COMPUTATIONS ON THE OPTICAL PROPERTIES OF THE ELECTROSTATIC SYMMETRIC QUADRUPOLE TRIPLET LENS

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Abstract

With the aid of transfer matrices method and personal computer a computational research has been carried out on the properties of a rectangular model representing the axial field of an electrostatic quadrupole triplet lens. The path of charged–particles beam traversing the rectangular fields has been determined by solving the trajectory equation of motion in Cartesian coordinates. A computer program has been written for this purpose.

The optical properties of the electrostatic quadrupole triplet lens have been computed with the aid of the beam trajectory along the lens axis. Computer programs in Fortran 77 have been written for determining the various optical properties. The computations have been mainly concentrated on determining the focal lengths and spherical aberration coefficients in both horizontal and vertical planes of the trajectory along the optical axis. The results have shown that the choice of the geometrical and operational parameters depends on the function of the electrostatic quadrupole triplet lens in a particular electron or ion–optical system.

Keywords: quadrupole, optical properties, electrostatic

I. Electrostatic Quadrupole Triplet Lens System

A quadrupole doublet is employed in electron–optical devices of moderate accelerating voltage when astigmatic focusing is required. Although the triplet follows the doublet in order of increasing complexity, it is not an inviting combination from the optical point of view. In general, regularity will be achieved by adjusting the three excitations and the two separations once the design of the individual components has been decided (in particular, their effective lengths). The symmetric triplet does, however, offer convenient means of converting a virtual image into a real one with unit magnification. Blewett in 1958 has tabulated few values of the parameters of symmetric triplets for which the imagery is stigmatic. Most published information is in graphical form; however, in 1961 Enge has plotted families of curves illustrating the behavior of a symmetric triplet producing pseudo–stigmatic imagery **[7]**.

Consider figure (1a). The length of the central quadrupole is L_2 and the outer components are of lengths L_1 and L_3. It is seen that the object and image distances (u and v respectively) are the same in both planes. The present work has been focused on symmetric triplets, which means that voltages applied to the first and third quadrupoles are the same. This reduction of the number of variable parameters makes the tuning of the triplet much easier than doublet while at the same time the triplet remains flexible enough to satisfy various requirements in the first order focusing properties. Voltages are applied to the triplet in such a way that in both x–z and y–z planes converging and diverging lenses alternate. The y–z plane has been taken to be the DCD (diverging–converging–diverging) plane and x–z plane to be the CDC (converging–diverging–converging) plane **[1]**.

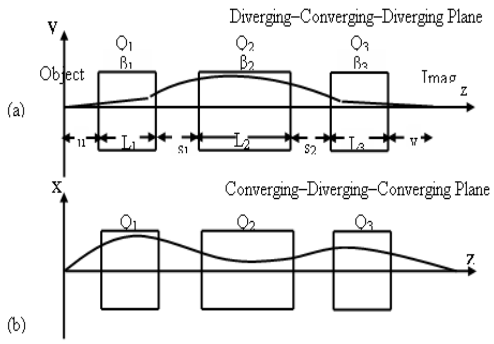


Figure (1). Particle trajectories in the two planes of three-element quadrupole lens system [5].

For the complete three–element lens system, the notations used are as follows: (a) the lens excitation parameter is denoted by β, (b) the field–free distance between the quadrupole lenses is denoted by s i.e. the distance between the first quadrupole lens Q_1 and second quadrupole lens Q_2 is s_1, and that between second quadrupole lens Q_2 and third quadrupole lens Q_3 is s_2 as shown in figure (1a), and (c) the object and image distance u and v respectively.

In stigmatic modes the beam crossover in the x–z plane and y–z plane concide. In this case the linear magnifications of a doublet are generally quite different in the x and y directions, which might be a limitation for some applications. The triplet allows the magnification ratio to be varied without changing the image position and it can be made equal to unity if necessary **[2]**.

