Journal of Al-Nahrain University Vol.12 (1), March, 2009, pp.121-125 Science

ON PARA-LINDELÖF AND SEMIPARA-LINDELÖF BITOPOLOGICAL SPACES

Afifa Yousif Jafar* Al-Faham and Heyam Kh. Al- Awadi**
*** Department of Mathematics, College of Science , Al-Kuffa University.**
**** Department of Mathematics, College of Science, Al-Kuffa University.**

Abstract

We define a para–Lindelöf bitopological space and semipar–Lindelöf bitopological space and we find some properties of these concepts and give the relation between these concepts.

Introduction

The concept of paracompactness is due to dieudonne[5]. The concept of par–Lindelöf is due to fleissner [6]. The term space (X,t,r) is referred to as a set X with two general non identical topologies t and r [3]. A collection of subset of X is locally finite (resp. locally countable) [6] with respect to the topology r if every $x \in X$ has a r –neighborhood meeting finitely many (resp. countable many) elements of the collection. A collection has the s –property [6] if it is the union of countablly many collection with the property. A cover (or covering) of a space (X,t)[5] is a collection of subset of x whose union is all of x. A t –open cover of x is a cover consisting of t –open sets, and other adjective applying to subset apply similarly to cover. If P and F are covers of x, we say F refines P [5] if each number of F is contained in some member of P , then we say F refines (or refinementof) P . A subset of a topological space (X,t) is an F_s with respect to the topology t [5] if it is a countable union of t –closed sets, and written by $t-F_s$.

1. para-Lindelöf Bitopological spaces

Defintion (1.1):

A bitopological space (X,t,r) is said to be $(t-r)$–compact (resp. Lindelöf) with respect to r if every t –open cover has a finite (resp. countable) r – open subcover.[1]([2])

Defintion (1.2):

A bitopological space (X,t,r) is said to be $(t-r)$–paracompact with respect to r if every t –open cover has a r –open refinement which is locally finite with respect to r .[4]

In the following definition, we extend the definition of para–Lindelöf topological space which was given in [6].

Defintion (1.3):

A bitopological space (X,t,r) is said to be $(t-r)$ –para–Lindelöf with respect to r if every t –open cover has a r –open refinement which is locally countable with respect to r .

Proposition (1.4):

If a bitopological space (X,t,r) is $(t-r)$–compact with respect to r then it is $(t-r)$–Lindelöf with respect to r .[2]

Proof:

This follows from the fact that every finite collection is countable.

Proposition (1.5)

If a bitopological space (X,t,r) is $(t-r)$–compact with respect to r then it is $(t-r)$–paracompact with respect to r .

Proof :

This follows from the fact that every finite collection is locally finite.

Proposition (1.6):

If a bitopological space (X,t,r) is $(t-r)$–Lindelöf with respect to r then it is $(t-r)$–para–Lindelöf with respect to r .

Proof:

This follows from the fact that every countable collection is locally countable.

Proposition (1.7):

If a bitopological space (X, t, r) is $(t-r)$–paracompact with respect to r then it is $(t-r)$–para–Lindelöf with respect to r.

Proof:

This follows from the fact that every locally finite collection is locally countable.

Corollary (1.8):

If a bitopological space (X, t, r) is $(t-r)$–compact with respect to r then it is $(t-r)$–para–Lindelöf with respect to r.

Proof:

This follows from Proposition (1.4) and Proposition (1.6).

Theorem (1.9):

If (X, t, r) is $(t-r)$–para–Lindelöf with respect to r, then the t–closed subspace (Y, t_Y, r_Y) it is $(t_Y - r_Y)$–para–Lindelöf with respect to r_Y.

Proof:

Let $F = \{U_l : l \in L\}$ be a t_Y–open cover of Y. Since each U_l is a t_Y–open subset of Y, there is a t–open subset V_l of X such that $U_l = V_l \cap Y$ for each $l \in L$. Let $P = \{V_l : l \in L\} \cup \{X / Y\}$. Then P is t–open cover of X. By hypothesis P has a r–open refinement $Y = \{W_g : g \in G\}$ which is locally countable with respect to r. Set $W = \{W_g \cap Y : g \in G\}$ then W is r_Y–open refinement of F which is locally countable with respect to the r_Y.

Theorem (1.10):

Let (X, t, r) be a bitopological space, and let $\sum = \{X_i : X_i \in t \cap r\}$ be a partition of X. The space (X, t, r) is $(t-r)$–para–Lindelöf with respect to r if and only if the space (X, t_i, r_i) is $(t_i - r_i)$–para–Lindelöf with respect to r_i.

