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# CERTAIN TYPES OF GENERALIZED CLOSED SETS

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## Abstract

In this paper, we introduce and study new types of generalized closed subsets of topological space, and we will summarize the relationships between them and we will proved every pointed on it. Also we will introduce and study new types of continuous functions on it, also, we will summarize the relationships between them, and proved every pointed on it. Several properties of these concepts are proved.

**Keywords: closed set, g-closed set, α-closed set, semi-closed set.**

## 1-Introduction

In 1970, Levine [4] introduced the concept of generalized closed sets in a topological space, shortly ($g$-closed) where he defined a subset $A$ of a topological space $X$ to be $g$-closed if $\mathrm{cl}(A)\subseteq U$ whenever $A\subseteq U$ and $U$ is open. In this paper we will introduce and investigate a new types of $g$-closed subsets of topological spaces, which we called it $gs^*$-closed set, $sg^*$-closed set, $g\alpha^*$-closed set, $\alpha g^*$-closed set, the relationships between them are summarized in the diagram, and we proved every pointed between them, also, we will give examples to show that the inverse pointed in the diagram is not true. Finally we will introduce and study new types of continuous functions on our new concepts, which we called it $gs^*$-continuous functions, $sg^*$-continuous functions, $g\alpha^*$-continuous functions and $\alpha g^*$-continuous functions, also, we will summarize the relationships among them, and proved every pointed on it. Several properties of these concepts are proved.

## 2-Preliminaries

Throughout this paper, for a subset $A$ of a topological space $(X,\tau)$ $cl(A)$ (resp. $\mathrm{int}(A), cl_s(A), cl_\alpha(A)$) will denote to the closure (resp. interior, smallest $semi$-closed set containing $A$, smallest $\alpha$-closed set containing $A$), also the symbol □ will indicate the end of a proof.

For the sake of convenience, we begin with some basic concepts, although most of these concepts can be found from the references of this paper.

**Definition (2.1) [3]**

A subset $A$ of a topological space $(X,\tau)$ is called:

1. $semi$-closed if $\mathrm{int}(cl(A))\subseteq A$,
2. $\alpha$-closed if $cl(\mathrm{int}(cl(A)))\subseteq A$.

**Definition (2.2)** [3]

The complement of $semi$-closed (resp. $\alpha$-closed) is called $semi$-open (resp. $\alpha$-open).

**Definition (2.3)**

A subset $A$ of a topological space $(X,\tau)$ is called:

1. generalized closed ($g$-closed) set [4] if $cl(A)\subseteq U$ whenever $A\subseteq U$ and $U$ is open,
2. generalized $semi$-closed ($gs$-closed) set [6] if $cl_s(A)\subseteq U$ whenever $A\subseteq U$ and $U$ is open,
3. $semi$-generalized closed ($sg$-closed) set [5] if $cl_s(A)\subseteq U$ whenever $A\subseteq U$ and $U$ is $semi$-open,
4. generalized $\alpha$-closed ($g\alpha$-closed) set [2] if $cl_\alpha(A)\subseteq U$ whenever $A\subseteq U$ and $U$ is $\alpha$-open,
5. $\alpha$-generalized closed ($\alpha g$-closed) set [1] if $cl_\alpha(A)\subseteq U$ whenever $A\subseteq U$ and $U$ is open.

**Definition (2.4)**

The complement of $g$-closed (resp. $gs$-closed, $sg$-closed, $g\alpha$-closed, $\alpha g$-closed) is called $g$-open (resp. $gs$-open, $sg$- open, $g\alpha$- open, $\alpha g$- open).

**Remark (2.5)**

The relationships between the concepts in definitions (2.1) and (2.3) summarized in the following diagram:

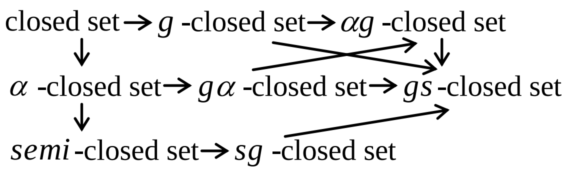


**Diagram (2-1)**

Now, we will prove every pointed in the above diagram in the following propositions:

**Proposition (2.6)**

Every closed subset of a topological space $(X,\tau)$ is $g$-closed.

