2,E) is a soft (1,2)*-$\tilde{D}_b$-set.

Subcase (b)

$\tilde{y}\tilde{\notin}(U_1,E)$. Since $\tilde{x}\tilde{\in}(U_1,E)\setminus(U_2,E)$ and $\tilde{x}\tilde{\notin}(U_3,E)$, then $\tilde{x}\tilde{\in}(U_1,E)\setminus((U_2,E)\tilde{\cup}(U_3,E))$. Since $\tilde{y}\tilde{\in}(U_3,E)\setminus(U_4,E)$ and $\tilde{y}\tilde{\notin}(U_1,E)$, then $\tilde{y}\tilde{\in}(U_3,E)\setminus((U_4,E)\tilde{\cup}(U_1,E))$. Observe also from Proposition (1.12), (iii), that $(U_2,E)\tilde{\cup}(U_3,E)$ and $(U_4,E)\tilde{\cup}(U_1,E)$ are soft (1,2)*-b-open sets. Hence $\tilde{x}\tilde{\in}(U_1,E)\setminus((U_2,E)\tilde{\cup}(U_3,E))$, $\tilde{y}\tilde{\in}(U_3,E)\setminus((U_4,E)\tilde{\cup}(U_1,E))$ and $((U_1,E)\setminus((U_2,E)\tilde{\cup}(U_3,E)))\tilde{\cap}((U_3,E)\setminus((U_4,E)\tilde{\cup}(U_1,E)))=\tilde{\phi}$.

In case (ii): $\tilde{x}\tilde{\in}(U_3,E)$ and $\tilde{x}\tilde{\in}(U_4,E)$. We have $\tilde{y}\tilde{\in}(U_3,E)\setminus(U_4,E)$, $\tilde{x}\tilde{\in}(U_4,E)$ and $((U_3,E)\setminus(U_4,E))\tilde{\cap}(U_4,E)=\tilde{\phi}$. Observe that $(U_4,E)\neq\tilde{X}$ since $(V,E)\neq\tilde{\phi}$, thus by Remarks (3.2),(i), (U_4,E) is a soft (1,2)*-$\tilde{D}_b$-set. Hence $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$ is a soft (1,2)*-b-$\tilde{D}_2$-space. Similarly, we can prove that $\tilde{X}$ is a soft (1,2)*-$\tilde{D}_1$-space if and only if it is a soft (1,2)*-$\tilde{D}_2$-space.

From theorem (3.4), no. (ii), and theorem (3.10), we get the following result.

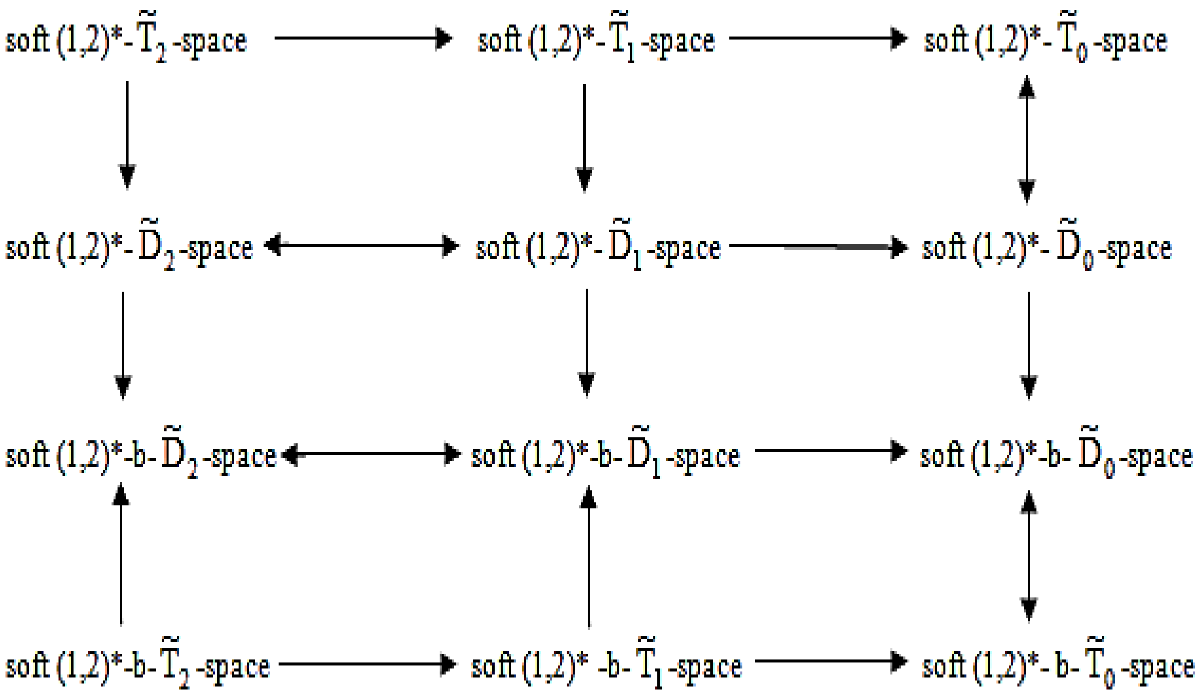
Corollary (3.12) :

