CALCULATION OF THE PARAMETERS FOR ATMOSPHERIC MODEL FOR THE EARTH

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Abstract

In this model we have used the balance forces theory for a vertical column of air for calculate the pressure equation, also calculate the variations of density, pressure, temperature and molecular weight with altitude from sea level up to 700 km. The equations parameters are derived analytically using the hydrostatic equation. In addition, the equations of the gravitational acceleration, speed of sound and scale height are derived. Equation of temperature and equation of molecular weight merged in single equation is called equation of molecular temperature. Results of these equations have been from solving by using computer. Finally these results are discussed, which represent the variation of atmospheric parameters with altitude. We can use this model for study effect of earth's atmosphere on airplanes and moving bodies inside it.

Introduction

Study of the upper atmosphere began before 50 years ago. When balloons were first employed for the determination of atmospheric conditions at altitudes up to 30 Km, did not become information regarding atmospheric conditions at higher altitudes available until ad vented rocket-sounding techniques, and entry rocket sounding constituted a new era in technology and scientific instrumentation, where was launched into higher altitudes for investigation atmospheric condition in this altitudes. in conventional cases old was research atmosphere properties at a high of 100 Km, and when entper rocket-sounding began sounding rocket arise into altitudes more from 200 Km, and become estimate atmosphere properties were purely extra polation from data obtained at the lower altitudes the atmosphere properties is density and temperature pressure, molecular weight an important affect on atmosphere properties resultant from solar activity [6]. The atmosphere is for the most part structure from nitrogen 78.09 % and oxygen 20.95 % and remainders the gases least than 1 % from structure atmosphere, the atmosphere is instable, and it is continuously changing its temperature, pressure, chemical constituent's particulate presence and electrical characteristics primarily due to changes in solar radiation. If converted variation in atmospheric properties at a fixed altitude to variations in altitude. It will be found that such altitude variations are perhaps negligible when compared to the altitude range over which flight takes place. We shall establish a simple atmosphere model. Begin from sea level in more than 700 km where calculate variation in density and pressure, molecular weight, temperature with variation altitude From zero up to 700 km [2].

Equations of model

Let us consider a vertical column of air with unit cross-sectional area A=1.

The mass (dm) of air between (z) and (z+dz) in the column is:

$$\rho(z)=\frac{m}{v}=\frac{dm}{Adz}$$

$$dm=\rho(z)Adz \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Where ρ (z) is the density of air at height (z) as shown in Fig. (1).

The force active on this column due to the weight of the region of the air is:

$$df_G=dm\ g(z)=g(z)\ \rho(z)\ Adz \quad \ldots\ldots(2)$$

Where g (z) is the acceleration due to gravity at height (z).

The changes in pressures on path surface (p→p) leads to up word pressure given as:

$$\Delta f_p=A\ p\ \Delta p \rightarrow \quad df_p=A[p-(p-dp)]$$

$$df_p=-A\ dp \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

(-dp) is a positive quantity

The balance of forces:

$$df_G = df_p$$

$$\frac{dp}{dz} = -\ g(z)\ \rho(z) \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

This equation is a term in the hydrostatic equation [2].

$$T = T_i - \left[\frac{(n-1)\, g_o\ M}{n\, R^*}\right](h - h_i)\ \ldots\ldots\ldots (5)$$

Where R^* is the universal gas constant, and M is the molecular weight of air, and the h_i, T_i indicates the values of (h) and (T) at the beginning of the (i) th layer. Equation (5) is use to calculate the temperature (T) throughout the layer as a function of geo potential altitude. This equation remains valid until the next layer is encountered [2].