Proof:

The "only if part", Since $X_i = X / \bigcup_{i \neq j} X_j$ is t–closed then the subspace (X, t_i, r_i) is $(t_i - r_i)$–para–Lindelöf with respect to r_i for every i by theorem (1.9).

The "if part". Let $F = \{U_l : l \in L\}$ be a t–open cover of X. The collection $P = \{U_l \cap X_i : l \in L\}$ be a t_i–open cover of X_i with cardinality $\leq m$ for every i. By hypothesis P has a r_i–open refinement $Y_i = \{W_{i_l} : l \in L\}$ which is locally countable with respect to r_i. Let $W = \left\{\bigcup_{i \in I} W_{i_l} : l \in L\right\}$ then W is r–open refinement of F which is locally countable with respect to the r.

Theorem (1.11):

If each t–open set in a $(t-r)$–para–Lindelöf with respect to r space (X, t, r) is $(t-r)$–para–Lindelöf with respect to r, then every subspace (Y, t_Y, r_Y) is $(t_Y - r_Y)$–para–Lindelöf with respect to r_Y.

Proof:

Let $F = \{U_l : l \in L\}$ be a t_Y–open cover of Y. Since each U_l is a t_Y–open subset of Y, there is a t–open subset V_l of X such that $U_l = V_l \cap Y$ for each $l \in L$. Then $G = \bigcup_{l \in L} W_l$ is a t–open set. Let $W = \{V_l : l \in L\}$ be a t–open cover of G. By hypothesis G is $(t-r)$–para–Lindelöf with respect to r, thus W has a r–open refinement $Y = \{W_g : g \in G\}$

which is locally countable with respect to r. Set

$$\Sigma = \{W_g \cap Y : g \in G\}.$$

Then Σ is r_Y–open refinement of F which is locally countable with respect to the r_Y.

2. Semipara - Lindelöf Bitopological spaces

Here we will introduce the concept of semipara–Lindelöf space and we give some properties of this space.

Definition (2.1):

A bitopological space (X, t, r) is said to be $(t-r)$–semiparacompact with respect to r, if each t–open cover of X has a r–open refinement which is s–locally finite with respect to r.[4]

Definition (2.2):

A bitopological space (X, t, r) is said to be $(t-r)$–semipara–Lindelöf with respect to r, if each t–open cover of X has a r–open refinement which is s–locally countable with respect to r.

Proposition (2.3):

If a bitopological space (X, t, r) is $(t-r)$–para–Lindelöf with respect to r then it is $(t-r)$–semipara–Lindelöf with respect to r.

Proof:

This follows from the fact that every locally countable collection is s–locally countable.

Proposition (2.4):

If a bitopological space (X, t, r) is $(t-r)$–paracompact with respect to r then it is $(t-r)$–semiparacompact with respect to r.[1]

Proposition (2.5):

If a bitopological space (X, t, r) is $(t-r)$–semiparacompact with respect to r then it is $(t-r)$–semipara–Lindelöf with respect to r.

Proof:

This follows from the fact that every s–locally finite collection is s–locally countable.

Proposition (2.6):

If a bitopological space (X, t, r) is $(t-r)$–paracompact with respect to r then it is $(t-r)$–semipara–Lindelöf with respect to r.

Proof:

This follows from Proposition (2.4) and Proposition (2.5).

Theorem (2.7):

If (X, t, r) is $(t-r)$– semipara – Lindelöf with respect to r, then the t–closed subspace (Y, t_Y, r_Y) it is $(t_Y - r_Y)$–semipara–Lindelöf with respect to r_Y.

Proof:

Let $F = \{U_l : l \in L\}$ be a t_Y–open cover of Y. Since each U_l is a t_Y–open subset of Y, there is a t–open subset V_l of Y such that $U_l = V_l \cap Y$. Let $P = \{V_l : l \in L\} \cup \{X / Y\}$. Then P is t–open cover of X. By hypothesis P has a r–open refinement W which is s–locally countable with respect to r, hence $W = \bigcup_n W_n$ where each then $W_n = \{W_{ng} : g \in G\}$ is locally countable with respect to the r. Set $Y = \bigcup_n Y_n$ where each $Y_n = \{W_{ng} \cap Y : g \in G\}$. Then Y is r–open refinement of F which is s–locally countable with respect to the r.