If $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$ is a soft (1,2)*-b-$\tilde{D}_1$-space (resp. soft (1,2)*-$\tilde{D}_1$-space), then it is a soft (1,2)*-b-$\tilde{T}_0$-space (resp. soft (1,2)*-$\tilde{T}_0$-space).

Remark (3.13) :

The converse of corollary (3.12), may not be true in general. In example (3.8), $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$ is a soft (1,2)*-b-$\tilde{T}_0$-space (resp. soft (1,2)*-$\tilde{T}_0$-space), but is not a soft (1,2)*-b-$\tilde{D}_1$-space (resp. soft (1,2)*-$\tilde{D}_1$-space).

The following diagram show the relations among the soft (1,2)*-$\tilde{D}_i$-spaces, soft (1,2)*-b-$\tilde{D}_i$-spaces, soft (1,2)*-$\tilde{T}_i$-spaces and soft (1,2)*-b-$\tilde{T}_i$-spaces, $i=0,1,2$.



Definition (3.14) :

Let $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$ be a soft bitopological space. A soft point $\tilde{x}\tilde{\in}\tilde{X}$ which has $\tilde{X}$ as the only soft (1,2)*-neighborhood (resp. soft (1,2)*-b-neighborhood) is called a soft (1,2)*-neat (resp. soft (1,2)*-b-neat) point.

Theorem (3.15)

For a soft (1,2)*-b-$\tilde{T}_0$-space (resp. soft (1,2)*-$\tilde{T}_0$-space) $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$ the following are equivalent:

(i) $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$ is a soft (1,2)*-b-$\tilde{D}_1$-space (resp. soft (1,2)*-$\tilde{D}_1$-space).

(ii) $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$ has no soft (1,2)*-b-neat (resp. soft (1,2)*-neat) point.

Proof:

$(i)\Rightarrow(ii)$. Since $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$ is a soft (1,2)*-b-$\tilde{D}_1$-space, then each soft point $\tilde{x}\tilde{\in}\tilde{X}$ is contained in a soft (1,2)*-$\tilde{D}_b$-set $(U,E)=(U_1,E)\setminus(U_2,E)$, where (U_1,E) and (U_2,E) are soft (1,2)*-b-open sets and thus in (U_1,E). By definition $(U_1,E)\neq\tilde{X}$. This implies that $\tilde{x}$ is not a soft (1,2)*-b-neat point.