***Proof:***

Let $A\subseteq X$ be closed set, and let $A\subseteq U$, where $U$ is open set, since $A$ is closed set then $cl(A)=A$, hence $cl(A)\subseteq U$, i.e. $A$ is $g$-closed.□

**Proposition (2.7)**

Every $g$-closed subset of a topological space $(X,\tau)$ is $\alpha g$-closed.

***Proof:***

Let $A\subseteq X$ be $g$-closed set, and let $A\subseteq U$, where $U$ is open set, since $A$ is $g$-closed set then $cl(A)\subseteq U$, and hence $\operatorname{int}(cl(A))\subseteq\operatorname{int}(U)$, but $U$ is open set, so $\operatorname{int}(cl(A))\subseteq U$. Since $cl_\alpha(A)$ is the smallest $\alpha$-closed set containing $A$, so, $cl_\alpha(A)=A\cup cl(\operatorname{int}(cl(A)))\subseteq A\cup cl(U)\subseteq U$, i.e. $A$ is $\alpha g$-closed.□

**Proposition (2.8)**

Every closed subset of a topological space $(X,\tau)$ is $\alpha$-closed.

***Proof:***

Let $A\subseteq X$ be closed set, then $cl(A)=A$, hence $\operatorname{int}(cl(A))=\operatorname{int}(A)$, but $\operatorname{int}(A)\subseteq A$, so $\operatorname{int}(cl(A))\subseteq A$, and $cl(\operatorname{int}(cl(A)))\subseteq cl(A)$. Then $cl(\operatorname{int}(cl(A)))\subseteq A$, i.e. $A$ is $\alpha$-closed.□

**Proposition (2.9)**

Every $\alpha$-closed subset of a topological space $(X,\tau)$ is $g\alpha$-closed.

***Proof:***

Let $A\subseteq X$ be $\alpha$-closed set, and let $A\subseteq U$, where $U$ is $\alpha$-open set, since $A$ is $\alpha$-closed set, then $cl(\operatorname{int}(cl(A)))\subseteq A\subseteq U$, since $cl_\alpha(A)$ is the smallest $\alpha$-closed set containing $A$, so,

$$cl_\alpha(A)=A\cup cl(\operatorname{int}(cl(A)))$$
$$\subseteq A\cup U$$
$$\subseteq U,$$

i.e. $A$ is $g\alpha$-closed.□

**Proposition (2.10)**

Every $g\alpha$-closed subset of a topological space $(X,\tau)$ is $gs$-closed.

***Proof:***

Let $A\subseteq X$ be $g\alpha$-closed set, and let $A\subseteq U$, where $U$ is open set, since $A$ is $g\alpha$-closed set, then $cl_\alpha(A)\subseteq U$, and since $cl_\alpha(A)=A\cup cl(\operatorname{int}(cl(A)))$, then $cl(\operatorname{int}(cl(A)))\subseteq cl_\alpha(A)\subseteq U$, but $\operatorname{int}(cl(A))\subseteq cl(\operatorname{int}(cl(A)))$ then $\operatorname{int}(cl(A))\subseteq U$. Since $cl_s(A)$ is the smallest $semi$-closed set containing $A$, so,

$$cl_s(A)=A\cup\operatorname{int}(cl(A))$$
$$\subseteq U,$$

i.e. $A$ is $gs$-closed.□

**Proposition (2.11)**

Every $g$-closed subset of a topological space $(X,\tau)$ is $gs$-closed.