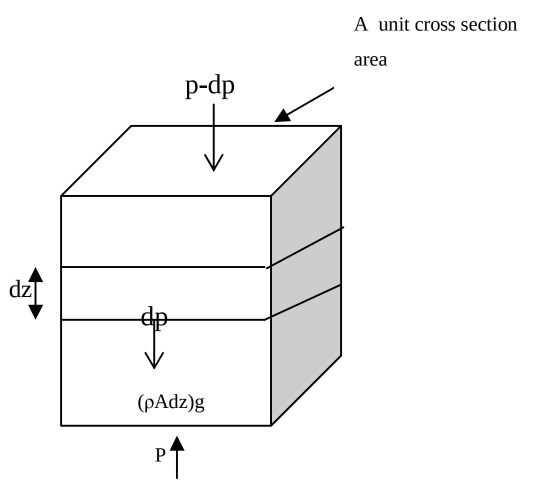

Fig. (1) : Atmospheric element [2].

1. Development of Analytic Atmosphere Model

Our aim is to establish atmosphere model calculate variation density, pressure, temperature, molecular weight with altitude from sea level into 700 km over sea level. The first step in the development of an analytic model of the atmosphere began with the derivation of the hydrostatic equation (4) and this equation function in pressure (p) and geometric altitude z. Integration of this equation requires to density ρ and gravitational constant (g) [7].

The density determination with the thermal equation of state of an ideal gas is ($\rho = \frac{MP}{R^* T}$) [2]. The gravitational acceleration constant expressed as a simple function of altitude enter an artificial altitude and which called the geo potential altitude. This altitude is constant at sea level. Introduction of the thermal equation state replaces a density variation with a thermal and molecular weight variation. Changes of molecular weight (M) with altitude are by passed introducing an artificial temperature T_m called the molecular temperature, which bases temperature into constant sea level value of molecular weight. This the main result of the entire exercise is to require a tabulation of the thermal gradient (dT_m/dz) or (dT_m/dh) depending upon whether we choose geometric or geo potential altitude as the independent variable by combine the hydrostatic and thermal state equations to get a relationship between pressure and altitude (geometric or geo potential) as:

$$dp = -\rho\, g_o\ dh$$

The equation of gravitational acceleration given as [2]:

$$g = g_o\left(\frac{RE}{RE + z}\right)^2 \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

$$g = g_o(1 - bz)$$

Where g_o=[GM / RE^2], RE is the Earth radius.

The equation of pressure ratio given as [2]:

$$\frac{P}{P_i} = \left[\left[\frac{L_z}{T_{Mi}}\right](z - z_i) + 1\right]^{-\left[\frac{g_o}{RL_z}\right] + b\left[\frac{T_{Mi}}{L_z} z_i\right]} \ldots\ldots (7)$$

$$\exp\left[\frac{g_o\ b}{RL_z}(z - z_i)\right]$$

The equation of density ratio given as [2]:

$$\frac{r}{r_i} = \left[\frac{L_z}{T_{Mi}}(z - z_i) + 1\right]^{-\left[\frac{g_o}{RL_z}\left[\frac{RL_z}{g_o} + (1-b)\right]\frac{T_{Mi}}{L_z} z_i\right]} \ldots.. (8)$$

$$\exp\left[\frac{g_o\ b}{RL_z}(z - z_i)\right]$$

2. The speed of sound

The speed of sound is an important factor in the study of high speed light. Small pressure disturbances are cause by all parts of an aircraft as it moves through the air. These disturbances transmitted in all directions through the air at the speed of sound; it varies with temperature as:

$$sp = sp_o\sqrt{\theta}\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

Where sp_o speed of sound at sea level standard day, $\theta = T/T_0$ temperature ratio [1].

3. Scale height

As in the case of growth and decay, where half life doubling time and time constant are of interest as a means of appreciating the scale of a problem, so also is there defined for the atmosphere a scale height which is a measure of rate of change of atmospheric pressure and density with altitude [5].

The pressure scale height H_p is form by:

$$H_p = \frac{RT_M}{g/g_o} \qquad (10)$$

Density scale height H_r is form by:

$$H_r = \frac{RT_M}{(g/g_o) + RL_z} \qquad (11)$$

The computer program for atmospheric model will be base upon Eq. (5), Eq. (6),Eq. (7), Eq. (8), Eq. (9), Eq. (10) and Eq. (11), Table (1) illustrates the parameters estimated from these equations.

