HYPERCYCLICITY AND SUPERCYCLICITYFOR SOME CLASSES OF OPERATORS

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Abstract

In this paper,we prove that, if T is the quotient of a decomposable on a separable Banach space (M–hyponormal operator on a real Hilbert space), then T is hypercyclic operators. We also show that these classes of operators are supercyclic operators.

Key words and phrases: Hypercyclic operator, supercyclic operator, decomposable operator, M–hyponormal operator , single valued extension property (SVEP), Bishop's property (β), Dunford's property (C), decomposition property (δ).

Introduction

Let X be a complex Banach space, and $\mathcal{L}(T)$ be the set of all bounded linear operators on X, we also denote as usual the spectrum of T by $\sigma(T)$. If $T \in \mathcal{L}(X)$, then a part of T is a bounded operator obtained by restricting T to an invariant closed subspace M, say $T|_M$, a part of the spectrum of T is denoted by, $\sigma(T|_M)$, where M is an invariant closed subspace of T.

An operator $T \in \mathcal{L}(X)$ is called *hypercyclic* if there is a vector $x \in X$ with dense orbit $\{x, Tx, T^2x, \ldots\}$, and is called *supercyclic* if there is a vector $x \in X$ $\{cT^n x : n \geq 0, c \in \mathbb{C}\}$, is dense in X, see[3]. $T \in \mathcal{L}(X)$ is said to be *decomposable* if every open cover $\mathbb{C} = U \cup V$of the complex plane $\mathbb{C}$ by two open sets U and V effects a splitting of the spectrum $\sigma(T)$ and of the space X, in the sense that there exist T–invariant closed linear subspaces Y and Z of X for which $\sigma(T|_Y) \subseteq U$, $\sigma(T|_Z) \subseteq V$, and $X = Y + Z$, for example, all normal operators on a Hilbert space, compact operators and generalized scalar operators on Banach spaces are decomposable, see[6].

Also, for a T-invariant closed subspace M of T, let $T/M \in \mathcal{L}(X/M)$ denote the operator induced by $T \in \mathcal{L}(X)$ on the quotient space X/M.and called it the quotient of operator, It is known that every quotient space is Banach space, if X is Banach space, see [6].

Following to[2], let H be a complex Hilbert space. $T \in \mathcal{L}(H)$, T is said to be *M–hyponormal* operator, if there exists a constant number $M > 0$ such that $\| (T - \lambda I)^* x \| \leq M \| (T - \lambda I)x \|$ for each complex number λ. It is known that every hyponormal operator and every normal operator are M–hyponormal operators.The purpose of the present paper is to study the quotient of a decomposable on a complex Banach space and the M–hyponormal operator on a real Hilbert space to be hypercyclic or supercyclic under sufficient conditions.

Preliminaries

An operator $T \in \mathcal{L}(X)$ is said to be have *single valued extension property* (SVEP) *at* λ_0 if for every open set $U \subseteq \mathbb{C}$ containing λ_0 the only analytic solution $f: U \to X$ of the equation

$$(T - \lambda I)f(\lambda) = 0 \qquad (\lambda \in U)$$

is the zero function., an operator T is said to have SVEP if T has SVEP at every $\lambda \in \mathbb{C}$, ([4], [6])

Given $T \in \mathcal{L}(X)$, the *local resolvent set* $\rho_T(x)$ of T at the point $x \in X$ is defined as the union of all open subsets $U \subseteq \mathbb{C}$ for which there is an analytic function $f: U \to X$ such that

$$(T-\lambda I)f(\lambda) = x \quad (\lambda \in U)$$

The *local spectrum* $\sigma_T(x)$ of T at x is then defined as

$$\sigma_T(x) = \mathbb{C} \setminus \rho_T(x)$$

For $T \in \mathcal{L}(X)$, we define the *local* (resp. *glocal*) *spectral subspaces* of T as follows. Given a set $F \subseteq \mathbb{C}$ (resp. a closed set $G \subseteq \mathbb{C}$).

$$X_T(F) = \{x \in X : \sigma_T(x) \subseteq F\}.$$

(resp.

$\mathcal{X}_T(G) = \{x \in X$: there exists an analytic function $f: \mathbb{C}\backslash G \to X$ such that $(T-\lambda I)f(\lambda) = x$ for all $\lambda \in \mathbb{C}\backslash G\}$).