$(ii)\Rightarrow(i)$. If $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$ is a soft (1,2)*-b-$\tilde{T}_0$-space, then for each distinct soft points $\tilde{x},\tilde{y}\tilde{\in}\tilde{X}$, at least one of them, say $\tilde{x}$ has a soft (1,2)*-b-neighborhood (U,E) containing $\tilde{x}$, but not $\tilde{y}$. Thus (U,E) is different from $\tilde{X}$ and therefore by Remark (3.2),(i), (U,E) is a soft (1,2)*-$\tilde{D}_b$-set. Since $\tilde{X}$ has no soft (1,2)*-b-neat point, then $\tilde{y}$ is not a soft (1,2)*-b-neat point. Thus there exists a soft (1,2)*-b-neighborhood (V,E) of $\tilde{y}$ such that $(V,E)\neq\tilde{X}$. Therefore, $\tilde{y}\tilde{\in}(V,E)\setminus(U,E)$, $\tilde{x}\tilde{\notin}(V,E)\setminus(U,E)$ and $(V,E)\setminus(U,E)$ is a soft (1,2)*-$\tilde{D}_b$-set. Hence $(X,\tilde{\tau}_1,\tilde{\tau}_2,E)$ is a soft (1,2)*-b-$\tilde{D}_1$-space.

Theorem (3.16) :

If $f:(X,\tilde{\tau}_1,\tilde{\tau}_2,E)\to(Y,\tilde{\sigma}_1,\tilde{\sigma}_2,E)$ is a soft (1,2)*-b-continuous (resp. soft (1,2)*-continuous) surjective function and (A,E) is a soft (1,2)*-$\tilde{D}$-set in $\tilde{Y}$, then the inverse image of (A,E) is a soft (1,2)*-$\tilde{D}_b$-set (resp. soft (1,2)*-$\tilde{D}$-set) in $\tilde{X}$.

Proof :

Let (A,E) be a soft (1,2)*-$\tilde{D}$-set in $\tilde{Y}$, then there are two soft $\tilde{\tau}_1\tilde{\tau}_2$-open sets (U,E) and (V,E) in $\tilde{Y}$ such that $(U,E)\neq\tilde{Y}$ and $(A,E)=(U,E)\setminus(V,E)$. Since f is soft (1,2)*-b-continuous, then $f^{-1}((U,E))$ and $f^{-1}((V,E))$ are soft (1,2)*-b-open sets in $\tilde{X}$. Since $(U,E)\neq\tilde{Y}$ and f is surjective, then $f^{-1}((U,E))\neq\tilde{X}$. Hence $f^{-1}((A,E))=f^{-1}((U,E))\setminus f^{-1}((V,E))$ is a soft (1,2)*-$\tilde{D}_b$-set in $\tilde{X}$. By the same way we can prove that other case.

Theorem (3.17) :

If $f:(X,\tilde{\tau}_1,\tilde{\tau}_2,E)\to(Y,\tilde{\sigma}_1,\tilde{\sigma}_2,E)$ is a soft (1,2)*-b-irresolute surjective function and (A,E) is a soft (1,2)*-$\tilde{D}_b$-set in $\tilde{Y}$, then the inverse image of (A,E) is a soft (1,2)*-$\tilde{D}_b$-set in $\tilde{X}$.

Proof :

Let (A,E) be a soft (1,2)*-$\tilde{D}_b$-set in $\tilde{Y}$, then there are two soft (1,2)*-b-open sets (U,E) and (V,E) in $\tilde{Y}$ such that $(U,E)\neq\tilde{Y}$ and $(A,E)=(U,E)\setminus(V,E)$. Since f is soft (1,2)*-b-irresolute, then $f^{-1}((U,E))$ and $f^{-1}((V,E))$ are soft (1,2)*-b-open sets in $\tilde{X}$. Since $(U,E)\neq\tilde{Y}$ and f is surjective, then $f^{-1}((U,E))\neq\tilde{X}$. Hence $f^{-1}((A,E))=f^{-1}((U,E))\setminus f^{-1}((V,E))$ is a soft (1,2)*-$\tilde{D}_b$-set in $\tilde{X}$.