II. Aim of the Project

A simple quadrupole lens forms a real image in its converging plane and a virtual image in its diverging plane. Since, in practice, a focusing system requires in general a real image in both planes, hence a quadrupole triplet lens has been considered to achieve such requirement. A system consisting of three quadrupole lenses at opposite polarity and separated by drift space is to be investigated in detail. The optical properties of the triplet and the beam trajectory along the fields are to be considered. The present work is mostly restricted to the symmetric triplet for its advantage of remaining flexible enough to satisfy various requirements in the first order focusing properties. The rectangular field model has been taken into consideration to represent the potential distribution of an electrostatic quadrupole triplet lens. Its distribution resembles the actual field of hyperbolic quadrupoles more accurately. It is aimed in the present work at determining the properties of such lens. The first order optical properties of this model will be computed by solving the trajectory equation of the charged–particles beam traversing each field distribution, taking into account the convergence and divergence planes. The effect of the lens excitation, image and object distance, lens length, and the separation between the lenses on the spherical aberration coefficients will be investigated in detail.

III. Field models for electrostatic quadrupole lenses

The field distribution of a quadrupole lens may be represented by various models shown in figure (2).

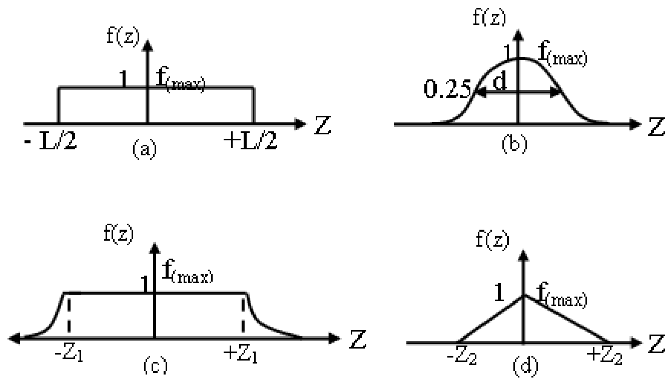


Figure (2). Field distribution of a quadrupole lens [4].

(a) Rectangular model,
(b) Bell–shaped model
(c) Modified bell–shaped model
(d) Triangular model.

The function f(z) for a rectangular field model of long quadrupole is represented in the following form **[7]**:

$f(z)=(f(z))_{max}=1$ when $-L/2\leq z\leq L/2$ **.................(1)**

For short quadrupole lens, Glaser's bell–shaped field model shown in figure (2b) is found to be more suitable and is represented by the following function **[7]**:

$f(z)=(f(z))_{max}/[1+ (z/d)^2]^2=1/ [1+ (z/d)^2]^2$ ....**(2)**

The modified bell–shaped field model shown in figure (2c) represents the intermediate case between the rectangular and the bell–shaped model such that the field distribution may be represented by the following function:

$f(z)=1/[1+((z-z_1))/d)^2]^2$ when $z>z_1$ ....**(3)**

$f(z)=1/[1+((z+z_1))/d)^2]^2$ when $z<-z_1$........**(4)**

The triangular field distribution model shown in figure (2d) is another model proposed by **[6]**; it is given by:

$f(z) = \Phi z+ z_2$ when $-z_2 \leq z \leq 0$...........**(5)**

$f(z) = -\Phi z+ z_2$ when $0 \leq z \leq z_2$.........**(6)**

$f(z) = (f(z))_{max} = 1$ at $z = 0$ ....**(7)**

where Φ is the slope of the two steep sides of the triangle.

IV. The equation of motion

The path of charged–particles beam traversing the fields can be determined by solving the trajectory equation of motion in Cartesian coordinates:

$x'' + \beta^2 f(z) x = 0$ ....**(8)**

$y'' - \beta^2 f(z) y = 0$ ....**(9)**

where β^2 is the lens excitation, given by the following relation:

$\beta^2 = U_1 k / c^2 U_o$ ....**(10)**

U_1 being the electrode voltage, U_o is the accelerating voltage, x'' and y'' are the second derivatives with respect to z, and k a coefficient accounting for the shape of electrode. Since the present work has been concentrated on the hyperbolic shape for the electrodes, thus $k = 1$ **[3], [4]**.