***Proof:***

Let $A\subseteq X$ be $g$-closed set, and let $A\subseteq U$, where $U$ is open set, since $A$ is $g$-closed set then $cl(A)\subseteq U$, and hence $\operatorname{int}(cl(A))\subseteq\operatorname{int}(U)$, but $U$ is open set, so $\operatorname{int}(cl(A))\subseteq U$. Since $cl_s(A)$ is the smallest $semi$-closed set containing $A$, so,

$$cl_s(A)=A\cup\operatorname{int}(cl(A))$$
$$\subseteq U,$$

i.e. $A$ is $gs$-closed.□

**Proposition (2.12)**

Every $g\alpha$-closed subset of a topological space $(X,\tau)$ is $\alpha g$-closed.

***Proof:***

Let $A\subseteq X$ be $g\alpha$-closed set, and let $A\subseteq U$, where $U$ is open set, since $A$ is $g\alpha$-closed set, then $cl_\alpha(A)\subseteq U$, i.e. $A$ is $\alpha g$-closed.□

**Proposition (2.13)**

Every $\alpha g$-closed subset of a topological space $(X,\tau)$ is $gs$-closed.

***Proof:***

Let $A\subseteq X$ be $\alpha g$-closed set, and let $A\subseteq U$, where $U$ is open set, since $A$ is $\alpha g$-closed set, then $cl_\alpha(A)\subseteq U$, and since $cl_\alpha(A)=A\cup cl(\text{int}(cl(A)))$, then $\text{cl}(\text{int}(\text{cl}(\text{A})))\subseteq \text{cl}_\alpha(\text{A})\subseteq \text{U}$, but $\text{int}(cl(A))\subseteq cl(\text{int}(cl(A)))$ then $\text{int}(cl(A))\subseteq U$. Since $cl_s(A)$ is the smallest $semi$-closed set containing $A$, so,

$$cl_s(A)=A\cup\text{int}(cl(A))$$
$$\subseteq U,$$

i.e. $A$ is $gs$-closed.□

**Proposition (2.14)**

Every $\alpha$-closed subset of a topological space $(X,\tau)$ is $semi$-closed.

***Proof:***

Let $A\subseteq X$ be $\alpha$-closed set, then $\text{cl}(\text{int}(\text{cl}(\text{A})))\subseteq \text{A}$, since $\text{int}(cl(A))\subseteq cl(\text{int}(cl(A)))$, so $\text{int}(cl(A))\subseteq A$, i.e. $A$ is $semi$-closed.□

**Proposition (2.15)**

Every $semi$-closed subset of a topological space $(X,\tau)$ is $sg$-closed.

***Proof:***

Let $A\subseteq X$ be $semi$-closed set, and let $A\subseteq U$, where $U$ is $semi$-open set, since $A$ is $semi$-closed set, then $\text{int}(cl(A))\subseteq A\subseteq U$. Since $cl_s(A)$ is the smallest $semi$-closed set containing $A$, so,

$$cl_s(A)=A\cup\text{int}(cl(A))$$
$$\subseteq U,$$

i.e. $A$ is $sg$-closed.□

**Proposition (2.16)**

Every $sg$-closed subset of a topological space $(X,\tau)$ is $gs$-closed.

***Proof:***

Let $A\subseteq X$ be $sg$-closed set, and let $A\subseteq U$, where $U$ is open set, since $A$ is $sg$-closed set, then $cl_s(A)\subseteq U$, i.e. $A$ is $gs$-closed.□

Now, we will give some example to show that the inverse pointed in the diagram (2.1) is not true

**Example (2.17)**

$g$-closed set $\not\rightarrow$ closed set.

Let $X=\{a,b,c\}$,
$\tau=\{X,\phi,\{a\},\{c\},\{a,c\}\}$, so
$\tau^c=\{\phi,X,\{b,c\},\{a,b\},\{b\}\}$, let $A=\{a\}$,
$U=\{a,b,c\}$ open set.

Now, since $cl(A)=\{a,b\}\subseteq U$, i.e. $A=\{a\}$ is $g$-closed set, but it is not closed set.

**Example (2.18)**

$\alpha$-closed set $\not\rightarrow$ closed set.

Let $X=\{a,b,c,d\}$,
$\tau=\{X,\phi,\{a\},\{c\},\{a,c\},\{a,b,d\}\}$, so
$\tau^c=\{\phi,X,\{b,c,d\},\{a,b,d\},\{b,d\},\{c\}\}$, let
$A=\{b,c\}$.