Theorem (3.18) :

Let $f:(X,\tilde{\tau}_1,\tilde{\tau}_2,E)\to(Y,\tilde{\sigma}_1,\tilde{\sigma}_2,E)$ be a soft (1,2)*-b-continuous bijective function. If $\tilde{Y}$ is a soft (1,2)*-$\tilde{D}_i$-space, then $\tilde{X}$ is a soft (1,2)*-b-$\tilde{D}_i$-space, $i=0,1,2$.

Proof:

Suppose that $\tilde{Y}$ is a soft (1,2)*-$\tilde{D}_2$-space. Let $\tilde{x},\tilde{y}\,\tilde{\in}\,\tilde{X}$ such that $\tilde{x}\neq\tilde{y}$. Since f is injective and $\tilde{Y}$ is a soft (1,2)*-$\tilde{D}_2$-space, then there exists disjoint soft (1,2)*-$\tilde{D}$-sets (A_1,E) and (A_2,E) of $\tilde{Y}$ such that $f(\tilde{x})\,\tilde{\in}\,(A_1,E)$ and $f(\tilde{y})\,\tilde{\in}\,(A_2,E)$. By theorem (3.16), $f^{-1}((A_1,E))$ and $f^{-1}((A_2,E))$ are soft (1,2)*-$\tilde{D}_b$-sets in $\tilde{X}$ such that $\tilde{x}\,\tilde{\in}\,f^{-1}((A_1,E))$, $\tilde{y}\,\tilde{\in}\,f^{-1}((A_2,E))$ and $f^{-1}((A_1,E))\tilde{\cap}f^{-1}((A_2,E))=\tilde{\phi}$. Hence $\tilde{X}$ is a soft (1,2)*-b-$\tilde{D}_2$-space. By the same way we can prove that other cases.

Theorem (3.19) :

Let $f:(X,\tilde{\tau}_1,\tilde{\tau}_2,E)\to(Y,\tilde{\sigma}_1,\tilde{\sigma}_2,E)$ be a soft (1,2)*-continuous bijective function. If $\tilde{Y}$ is a soft (1,2)*-$\tilde{D}_i$-space, then so is $\tilde{X}$, $i=0,1,2$.

Proof **:**

Obvious.

Theorem (3.20):

Let $f:(X,\tilde{\tau}_1,\tilde{\tau}_2,E)\to(Y,\tilde{\sigma}_1,\tilde{\sigma}_2,E)$ be a soft (1,2)*-b-irresolute bijective function. If $\tilde{Y}$ is a soft (1,2)*-b-$\tilde{D}_i$-space, then so is $\tilde{X}$, $i=0,1,2$.

Proof **:**

Suppose that $\tilde{Y}$ is a soft (1,2)*-b-$\tilde{D}_2$-space. Let $\tilde{x},\tilde{y}\,\tilde{\in}\,\tilde{X}$ such that $\tilde{x}\neq\tilde{y}$. Since f is injective and $\tilde{Y}$ is a soft (1,2)*-b-$\tilde{D}_2$-space, then there exists disjoint soft (1,2)*-$\tilde{D}_b$-sets (A_1,E) and (A_2,E) of $\tilde{Y}$ such that $f(\tilde{x})\,\tilde{\in}\,(A_1,E)$ and $f(\tilde{y})\,\tilde{\in}\,(A_2,E)$. By theorem (3.17), $f^{-1}((A_1,E))$ and $f^{-1}((A_2,E))$ are soft (1,2)*-$\tilde{D}_b$-sets in $\tilde{X}$ such that $\tilde{x}\,\tilde{\in}\,f^{-1}((A_1,E))$, $\tilde{y}\,\tilde{\in}\,f^{-1}((A_2,E))$ and $f^{-1}((A_1,E))\tilde{\cap}f^{-1}((A_2,E))=\tilde{\phi}$. Hence $\tilde{X}$ is a soft (1,2)*-b-$\tilde{D}_2$-space. By the same way we can prove that other cases.

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