Note that T has SVEP if and only if $X_T(F) = \mathcal{X}_T(F)$ for all closed sets $F \subseteq \mathbb{C}$, [6, Proposition (3.3.2)].

An operator $T \in \mathcal{L}(X)$ has *Dunford's property* (C) if the local spectral subspace $X_T(F)$ is closed for every closed set $F \subseteq \mathbb{C}$. We also say that T has *Bishop's property* (β) if for every sequence $f_n: U \to X$ such that $(T-\lambda I)f_n(\lambda) \to 0$ uniformly on compact subsets in U, it follows that $f_n \to 0$ uniformly on compact subsets in U. It is well known [6, 7] that Bishop's property $(\beta) \Longrightarrow$ Dunford's property (C) $\Longrightarrow$ SVEP.

Moreover, an operator $T \in \mathcal{L}(X)$ has *decomposition property* (δ) if $X = \mathcal{X}_T(\overline{U}) + \mathcal{X}_T(\overline{V})$ for every open cover $\{U, V\}$ of $\mathbb{C}$.

As shown in [1], an operator $T \in \mathcal{L}(X)$ has property (δ) iff it is the quotient of a decomposable operator. Moreover properties (β) and (δ) are dual to each other, in the sense that an operator $T \in \mathcal{L}(X)$ has *property* (β) iff its adjoint has property (δ), and conversely, T has property (δ) iff its adjoint has property (β).

The following result from Feldman, Miller and Miller [3], gives the relation between parts of the spectrum and the local spectra of an operator with Dunford's property (C).

Proposition (2.1):

If $T \in \mathcal{L}(X)$ has Dunford's property (C), then $\sigma_T(x) = \sigma(T|_{X_T(F)})$ whenever $F = \sigma_T(x)$ for some nonzero $x \in X$.

The following result from Feldman, Miller and Miller [3], gives sufficient condition for an operator to be hypercyclic, we denote the interior and exterior of the unit circle by $\mathbb{D}$, $\mathbb{C}\backslash\overline{\mathbb{D}}$ respectively.

Corollary (2.2)

Let X be a complex Banach space and suppose that $T \in \mathcal{L}(X)$ has the decomposition property (δ). If $\sigma_{T^*}(x^*) \cap \mathbb{D} \neq \emptyset$ and $\sigma_{T^*}(x^*) \cap (\mathbb{C}\backslash\overline{\mathbb{D}}) \neq \emptyset$ for every nonzero $x^* \in X^*$. Then T is hypercyclic.

If A is a compact set in the complex plane and $\epsilon > 0$, then $B(A, \epsilon)$ denote the ϵ −neighborhood of A, that is, $B(A, \epsilon) = \{z : dist(z, A) < \epsilon\}$. For the proof of the following classic result see Newman [7], Corollary 1.

Lemma (2.3)

If K is any compact set in the complex plane, A is a component of K, and $\epsilon > 0$, then there exists disjoint open sets U, V such that $K \subseteq U \cup V$ and $A \subseteq U \subseteq B(A, \epsilon)$.

If $\rho \geq 0$, we denote the circle $\{z \in \mathbb{C} : |z| = \rho\}$ by Γ_ρ. The interior and exterior of Γ_ρ are the regions $int\,\Gamma_\rho = \{z \in \mathbb{C} : |z| < \rho\}$ and $ext\,\Gamma_\rho = \{z \in \mathbb{C} : |z| > \rho\}$. Recall that an operator T of is said to be ρ −outer (outer with respect to Γ_ρ) or ρ −inner (inner with respect to Γ_ρ) provided that T satisfies conditions either (a) $X_T(ext\,\Gamma_\rho)$ is dense and for every $\epsilon > 0$, $X_T(\text{int}X_{\rho-\epsilon})$, or (b) $X_T(int\Gamma_\rho)$ is dense and for every $\epsilon > 0$, $X_T(ext\Gamma_{\rho-\epsilon})$ is dense, respectively.

The following Theorem is a stronger form of a result due to Herrero [5, Proposition (3.1)].