Now, since $cl(A)=\{b,c,d\}\subseteq U$, and $\text{int}(cl(A))=\{c\}$, $cl(\text{int}(cl(A)))=\{c\}\subseteq A$ i.e. $A=\{b,c\}$ is $\alpha$-closed set, but it is not closed set.

**Example (2.19)**

$\alpha g$-closed set $\not\rightarrow$ $g$-closed set.

Let $X=\{a,b,c\}$, $\tau=\{X,\phi,\{a\},\{c\},\{a,c\}\}$,
so $\tau^c=\{\phi,X,\{b,c\},\{a,b\},\{b\}\}$, let
$A=\{c\}$,.$U=X$ open set.

Now, since $cl(A)=\{b,c\}\subseteq U$, and $\text{int}(cl(A))=\{c\}$, $cl(\text{int}(cl(A)))=\{b,c\}\subseteq U$ i.e. $A=\{c\}$ is $\alpha g$-closed set, but it is not $g$-closed set, Since if we take $U=\{a\}$, $cl(A)=\{a,b\}\not\subseteq A$.

**Example (2.20)**

$gs$-closed set $\not\rightarrow$ $g$-closed set.

Let $X=\{a,b,c\}$, $\tau=\{X,\phi,\{a\},\{c\},\{a,c\}\}$, so $\tau^c=\{\phi,X,\{b,c\},\{a,b\},\{b\}\}$, let $A=\{c\}$,. $U=X$ open set.

Now, by proposition (2.10) we have $A=\{c\}$ is $gs$-closed, but it is not $g$-closed set.

**Example (2.21)**

$gs$-closed set $\not\rightarrow$ $\alpha g$-closed set.

Let $X=\{a,b,c\}$, $\tau=\{X,\phi,\{a\},\{b\},\{a,b\}\}$, so $\tau^c=\{\phi,X,\{b,c\},\{a,c\},\{c\}\}$, let $A=\{b\}$,. $U=X$ open set.

Now, since $cl(A)=\{b,c\}\subseteq U$, and $\text{int}(cl(A))=\{b\}$, so $cl_s(A)=\{b,c\}\subseteq U$ i.e. $A=\{b\}$ is $gs$-closed set, but it is not $\alpha g$-closed set, Since if we take $U=\{a\}$, $cl(A)=\{c,b\}$, and. $\text{int}(cl(A))=\{b\}$, $cl(\text{int}(cl(A)))=\{c,b\}$, so $cl_\alpha(A)=\{c,b\}\not\subseteq U$.

## 3-( $gs^*, sg^*, g\alpha^*$ and $\alpha g^*$ )-Closed Sets and Continuous Functions on it

In this section we will introduce and investigate new types of $g$-closed subsets of topological space, which we called it $gs^*$-closed set, $sg^*$-closed set, $g\alpha^*$-closed set, $\alpha g^*$-closed set, the relationships between them are summarized in the diagram (3-1), and we will proved every pointed between them, also, we will give examples for these concepts. Finally we will define new types of continuous functions on our new concepts, also the relationships between them are summarized in the diagram (3-2), and we will proved every pointed between them. We well prove several propositions about these concepts.

**Definition (3.1)**

A subset $A$ of a topological space $(X,\tau)$ is called:

1. $gs^*$-closed set if $cl_s(A)\subseteq U$ whenever $A\subseteq U$ and $U$ is $gs$-open,
2. $sg^*$-closed set if $cl_s(A)\subseteq U$ whenever $A\subseteq U$ and $U$ is $sg$-open,
3. $g\alpha^*$-closed set if $cl_\alpha(A)\subseteq U$ whenever $A\subseteq U$ and $U$ is $g\alpha$-open,
4. $\alpha g^*$-closed set if $cl_\alpha(A)\subseteq U$ whenever $A\subseteq U$ and $U$ is $\alpha g$-open.

