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SIMULATING AN EVOLUTION STRATEGY TO FORECAST TIME SERIES ARMA(1,1) MODEL

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Abstract

This paper presents a multi-member evolution strategy $ES^{(\mu+\lambda)}$ to forecast future value of observed time series $ARMA(1,1)$ model. The proposed method is simple and straight forward and doesn't required any problem specific parameter tuning of the problem. The experiments designed based on simulate $ES^{(\mu+\lambda)}$ for different values of sample size (n=25,50,100),model parameters ϕ set $(\pm 0.05,\pm 0.3,\pm 75)$ and θ set to $(\pm 0.1,\pm 0.4,\pm .9)$ and use lead time for forecasting future value equal to (l=1,2,3).The value of μ,λ take equal to (15,100) beside this, there is anther experiment designed for simulating one of method which is known as Box – Jenkins with same values of sample size, model parameters and leads time(l) for number of replicate (RR=1000). Results of this study has cleared by numbers of figures and tables, which are made to clear compression between ES-algorithm and B.J method based on computing values of FMSE (Forecasting Mean Square Error) & Thiels' (U- statistic) ,statistics used as tools to measures reliability of ES- algorithm and also used to clear accuracy of ES algorithm results. Table(1), tables (2-7) and figures (4 -9) results of statistics show the reliability of $ES^{(\mu+\lambda)}$ algorithm to producing individuals which give reasonably predictions of future values of time series for different values of sample size and lead time values of model parameters.

Key Words: Time series , forecasting function ,evalution strategy ,ARMA model, Box- Jenkins (B.J) forecasting, likelihood function, mution ,crossover, Thiels' (U- statistic).

1.Introduction

Time-series forecasting is a forecasting method which use a set of historical values to predict an outcome. These historical values, often referred to as a "time series", are spaced equally over time and can represent anything from monthly sales data to daily electricity consumption to hourly call volumes.A common goal of time series analysis is extrapolating past behavior into the future. The statgraphics forecasting procedures include random walks, moving averages, trend models, simple, linear, quadratic, and seasonal exponential smoothing, and ARIMA parametric time series models. Users may compare various models by holding samples at the end of the time series for validation purposes. Forecasting use at time t of available observations from a time series to forecast its value at some futuere time $(t+l)$ can provide a basis for economic & business planning, production planning ,inventory & production control and control & optimization of industrial processes. Forecasts are usually needed over a period known as the lead time ,which varies with each problem .Assuming that observations are available at a discrete time (equispaced intervals) of time, so if $z_{t-1}, z_{t-2},...$ are observations of previous time, these observations may be used to forecast future value for lead times $l = 1,2,3...$ ahead , $\hat{z}_t(l)$ denote the forecast made at origin t of the z_{t+l} at some future time $(t+l)$,that is at lead time l.The function $\hat{z}_t(l), l = 1,2,...$ that provides the forecasts at origin t for all future lead times will called the forecast function at origin t. Our objective is to obtain a forecast function which is such that the mean square of the deviations $(z_{t+l} - \hat{z}_t(l))$ between the actual and forecasted values is as small as possible for each lead time l . The accuracy of the forecasts may be expressed by calculating probability limits on either side of each forecast ,these limits may be calculated for any convenient set of probabilities, for example 50% and 95% [2]. They are such that the realized value of the time series, when it eventually occurs, will be included within these limits with the stated probability. A model which describes the probability structure of a sequence of observation is called a stochastic process $\bar{z}=(z_1,z_2,....z_N)$ of N successive observations is regard as a sample realization, from an infinite population of such samples, which could have been generated by this process. A major objective of statistical investigation is to infer the properties of the population from those of sample ,making a forecast is to infer the probability distribution of future observation from the population. Given a sample z of past value we need a ways of describing stochastic processes and time series ,and also need classes of stochastic models which are capable of describing practically occurring situation. A particular stationary stochastic processes of value modeling ,which is One of the main topics of this work, is the B.J forecasting methodology which is developed by G.E.P.Box and G.M. Jenkins consist of four basic step, involves tentatively identifying a model and sample partial autocorrelation function Once model tentatively identified ,we estimate the model parameters in the second step .This is called estimation step .In the third step which is called the diagnostic checking step, here see whether or not the model we have tentatively identified and estimated is adequate .If the model proved to be inadequate ,it must be modified and improved. The diagnostic methods employed will help us to decide how the model can improved .When the final model is determined ,we use the model to forecast future time series values. This fourth step is called the forecast step. B.J methodology is an iterative procedure because steps of tentative identification, estimation, and diagnostic checking is iterative until find the adequate final model, which is used to compute forecasts of future time series values. Figure (1) depicts time series analysis steps [8].