V. The Quadrupole Triplet Lens and Its Field

Figure (3) depicts a diagram of the electrostatic quadrupole triplet lens under consideration. It consists of three coaxial quadrupole lenses Q_1, Q_2, and Q_3. The present investigation takes into consideration that the field distribution of each lens is of a rectangular shape of length L_1, L_2, and L_3 respectively as shown in **figure (4).** In fact it is assumed that the effective length of each field is equal to the length of the corresponding quadrupole lens. The axial distance s_1 separates the lenses Q_1 and Q_2 and s_2 separates Q_2 and Q_3. This quadrupole triplet lens takes into account the focusing of an accelerated charged–particles beam of positive ions traveling from the left to the right–hand–side. Thus the object is assumed to be situated on the left–hand–side at an axial distance u from the starting point of the field of lens Q_1 and the image position is on the right–hand–side at a distance v from the end point of the field of lens Q_3. Since the positive potential (+U) is applied on the x–axis electrodes of lenses Q_1 and Q_3, then the positively charged particles will be repelled along the x–axis and hence will be directed towards the z–axis. The positively charged particles will be attracted by the negative potential (–U) along the y–axis of Q_1 and Q_3. Therefore, in lenses Q_1 and Q_3 the convergence process takes place in the x–axis and the divergence process takes place in the y–axis; whereas in lens Q_2 the convergence and divergence processes take place in the y– and x–axis respectively. In the present work the polarities of the electrodes of each lens in the x and y axes shown in figure (3) are taken into account. Hence, the positively charged particles traverse the three electrostatic fields of the quadrupole triplet lens in two planes, namely the x–z plane where the convergence–divergence–convergence (CDC) process takes place and the y–z plane where the divergence–convergence–divergence (DCD) process takes place.

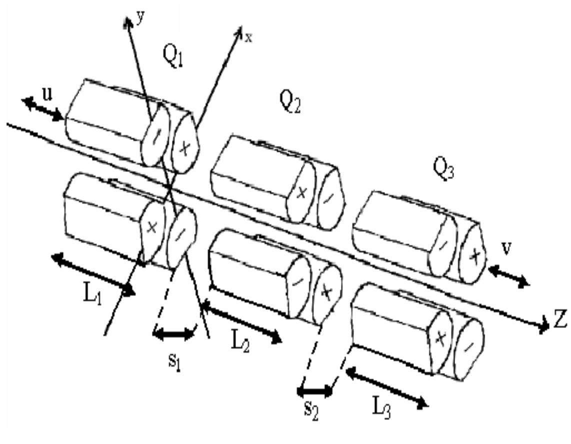


Figure (3). Diagram of an electrostatic quadrupole triplet lens.

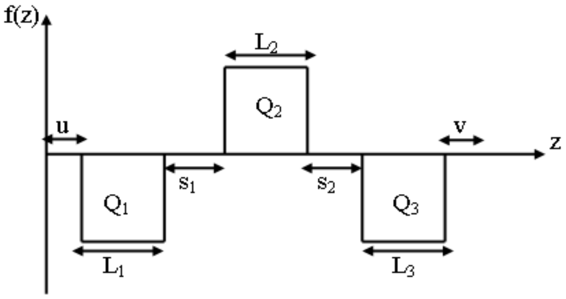


Figure (4).Field distribution of an electrostatic quadrupole triplet lens (rectangular model).