**Remark (3.2)**

The relationships between the concepts in definition (3.1) summarized in the following diagram:

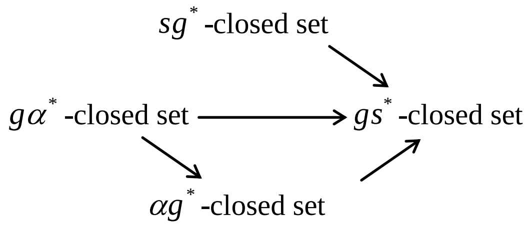


**Diagram (3-1)**

Now, we will prove every pointed in the above diagram in the following propositions:

**Proposition (3.3)**

Every $sg^*$-closed subset of a topological space $(X,\tau)$ is $gs^*$-closed.

***Proof:***

Let $A\subseteq X$ be $sg^*$-closed set, then $cl_s(A)\subseteq U$ where $A\subseteq U$ and $U$ is $sg$-open, then $U^c$ is $sg$-closed, hence $U^c$ is $gs$-closed, then $U$ is $gs$-open and $cl_s(A)\subseteq U$, i.e. $A$ is $gs^*$-closed.□

**Proposition (3.4)**

Every $g\alpha^*$-closed subset of a topological space $(X,\tau)$ is $gs^*$-closed.

***Proof:***

Let $A\subseteq X$ be $g\alpha^*$-closed set, then $cl_s(A)\subseteq U$ where $A\subseteq U$ and $U$ is $g\alpha$-open, since $cl_\alpha(A)=A\cup cl(\text{int}(cl(A)))$, for any $A\subseteq X$ then $cl(\text{int}(cl(A)))\subseteq cl_\alpha(A)\subseteq U$, but $\text{int}(cl(A))\subseteq cl(\text{int}(cl(A)))$ then $\text{int}(cl(A))\subseteq U$. Since $cl_s(A)$ is the smallest $semi$-closed set containing $A$, so, $cl_s(A)=A\cup\text{int}(cl(A))\subseteq U$, and since $U$ is $g\alpha$-open, then $U^c$ is $g\alpha$-closed, hence $U^c$ is $gs$-closed, then $U$ is

$gs$-open and $cl_s(A)\subseteq U$, i.e. $A$ is $gs^*$-closed.□

**Proposition (3.5)**

Every $g\alpha^*$-closed subset of a topological space $(X,\tau)$ is $\alpha g^*$-closed.

***Proof:***

Let $A\subseteq X$ be $g\alpha^*$-closed set, then $cl_\alpha(A)\subseteq U$ where $A\subseteq U$ and $U$ is $g\alpha$-open, then $U^c$ is $g\alpha$-closed, hence $U^c$ is $\alpha g$-closed, then $U$ is $\alpha g$-open and $cl_\alpha(A)\subseteq U$, i.e. $A$ is $\alpha g^*$-closed.□

**Proposition (3.6)**

Every $\alpha g^*$-closed subset of a topological space $(X,\tau)$ is $gs^*$-closed.

***Proof:***

Let $A\subseteq X$ be $\alpha g^*$-closed set, then $cl_\alpha(A)\subseteq U$ where $A\subseteq U$ and $U$ is $\alpha g$-open, since $cl_\alpha(A)=A\cup cl(\text{int}(cl(A)))$, then $cl(\text{int}(cl(A)))\subseteq cl_\alpha(A)\subseteq U$, but $\text{int}(cl(A))\subseteq cl(\text{int}(cl(A)))$ then $\text{int}(cl(A))\subseteq U$. Since $cl_s(A)$ is the smallest *semi*-closed set containing $A$, so, $cl_s(A)=A\cup\text{int}(cl(A))\subseteq U$, and since $U$ is $\alpha g$-open, then $U^c$ is $\alpha g$-closed, hence $U^c$ is $gs$-closed, then $U$ is $gs$-open and $cl_s(A)\subseteq U$, i.e. $A$ is $gs^*$-closed.□

Now, we will give examples of our new concepts.