Results and Discussion

Fig.(2) illustrates the variation of gravitational acceleration with altitude and decreases liner with increase altitude. The speed of sound increases slowly with increase altitude but in the high altitude, increasing becomes very high because the speed of sound is a function of temperature, these variations are shown in Fig.(3).

Fig. (4) indicates the variation of the molecular weight with altitude. Since both the molecular weight and temperature vary with altitude, we can combine both variations together in a single variable called the molecular temperature as shown in Fig.(9).

At sea, level on a standard day the temperature ratio equal to 1.0. Temperature decreases with altitude until the tropopause is reach 10935.2 m then it remains constant until about 25757.6 m, the temperature is -69.7^o F and temperature ratio equal to 0.7519 at the tropopause as shown in Fig.(5). In Fig.6 and Fig.(7), we have plotted density vs. altitude. For the atmospheric portion below about 200 km atmospheric effects are consider important above approximately this altitude, we usually can ignore such effects. Thus, we may fit a straight line to the logarithm of the density for altitudes below 200 km and ignore the fit above this altitude. Fig. (8) shows the variation between static pressure and altitude. Static pressure reduced, when altitude is increase because there is a less weight of air above. At 15 km altitude, the static pressure is about half that at sea level.

In the definition of the standard atmosphere, the lapse rates change discontinuously while molecular temperature is continuous. Consequently, density scale height varies continuously with altitude whereas density scale height, which is a function of lapse rate, varies in manner, which is discontinuously smooth. Fig. (10) shows the variation of density scale height with altitude. The parameters of atmospheric model illustrates in Table (1).

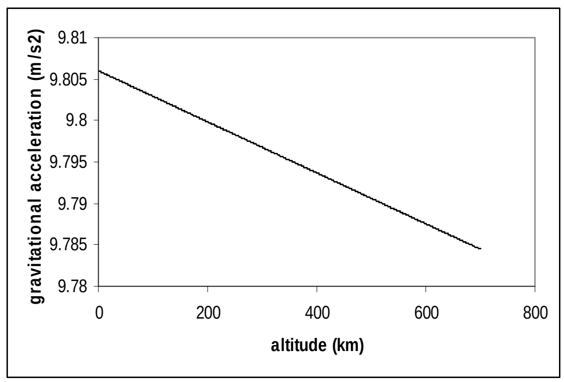

Fig.(2) : Gravitational acceleration vs. Altitude.

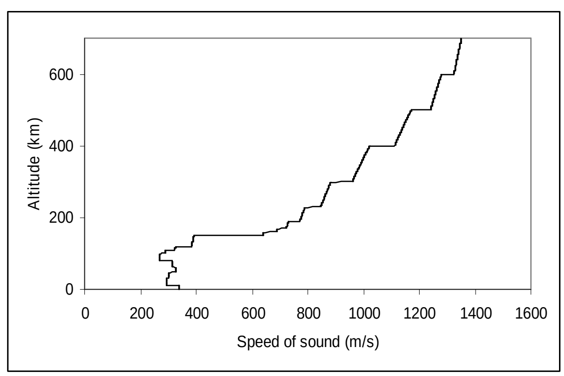

Fig.(3) : Speed of sound vs. Altitude.

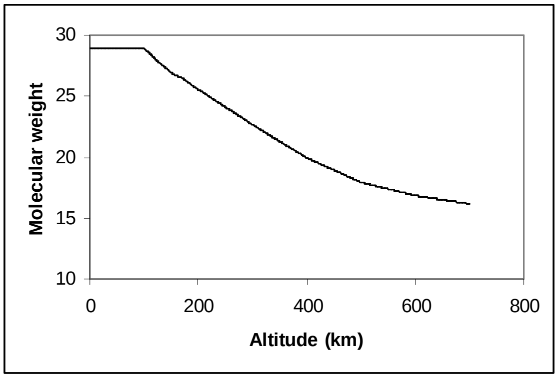

Fig.(4) : Molecular weight vs. Altitude.