Theorem (2.4)

If $T \in \mathcal{L}(X)$ is a supercyclic operator on a separable Banach space X, then there exists a circle Γ_ρ, $\rho \geq 0$, such that $\sigma(T^*|_M) \cap \Gamma_\rho \neq \emptyset$ for every nonzero weak*closed T^*–invariant subspace M of X^*.

In particular, every component of the spectrum of T intersects Γ_ρ.

If T is a supercyclic operator, then any circle as in Theorem (2.4) will be called a *supercyclicity circle* for T.

The following result from Feldman, Miller and Miller [3], gives sufficient condition for an operator to be supercyclic

Corollary (2.5)*:*

Let X be a complex Banach space and assume that $T \in \mathcal{L}(X)$ has the decomposition property (δ) .If there exists a circle Γ_ρ, $\rho \geq 0$, satisfying ***either****:*

a– For every nonzero $x^* \in X^*$, $\sigma_{T^*}(x^*)$ intersects
both Γ_ρ and $int\Gamma_\rho$, **or**

b– For every nonzero $x^* \in X^*$, $\sigma_{T^*}(x^*)$ intersects
both Γ_ρ and $ext\Gamma_\rho$.
Then T is supercyclic.

Main Results For hypercyclicity

Proposition (3.1):

If T is a quotient of a decomposable operator on a complex Banach space X, and $\sigma(T^*|_{M^*}) \cap \mathbb{D} = \emptyset$ and $\sigma(T^*|_{M^*}) \cap (\mathbb{C}\backslash\overline{\mathbb{D}}) = \emptyset$ for every hyperinvariant $M^*$of T^*, then T is hypercyclic.

***Proof*:**

Let T be a quotient of a decomposable operator on X, then T has property (δ). Hence, T^* has property (β), and so T^* has property (C). Since $\sigma(T^*|_{M^*}) \cap \mathbb{D} = \emptyset$ and $\sigma(T^*|_{M^*}) \cap (\mathbb{C}\backslash\overline{\mathbb{D}}) = \emptyset$ for every hyperinvariant M^* of T^*, and $X^*_{T^*}(F)$ is hyperinvariant for every closed set $F \subseteq \mathbb{C}$, then $\sigma\left(T^*|_{X^*_{T^*}(F)}\right) \cap \mathbb{D} = \emptyset$ and $\sigma\left(T^*|_{X^*_{T^*}(F)}\right) \cap (\mathbb{C}\backslash\overline{\mathbb{D}}) = \emptyset$ Since T^* has property (C), then $\sigma_{T^*}(x^*) = \sigma\left(T^*|_{X^*_{T^*}(F)}\right)$ whenever $F = \sigma_{T^*}(x^*)$ for some nonzero $x^* \in X^*$ by Proposition (2.1), it follows that $\sigma_{T^*}(x^*) \cap \mathbb{D} = \emptyset$ and $\sigma_{T^*}(x^*) \cap (\mathbb{C}\backslash\overline{\mathbb{D}}) = \emptyset$ for every nonzero $x^* \in X^*$. Thus Corollary (2.2) applies to give that T is hypercyclic.

Corollary (3.2):

If T is a quotient of a decomposable operator on X, and T is both inner and outer with respect to a circle Γ_ρ (where inner and outer with respect to a circle Γ_ρ is defined above), $\rho > 0$, then a multiple of T is hypercyclic.

Proof:

If T is both inner and outer with respect to Γ_ρ, then $\frac{1}{\rho}\,T$ will be hypercyclic by Propsition (3.1).

We now present new results for M–hyponormal operator on a real Hilbert space which is needed, then later

Proposition (3.3):

If T is M–hyponormal operator on a real Hilbert space H, then T^* has Bishop's property (β).

Proof:

Let $U \subseteq \mathbb{C}$ be an open set, and consider a sequence of analytic functions $f_n : U \to H$ for which $(T^* - \lambda I)f_n(\lambda) \to 0$ as $n \to \infty$ locally uniformly on U.We want to show that $f_n \to 0$ as $n \to \infty$, again locally uniformly on U. Since T is M–hyponormal operator, then T has property (β), by [6, Proposition (2.4.9)]. Hence $f_n \to 0$ as $n \to \infty$ uniformly on all compact subsets of U, for every sequence of analytic functions $f_n : U \to H$ for which $(T - \lambda I)f_n(\lambda) \to 0$ as $n \to \infty$ uniformly on all compact subsets of U, but we need $f_n \to o$ as $n \to \infty$ locally uniformly on U, when $(T^* - \lambda I)f_n(\lambda) \to 0$ as $n \to \infty$ locally uniformly on U Again, since T is M–hyponormal