**Example (3.7)**

Let $(X,\tau)$ be the usual topological space, let $U=(a,b)$ be an open interval, then $U$ is $sg$-closed [], now let $A=(c,d)$ such that $a<c<d<b$, since $cl(A)=[c,d]$, so $\text{int}(cl(A))=(c,d)$, and since

$$cl_s(A)=A\cup\text{int}(cl(A))$$
$$=(c,d)\cup(c,d)$$
$$=(c,d)$$
$$\subseteq(a,b)$$

i.e. $A=(c,d)$ is $sg^*$-closed.

**Example (3.8)**

Consider the above example, and by proposition (3.3) we have $A=(c,d)$ is also $gs^*$-closed.

**Example (3.9)**

Consider the example (2.18), we have the set $\{b,c\}$ is $\alpha$-closed, say $U$ where $X=\{a,b,c,d\}$ and $\tau=\{X,\phi,\{a\},\{c\},\{a,c\},\{a,b,d\}\}$, so by proposition (2.9) we have $U=\{b,c\}$ is $g\alpha$-closed. Now let $A=\{c\}$, so $cl(A)=\{c\}$, $\text{int}(cl(A))=\{c\}$ and $cl(\text{int}(cl(A)))=\{c\}$, but $cl_\alpha(A)=A\cup cl(\text{int}(cl(A)))=\{c\}\subseteq U$ i.e. $A=\{b,c\}$ is $g\alpha^*$-closed set.

**Example (3.10)**

Consider the above example, and by proposition (3.5) we have $A=\{b,c\}$ is also $\alpha g^*$-closed.

**Definition (3.11)**

A function $f:X\to Y$ is called:

(1) $gs^*$-continuous if the inverse image of every $g$-closed is $gs^*$-closed.

(2) $sg^*$-continuous if the inverse image of every $g$-closed is $sg^*$-closed.

(3) $g\alpha^*$-continuous if the inverse image of every $\alpha$-closed is $g\alpha^*$-closed.

(4) $\alpha g^*$-continuous if the inverse image of every $\alpha$-closed is $\alpha g^*$-closed.

**Remark (3.12)**

The relationships between the concepts in definition (3.11) summarized in the following diagram:

$sg^*$-continuous → $gs^*$-continuous

$g\alpha^*$-continuous → $\alpha g^*$-continuous

**Diagram (3-2)**

Now, we will prove every pointed in the above diagram in the following propositions:

**Proposition (3.13)**

Every $sg^*$-continuous function is $gs^*$-continuous function.

***Proof:***

Define $f: X \to Y$ which is $sg^*$- continuous function, and let $A$ be a $g$-closed in $Y$, since $f$ is $sg^*$-continuous, then by definition (3.11.②), we have $f^{-1}(A)$ is $sg^*$-closed, and by proposition (3.3) $f^{-1}(A)$ is $gs^*$-closed, i.e $f$ is $gs^*$-continuous function.□

**Proposition (3.14)**

Every $g\alpha^*$- continuous function is $\alpha g^*$-continuous function.

***Proof:***

Define $f: X \to Y$ which is $g\alpha^*$-continuous function, and let $A$ be a $\alpha$-closed in $Y$, since $f$ is $g\alpha^*$-continuous, then by definition (3.11.3), we have $f^{-1}(A)$ is $g\alpha^*$-closed, and by proposition (3.5) $f^{-1}(A)$ is $\alpha g^*$-closed, i.e $f$ is $\alpha g^*$-continuous function.□

**Proposition (3.15)**

Let $f: X \to Y$ and $h: Y \to Z$ are two $gs^*$-continuous functions then $h \circ f: X \to Z$ is also $gs^*$- continuous function, if every $gs^*$-closed in $Y$ is $g$-closed

***Proof:***

Let $f: X \to Y$ and $h: Y \to Z$ are two $gs^*$-continuous functions, and let $A$ be $g$-closed in $Z$, $h$ is $gs^*$-continuous function then $h^{-1}(A)$ is $gs^*$-closed in $Y$, so by hypothesis $h^{-1}(A)$ is $g$-closed in $Y$, since $f$ is $gs^*$-continuous function then $f^{-1}(h^{-1}(A)) = (f^{-1} \circ h^{-1})(A) = (h \circ f)^{-1}(A)$ is $gs^*$-closed, i.e. $h \circ f$ is $gs^*$-continuous function.□

**Proposition (3.16)**

Let $f: X \to Y$ and $h: Y \to Z$ are two $sg^*$-continuous functions then $h \circ f: X \to Z$ is also $sg^*$- continuous function, if every $sg^*$-closed in $Y$ is $g$-closed.