Table (1)
Parameters of the quadrupole triplet lens for stigmatic focusing under the following conditions:
β_1=β_3= 0.0268 mm^{-1}, β_2=0.0487 mm^{-1}, L_1=L_2 =L_3 = 5 mm, u=v =20 mm, and s_1= s_2 = 16

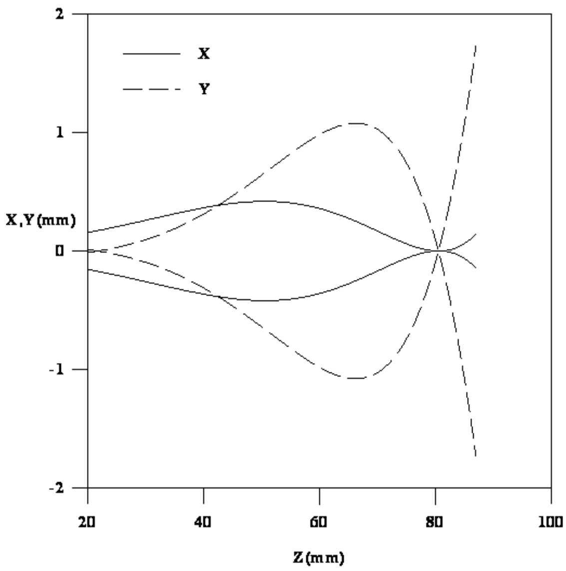
P_{CDC} = -4.51 mm	$\lvert P_{CDC}/ F_{CDC}\rvert$ = 14.88
P_{DCD} = -28.85 mm	$\lvert P_{DCD}/ F_{DCD}\rvert$ = 31.70
S_{CDC} = 53.97 mm	$\lvert S_{CDC}/ F_{CDC}\rvert$ = 178.12
S_{DCD} = 31.10 mm	$\lvert S_{DCD}/ F_{DCD}\rvert$ = 34.06
F_{CDC} = -0.30 mm	M_{CDC} = 1.00
F_{DCD} = -0.91 mm	M_{DCD} = 1.00
Z_{iCDC} = -76.35 mm	M = 1.00
Z_{iDCD} = -33.89 mm	

VI. Single Focus Quadrupole Triplet Lens

Application of quadrupole triplet lenses in an ion–optical instrument such as an ion implanter is usually intended to produce a single focus common for the x–z and y–z planes. Computations have been carried out to achieve this special case. In the present work it has been found that a single focus (or stigmatic focusing) can be obtained under the following conditions: L_1 =L_2 =L_3 = 5 mm, β_1 = β_3 = 0.0268 mm^{-1}, β_2 = 0.0487 mm^{-1}, s_1 = s_2 = 16 mm and u = v = 20 mm. It is seen that the excitation parameter of lens Q_2 is higher than that applied on lenses Q_1 and Q_3.

Figure (5) shows the single focus beam trajectory in two and three dimensions respectively. Figure(6) depicts a three–dimensional diagram of the paraxial charged–particles beam traversing the three excited quadrupole lenses that gave rise to a single focus situated in the field–free region outside the field of lens Q_3 under the above mentioned conditions.

The optical properties of the triplet lens under stigmatic focusing are listed in Table (1).



(b) (a)

Figure (5). (a) Beam trajectory in a quadrupole triplet lens for a single focus in both x–z and y–z planes (stigmatic focusing).
(b) The corresponding three–dimensional diagram of the beam trajectory determined by AutoCAD under the following conditions:
L_1 =L_2 =L_3 =5 mm, u = v = 20 mm, s_1= s_2 = 16 mm, β_1 = β_3 = 0.0268 mm^{-1}, and β_2 = 0.0487 mm^{-1}