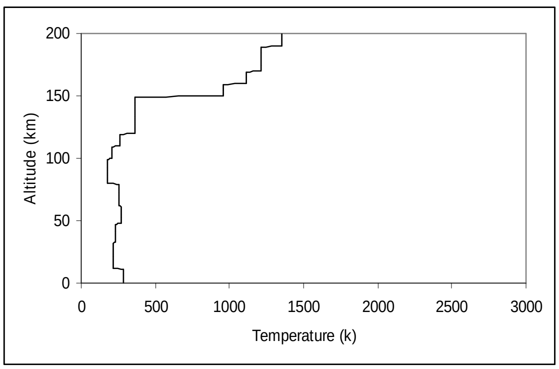

Fig. (5) : Temperature vs. Altitude.

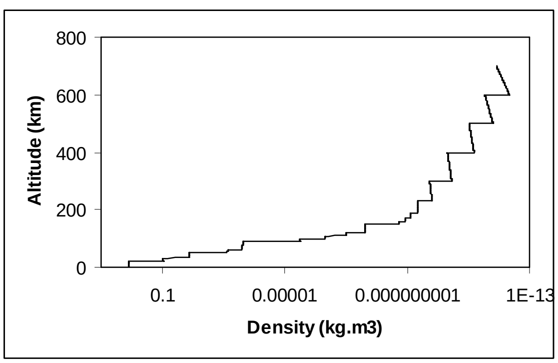

Fig.(6) : Density vs. Altitude.

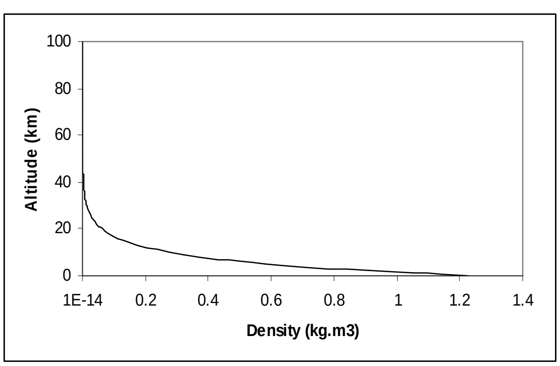

Fig.(7) : Density for exponential method vs. Altitude.

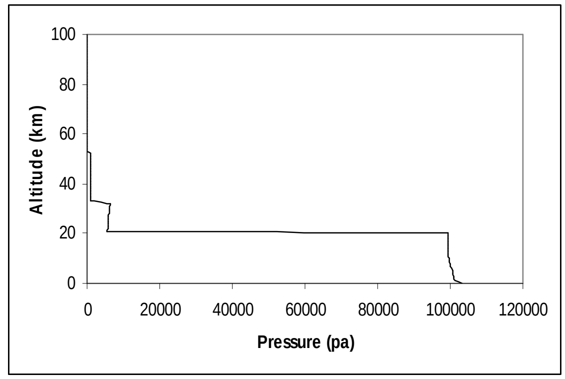

Fig.(8) :Pressure vs. Altitude.

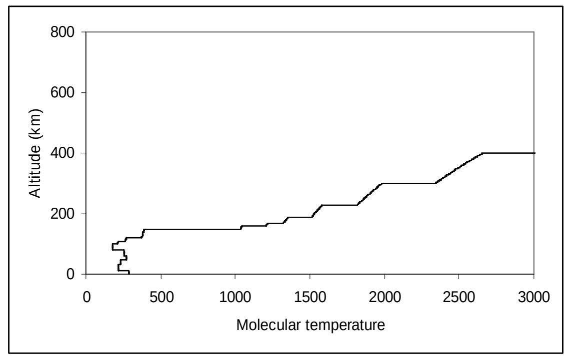

Fig.(9) : Molecular temperature vs. Altitude.