***Proof:***

Let $f: X \to Y$ and $h: Y \to Z$ are two $sg^*$-continuous functions, and let $A$ be $g$-closed in $Z$, $h$ is $sg^*$-continuous function then $h^{-1}(A)$ is $sg^*$-closed in $Y$, so by hypothesis $h^{-1}(A)$ is $g$-closed in $Y$, since $f$ is $sg^*$-continuous function then $(h \circ f)^{-1}(A)$ is $sg^*$-closed, i.e. $h \circ f$ is $sg^*$-continuous function.□

**Proposition (3.17)**

Let $f: X \to Y$ and $h: Y \to Z$ are two $g\alpha^*$-continuous functions then $h \circ f: X \to Z$ is also $g\alpha^*$- continuous function, if every $g\alpha^*$-closed in $Y$ is $\alpha$-closed.

***Proof:***

Let $f: X \to Y$ and $h: Y \to Z$ are two $g\alpha^*$-continuous functions, and let $A$ be $\alpha$-closed in $Z$, $h$ is $g\alpha^*$-continuous function then $h^{-1}(A)$ is $g\alpha^*$-closed in $Y$, so by hypothesis $h^{-1}(A)$ is $\alpha$-closed in $Y$, since $f$ is $g\alpha^*$-continuous function then $(h \circ f)^{-1}(A)$ is $g\alpha^*$-closed, i.e. $h \circ f$ is $g\alpha^*$-continuous function.□

**Proposition (3.18)**

Let $f: X \to Y$ and $h: Y \to Z$ are two $\alpha g^*$-continuous functions then $h \circ f: X \to Z$ is also $\alpha g^*$- continuous function, if every $\alpha g^*$-closed in $Y$ is $\alpha$-closed.

***Proof:***

Let $f: X \to Y$ and $h: Y \to Z$ are two $\alpha g^*$-continuous functions, and let $A$ be $\alpha$-closed in $Z$, $h$ is $\alpha g^*$-continuous function then $h^{-1}(A)$ is $\alpha g^*$-closed in $Y$, so by hypothesis $h^{-1}(A)$ is $\alpha$-closed in $Y$, since $f$ is $\alpha g^*$-continuous function then $(h \circ f)^{-1}(A)$ is $\alpha g^*$-closed, i.e. $h \circ f$ is $\alpha g^*$-continuous function.□

## References

[1] H. Maki, K. Balachandran and R. Devi, *"Associated topologies of generalized $\alpha$-closed sets and $\alpha$-generalized closed sets"*, Mem. Fac. Sci. Kochi Univ. Ser. A, Math., 15 (1994), 51-63.

[2] H. Maki, R. Devi and K. Balachandran, *"Generalized $\alpha$-closed sets in topology"*, Bull. Fukuoka Univ. Ed. Part III, 42 (1993), 13-21.

[3] N. Biswas, *"On Characterization of Semi-Continuous Function"*, Atti. Accad. Naz. Lincei Rend. Cl. Sci. Fis. Mat. Natur., 48(8) (1970), 399-402.

[4] N. Levine, *"Generalized closed sets in topological spaces"*, Rend. Circ. Mat. Palermo 19 (1970), 89-96.

[5] P. Bhattacharya and B. K. Lahiri, *"Semi-generalized closed sets in topology"*, Indian J. Math., 29 (1987), 375-382.

[6] S. Arya and T. Nour, *"Characterizations of s-normal spaces"*, Indian J. Pure Appl. Math., 21 (1990), 717-719.