operator, then there exists a constant number $M > 0$ such that
$\| (T - \lambda I)^{*}x \| \leq M \| (T - \lambda I)x \|$ for all $\lambda \in \mathbb{R}$, $x \in H$. Thus we have
$\| (T - \lambda I)^{*} f_n(\lambda) \| \leq M \| (T - \lambda I) f_n(\lambda) \|$ for all $\lambda \in U$, $f_n(\lambda) \in H$. So $(T - \lambda I)^{*} f_n(\lambda) \to 0$ as $n \to \infty$ Therefore T^{*} has Bishop's property (β).

Remark (3.4):

Proposition (3.3) is not true if H is a complex Hilbert space.

Now we shall prove that every M–hyponormal operator on a real Hilbert space is hypercyclic.

Proposition (3.5):

IF T is M–hyponormal operator on a real Hilbert space H, and $\sigma(T^{*}|_{M}) \cap \mathbb{D} = \emptyset$ and $\sigma(T^{*}|_{M}) \cap (\mathbb{C}\backslash\overline{\mathbb{D}}) = \emptyset$ for every hyperinvariant $M$of T^{*}. Then T is hypercyclic.

Proof:

If T is M–hyponormal operator on a real space H, then T^{*} has property (β), by Proposition (3.3). Thus T^{*} has property (C),.and so T has property (δ). Now, since $\sigma(T^{*}|_{M}) \cap \mathbb{D} = \emptyset$ and $\sigma(T^{*}|_{M}) \cap (\mathbb{R}\backslash\overline{\mathbb{D}}) = \emptyset$ for every hyperinvariant M of T^{*}, and $H_{T^{*}}(F)$ is hyperinvariant for every closed set $F \subseteq \mathbb{R}$, then $\sigma(T^{*}|_{H_{T^{*}}(F)}) \cap \mathbb{D} = \emptyset$ and $\sigma(T^{*}|_{H_{T^{*}}(F)}) \cap (\mathbb{R}\backslash\overline{\mathbb{D}}) = \emptyset$ Since T^{*} has property (C), then $\sigma_{T^{*}}(x) = \sigma(T^{*}|_{H_{T^{*}}(F)})$

whenever $F = \sigma_{T^{*}}(x)$ for some nonzero $x \in H$, by Proposition (2.1), it follows that $\sigma_{T^{*}}(x) \cap \mathbb{D} = \emptyset$ and $\sigma_{T^{*}}(x^{*}) \cap (\mathbb{R}\backslash\overline{\mathbb{D}}) = \emptyset$ for every nonzero $x \in H$.Therefore T is hypercyclic by Corollary (2.2).

Corollary (3.6):

If T is M–hyponormal operator on a real Hilbert space H, and T is both inner and outer with respect to a circle Γ_{ρ} (where inner and outer with respect to a circle Γ_{ρ} is defined above), $\rho > 0$, then a multiple of T is hypercyclic.

Proof:

If T is both inner and outer with respect to Γ_{ρ}, then $\frac{1}{\rho}T$ will be hypercyclic by Proposition (3.5).

Main Results for Supercyclic

Proposition (4.1):

If T is a quotient of a decomposable operator on a complex Banach space X, and there exists a circle Γ_{ρ}, $\rho \geq 0$, such that ***either****:*

a–For every hyperinvariant subspace M^{*} of T^{*}, $\sigma(T^{*}|_{M^{*}})$ intersects Γ_{ρ} and $int\,\Gamma_{\rho}$, ***or***

b–For every hyperinvariant subspace M^{*} of T^{*}, $\sigma(T^{*}|_{M^{*}})$ intersects Γ_{ρ} and $ext\,\Gamma_{\rho}$.

Then T is supercyclic.