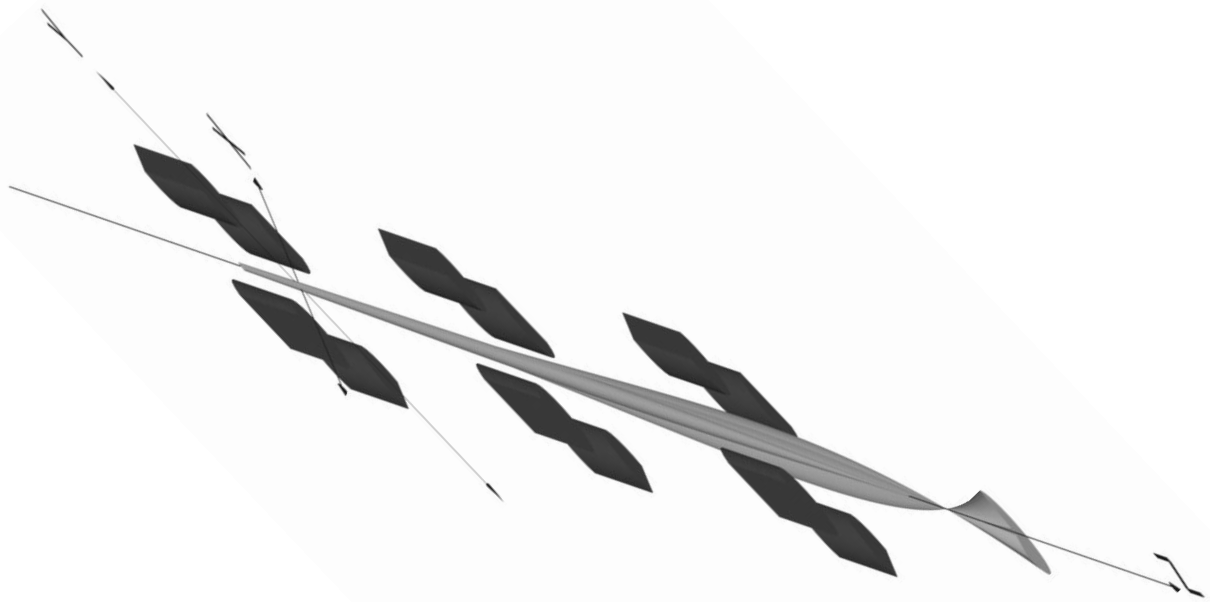


Figure (6). A three–dimensional diagram of the electrodes of the electrostatic quadrupole triplet lens with trajectory that gave rise to a common focus in x–z and y–z planes under the following conditions:
$L_1 = L_2 = L_3 = 5$ mm, $s_1 = s_2 = 16$ mm, u = v = 20 mm, $\beta_1 = \beta_3 = 0.0268\ mm^{-1}$, and $\beta_2 = 0.0487\ mm^{-1}$.

VII. Conclusions

It appears that the simple rectangular field model may be used to represent the axial field distribution of each electrostatic quadrupole lens constituting the triplet lens under investigation. The three–element lens system has many variable geometrical and operational parameters; thus a conclusive result is rather difficult.

A symmetrical triplet lens that has a common focus on the optical axis for the horizontal and vertical planes of the charged–particles beam trajectory has been achieved under certain geometrical and operational conditions. This single focus lens is found to have optically acceptable spherical aberration coefficients at unit magnification.

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الخلاصة:

إستعانه بطريقة المصفوفات الانتقاليه والحاسوب الشخصي فقد أجري بحث حاسوبي عن خواص نموذج مستطيلي يمثل المجال المحوري لعدسه ثلاثية كهروسكونيه رباعية الاقطاب، إن مسار حزمة الجسيمات المشحونه الماره خلال المجالات المستطيلية قد تم ايجاده بحل معادلة مسار الحركه بالاحداثيات الكارتيزيه إذ كتب برنامج حاسوبي لهذا الغرض. لقد حسبت الخواص البصريه للعدسة الثلاثية الكهروسكونيه بالاستعانه بمسار الحزمه على امتداد محور العدسه. وكتبت برامج حاسوبيه بفورتران ٧٧ لايجاد مختلف الخواص البصريه. وتم تركيز الحسابات على ايجاد الابعاد البؤرية ومعاملات الزيوغ الكروية في كلا المستويين الافقي والعمودي للمسار على امتداد المحور البصري. وقد أظهرت النتائج أن إختيار المعلمات الهندسية والتشغيلية يعتمد على وظيفة العدسه الثلاثية الكهروسكونية الرباعية الاقطاب في منظومة بصريه ألكترونيه او أيونيه معينة.