Theorem (2.8):

Let (X, t, r) be a bitopological space, and let $\Sigma = \{X_i : X_i \in t \cap r\}$ be a partition of X. The space (X, t, r) is $(t-r)$ – semipara – Lindelöf with respect to r if and only if the space (X, t_i, r_i) is $(t_i - r_i)$–semipara–Lindelöf with respect to r_i.

Proof:

The "only if part", Since $X_i = X / \bigcup_{i \neq j} X_j$ is t–closed then the subspace (X, t, r) is $(t - r)$–semipara–Lindelöf with respect to r for every i by theorem (2.7).

The "if part". Let $F = \{U_l : l \in L\}$ be a t–open cover of X. The collection $P = \{U_l \cap X_i : l \in L\}$ be a t_i–open cover of X_i for every i. By hypothesis P has a r_Y–open refinement Y_i which is s–locally countable with respect to r_i. So $Y_i = \bigcup_n Y_{in}$ where each $Y_{in} = \{W_{inl} \cap Y : l \in L\}$ is locally countable with respect to the r_i. Set $W = \bigcup_n W_n$ where Then W is r–open refinement of F which is s–locally countable with respect to the r.

Theorem (2.9):

If (X, t, r) is $(t - r)$–semipara–Lindelöf with respect to r, then the $t - F_s$ subspace (Y, t_Y, r_Y) it is $(t_Y - r_Y)$–semipara–Lindelöf with respect to r_Y.

Proof:

Let $F = \{U_l : l \in L\}$ be a t_Y–open cover of Y. Since each U_l is a t_Y–open cover of Y then there exists a t–open set V_l of Y such that $U_l = V_l \cap Y$. For each fixed n the collection

$$P_n = \{V_l : l \in L\} \cup \{X / Y_n\}$$

from a t–open cover of X. By hypothesis P_n has a r–open refinement W which is s–locally countable with respect to r, hence $W = \bigcup_n W_n$ where each $W_n = \{W_{ng} : g \in G\}$ is locally countable with respect to the r. For each n, let $Y = \bigcup_n Y_n$ such that $Y_n = \{W_{ng} \cap Y : W_{ng} \cap Y \neq \phi, g \in G\}$. Then Y is r–open refinement of F which is s–locally countable with respect to the r_Y.

Theorem (2.10):

If (X, t, r) is $(t - r)$–para–Lindelöf with respect to r, then the $t - F_s$ subspace (Y, t_Y, r_Y) it is $(t_Y - r_Y)$–semipara–Lindelöf with respect to r_Y.

Proof:

Suppose that Y is $t - F_s$ set. Then $Y = \bigcup_n Y_n$, where each Y_n is t–closed. Let $F = \{U_l : l \in L\}$ be a t_Y–open cover of Y. Since each U_l is a t_Y–open cover of Y then there exists a t–open set V_l of X such that $U_l = V_l \cap Y$ for each l. For each fixed n the collection

$$P_n = \{V_l : l \in L\} \cup \{X / Y_n\}$$

from a t–open cover of X. By hypothesis P_n has a r–open refinement W which is locally countable with respect to r. Then $W = \{W_{l_n} : (l, n) \in L \times N\}$ is locally countable with respect to the r. For each n, let $Y_n = \{W_{ng} \cap Y : W_{ng} \cap Y \neq \phi, g \in G\}$. Let $Y = \bigcup_n Y_n$. Then Y is r_Y–open refinement of F which is s–locally countable with respect to the r_Y.

References

[1] I. J. Kadhim. "On Paracompactness in Bitopological Spaces and Tritopolgical Spaces", Mcs. Thesis, University of Babylon (2006).

[2] "On Compactness in Bitopological Spaces", to apper.

[3] J. C. Kell. "Bitopological Spaces", proc. London Math.Soc. 13(1963), pp. 71-89.

[4] M. M. Kovar. "On 3-topological Version of θ-regularity", Internet. J.Math.Sci. 23(1998). No. 6, pp. 393-398.

[5] S. Willard, "General Topology", Addison-Wesry Pub. Co., Inc. (1970).

[6] W. G. Fleissner, "Normal, Not Paracompact Spaces", American Math.Soc. 7(1982). No.1, pp. 233-236.

الخلاصة

عرفنا في هذا البحث البارا-لاندلوف وشبه البارا-لاندلوف على الفضاء التبولوجي الثنائي، وأوجدنا بعض خصائص هذه المصطلحات والعلاقة التي تربط بينهما.