Proof:

Since T is a quotient of a decomposable operator on X, then T has property (δ). Hence T^{*} has property (β), and so T^{*} has property (C). If $\sigma(T^{*}|_{M^{*}})$ intersects Γ_{ρ} and Γ_{ρ} , for every hyperinvariant subspace M^{*} of T^{*}. And since $X^{*}{}_{T^{*}}(F)$ is hyperinvariant for every closed set $F \subseteq \mathbb{C}$, then $\sigma(T^{*}|_{X^{*}_{T^{*}}(F)})$ intersects Γ_{ρ} and $int\,\Gamma_{\rho}$. Since T^{*} has property (C), then $\sigma_{T^{*}}(x^{*}) = \sigma(T^{*}|_{X^{*}_{T^{*}}(F)})$ whenever $F = \sigma_{T^{*}}(x^{*})$ for some nonzero $x^{*} \in X^{*}$, by Proposition (2.1), it follows that $\sigma_{T^{*}}(x^{*})$ intersects both Γ_{ρ} and $int\,\Gamma_{\rho}$, for every nonzero $x^{*} \in X^{*}$. Thus Corollary (2.5) applies to give that T is supercyclic. Similarly, if $\sigma(T^{*}|_{M^{*}})$ intersects Γ_{ρ} and $ext\,\Gamma_{\rho}$ for every hyperinvariant subspace M^{*} of T^{*}, then T is supercyclic.

We say that an operator is *purely supercyclic*, if it pure (It mains the restriction of operator on any nontrivial invariant subspace is not normal), supercyclic and no multiple of it is hypercyclic.

Corollary (4.2):

If T is a quotient decomposable operator on X, and T is purely supercyclic, then T has unique supercyclicity circle.

Proof:

If there are two supercyclicity circles, Γ_{ρ_1} and Γ_{ρ_2} with $0 \leq \rho_1 < \rho_2$, then every part of the spectrum of T^* intersects both Γ_{ρ_1} and Γ_{ρ_2}. Now choose a ρ such that $\rho_1 < \rho < \rho_2$. An application of Lemma (2.3) and the fact that every part of $\sigma(T^*)$ must intersect both Γ_{ρ_1} and Γ_{ρ_2}, imply that every part of $\sigma(T^*)$ will intersect Γ_ρ, as well as the interior and exterior of Γ_ρ. Thus, T is both ρ −inner and ρ −outer, and the previous result implies that a multiple of T is hypercyclic, contrary to our assumption.

Corollary (4.3):

If $\{T_n\}$ is a bounded sequence of quotient of decomposable operators such that for every n, T_n is supercyclic, then $\oplus_n T_n$ is supercyclic if and only if there is a common supercyclicity circle, Γ_ρ, $\rho \geq 0$, and T_n is ρ −inner for every n or T_n is ρ −outer for every n.

Proof:

Let $T = \oplus_n T_n$. If T is supercyclic, then a supercyclicity circle for T will be a supercyclicity circle for each T_n. Similarly, if T is ρ −inner (or ρ −outer), then T_n is ρ −inner (or ρ −outer) for each n. Conversely, suppose Γ_ρ, is a supercyclicity circle for each T_n and each T_n is ρ −outer. We need to check that if M^* is a hyperinvariant subspace for T^*, then $\sigma(T^*|_{M^*})$ intersects both Γ_ρ and $ext\,\Gamma_\rho$. However, since M^* is hyperinvariant, it must be invariant under every coordinate projection. Thus $M^* = \oplus_n M_n^*$ where M_n^* is a hyperinvariant subspace of T_n^*. Thus, $\sigma(T^*|_{M^*}) \supseteq \sigma(T_n^*|_{M_n^*})$ for each n. So, if n is such that $M_n^* \neq \{0\}$, then by assumption $\sigma(T_n^*|_{M_n^*})$ intersects both Γ_ρ and $ext\,\Gamma_\rho$. Thus $\sigma(T^*|_{M^*})$ also intersects both Γ_ρ and $ext\,\Gamma_\rho$. So, Theorem (4.1) implies that T is supercyclic. If each T_n is ρ −inner, then the proof is similar.

Proposition (4.4):

If T is M–hyponormal operator on a real Hilbert space H, and there exists a circle Γ_ρ, $\rho \geq 0$, such that ***either****:*

a– For every hyperinvariant subspace M of T^*,
$\sigma(T^*|_M)$ intersects Γ_ρ and $int\,\Gamma_\rho$, ***or***

b– For every hyperinvariant subspace M of T^*,
$\sigma(T^*|_M)$ intersects Γ_ρ and $ext\,\Gamma_\rho$.