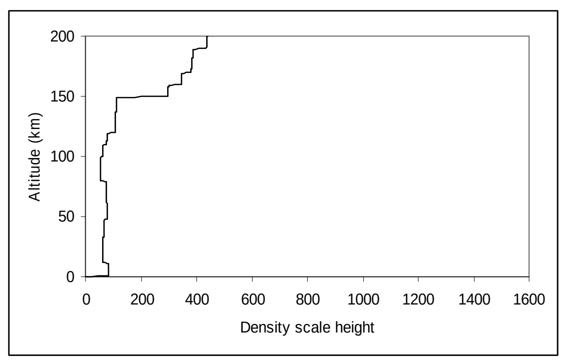

Fig. (10) : Density scale height vs. Altitude.

Table (1)
Parameters of atmospheric model.

Altitude (km)	Gravitational acceleration (m/s^2)	Temperature (k^o)	Speed of sound (m/s)	Density scale height	Molecular temperature	Density (gm/m^3)
0	9.806	289.1	337.1405	0	289.1	1.243323
2000	9.805938	288.1	336.5569	82.70165	288.1	1.221331
10000	9.805692	288.1	336.5569	82.70373	288.1	1.205276
20000	9.805384	216.65	291.8543	62.19481	216.65	1.203285
30000	9.805077	216.65	291.8543	62.19675	216.65	0.0991
40000	9.804769	228.65	299.8282	65.64383	228.65	1.36E-02
50000	9.804461	270.65	326.2052	77.70418	270.65	1.40E-02
60000	9.804152	270.65	326.2052	77.70662	270.65	7.29E-04
70000	9.803845	252.65	315.1712	72.54089	252.65	2.45E-04
80000	9.803537	180.65	266.5053	51.86987	180.65	2.39E-04
90000	9.803229	181.65	267.2419	52.15864	181.65	2.96E-06
100000	9.802921	211.65	288.8861	60.95147	212.2656	4.82E-07
110000	9.802614	261.65	322.9962	76.19724	265.3512	9.62E-08
120000	9.802306	361.65	383.0356	107.1609	373.1682	2.42E-08
130000	9.801998	360.65	385.1445	108.3476	377.2887	2.45E-08
140000	9.801691	360.65	387.8388	109.8722	382.5858	2.46E-08
150000	9.801383	961.65	637.8038	297.1483	1034.667	1.83E-09
160000	9.801076	1111.6	689.0652	346.8432	1207.666	1.15E-09
170000	9.800768	1211.65	721.5754	380.3556	1324.311	8.00E-10
180000	9.80046	1210.65	725.7417	384.7726	1339.647	8.16E-10

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الخلاصة

في هذا الموديل تم استخدام نظرية توازن القوى لمقطع عامودي من الهواء لحساب معادلة الضغط وكذالك حساب تغيرات الكثافة والضغط ودرجة الحرارة والوزن الجزيئي مع الارتفاع من مستوى سطح البحر الى ارتفاع 700 كيلو متر. تم اشتقاق معادلات عناصر الغلاف الغازي للارض تحليليا باستخدام المعادلة الهايدروستاتيكية وكذالك اشتقاق معادلة التعجيل الارضي ومعادلة سرعة الصوت ومعادلة الارتفاع القياسي وتم دمج معادلة درجة الحرارة مع معادلة الوزن الجزيئي بمعادلة واحدة تدعى معادلة درجة حرارة الجزيئات وببرمجة هذه المعادلات بستخدام الحاسبة الالكترونية تم الحصول على النتائج ومناقشتها وهذه النتائج تمثل تغيرات عناصر الغلاف الغازي مع الارتفاع. ويمكن استخدام هذا الموديل في دراسة تأثير الغلاف الغازي الارضي على الطائرات والاجسام المتحركة داخله.