Then T is supercyclic.

Proof:

Since T is M–hyponormal operator on a real Hilbert space H, then T^* has property (β), by Proposition (3.3). Thus T^* has property (C), and so T has property (δ). If $\sigma(T^*|_M)$ intersects Γ_ρ and Γ_ρ, for every hyperinvariant subspace M of T^*. And since $H_{T^*}(F)$ is hyperinvariant for every closed set $F \subseteq \mathbb{C}$, then $\sigma(T^*|_{H_{T^*}(F)})$ intersects Γ_ρ and $int\,\Gamma_\rho$. Now since T^* has property (C), then $\sigma_{T^*}(x^*) = \sigma(T^*|_{H_{T^*}(F)})$ whenever $F = \sigma_{T^*}(x^*)$ for some nonzero $x^* \in H$ by Proposition (2.1), it follows that and $\sigma_{T^*}(x^*)$ intersects both Γ_ρ and $int\,\Gamma_\rho$, for every nonzero $x^* \in H$. Thus Corollary (2.5) applies to give that T is supercyclic.Similarly, if $\sigma(T^*|_{M^*})$ intersects Γ_ρ and $ext\,\Gamma_\rho$ for every hyperinvariant subspace M of T^*, then T is supercyclic.

Corollary (4.5):

If T is M–hyponormal operator on a real Hilbert space H, and T is purely supercyclic, then T has unique supercyclicity circle.

Proof:

If there are two supercyclicity circles, Γ_{ρ_1} and Γ_{ρ_2} with $0 \leq \rho_1 < \rho_2$, then every part of the spectrum of T^* intersects both Γ_{ρ_1} and Γ_{ρ_2}. Now choose a ρ such that $\rho_1 < \rho < \rho_2$. An application of Lemma (2.3) and the fact that every part of $\sigma(T^*)$ must intersect both Γ_{ρ_1} and Γ_{ρ_2}, imply that every part of $\sigma(T^*)$ will intersect Γ_ρ, as well as the interior and exterior of Γ_ρ.Thus, T is both ρ −inner and ρ −outer, and the previous result implies that a multiple of T is hypercyclic, contrary to our assumption.

Corollary (4.6)*:*

If $\{T_n\}$ is a bounded sequence of M–hyponormal operators on a real Hilbert space H such that for every n, T_n is supercyclic, then $\oplus_n T_n$ is supercyclic if and only if there is a common supercyclicity circle, Γ_ρ, $\rho \geq 0$, and T_n is ρ −inner for every n or T_n is ρ −outer for every n.

Proof:

Let $=\oplus_n T_n$. If T is supercyclic, then a supercyclicity circle for T will be a supercyclicity circle for each T_n. Similarly, if T is ρ −inner (or ρ −outer), then T_n is ρ −inner (or ρ −outer) for each n. Conversely, suppose Γ_ρ, is a supercyclicity circle for each T_n and each T_n is ρ − outer. We need to check that if M is a hyperinvariant subspace for T^*, then $\sigma(T^*|_M)$ intersects both Γ_ρ and $ext\,\Gamma_\rho$. However, since M is hyperinvariant, it must be invariant under every coordinate projection. Thus $M = \oplus_n M_n$ where M_n is a hyperinvariant subspace of T_n^*. Thus, $\sigma(T^*|_M) \supseteq \sigma(T_n^*|_{M_n})$ for each n. So, if n is such that $M_n \neq \{0\}$, then by assumption $\sigma(T_n^*|_{M_n})$ intersects both Γ_ρ and $ext\,\Gamma_\rho$. Thus $\sigma(T^*|_M)$ also intersects both Γ_ρ and $ext\,\Gamma_\rho$. So, Theorem (4.4) implies that T is supercyclic. If each T_n is ρ −inner, then the proof is similar.

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الخلاصة

في هذا البحث نثبت بان المؤثر كوشي القابل للتحليـــل T على فضاء غير منتهي وقابل للفصل بناخ X المعرف علـــى حقل الاعداد العقدية والمؤثر M-فوق السوية T على فضـــاء غير منتهي هلبرت H المعرف على حقل الاعداد الحقيقيةهما مؤثران فوق الدائرية وكذلك هما مؤثران فائق الدائرية.