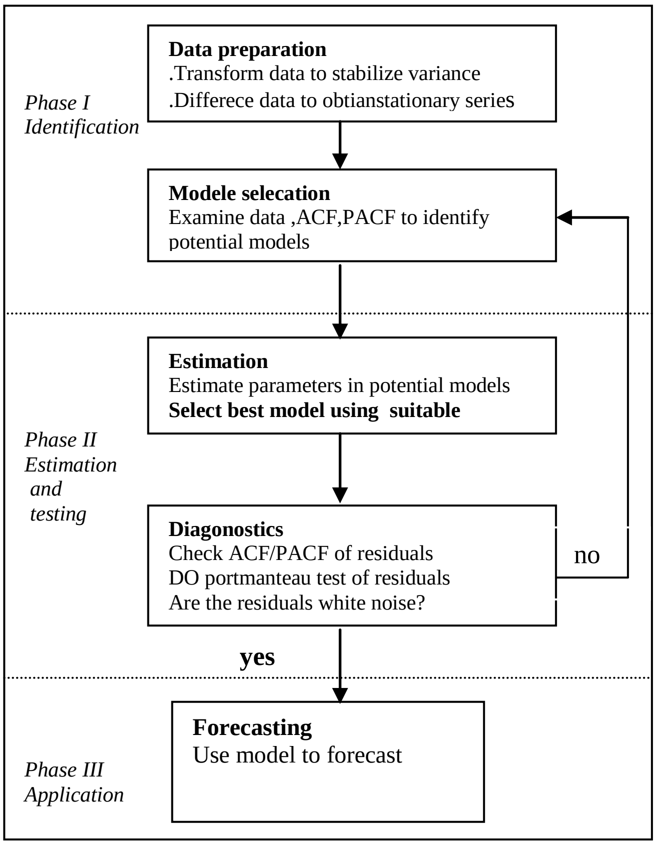

Figure 1: Box-Jenkins methodology for time series modeling.

2. Mathematical part

This section present many of mathematical definitions for ARMA models, forecasting function, forecasting errors and likelyhood function ,which is used to estimate model parameters, the following subsection illustrate these definitions in som details:

2.1 Box –Jenkins Models

B.J models represent a family of models. These models can be grouped in the following categories:

1-Autoregrassive Models AR(p)

$$z_t=\mu+\phi_1 z_{t-1}+...+\phi_p z_{t-p}+a_t \quad \textbf{..........(1)}$$

which employs $(p+2)$ unknown parameters: $\mu,\phi_1,...,\phi_p$ and σ_a^2 to be estimated from data series.

2- Moving Average Models MA(q)

$$z_t=\mu+a_t-\theta_1 a_{t-1}-...-\theta_q a_{t-q} \quad \textbf{....(2)}$$

which employs $(q+2)$ unknown parameters: $\mu,\theta_1,...,\theta_q$ and σ_a^2 to be estimated from data series.

3- Mixed Model (Autoregrassive-Movig Average Models ARMA(1,1)

$$z_t = \mu + \phi_1 z_{t-1} + ... + \phi_p z_{t-p} + a_t - \theta_1 a_{t-1} - ... - \theta_q a_{t-q} \quad \textbf{....... (3)}$$

which has $(p+q+2)$ unknown parameters: $\mu,\phi_1,...,\phi_p,\theta_1,...,\theta_q$ and σ_a^2 to be estimated from data series. where a_t is the error term which has $(i.i.d)N(0,\sigma_a^2)$ distribution.

Therefore, all models mentioned above are subset of ARMA models illustrated in equation [1].

$$w_t = \mu + \frac{\theta(B)}{\phi(B)} a_t \quad \textbf{.................................... (4)}$$

where

w_t=is the response series or a difference of the response series,

μ =a constant or intercept,

B=the backshift operator (i.e. $Bz_t=z_{t-1}$) ,

$\phi(B)$=the autoregressive operator, represent as a polynomial in the backshift operator :

$\phi(B) = 1 - \phi_1 B - ... - \phi_p B^p$

$\theta(B)$=the moving average operator, represent as a polynomial in the backshift operator :

$\theta(B) = 1 - \theta_1 B - ... - \theta_q B^q$

a_t=the random error or shock.

This paper interest on stationary ARMA(1,1) model if we suppose $\mu = 0$

$$w_t = \phi_1 w_{t-1} + a_t - \theta_1 a_{t-1} \quad \textbf{........... (5)}$$

or

$$(1-\phi_1 B) w_t = (1-\theta_1 B) a_t \quad (6)$$

$$\phi(B) w_t = \theta(B) a_t \quad \textbf{........................... (7)}$$

This model is stationary if the root of $(1-\phi_1 B)=0$ lies outside the unit circle and invertible if the root of $(1-\theta_1 B)=0$ lies outside the unit, so we can get that $|\phi_1|<1$, and $|\theta_1|<1$ too.

2.2 Estimation

Likilyhood function that is used to estimate model paramaters is formulated in eq (8), [9],

$$ln(L) = -\frac{n}{2}(2\pi\,\sigma_a) + \frac{1}{2} ln(|M^{(1,1)}|) - \frac{s(\phi_1,\theta_1)}{2\sigma_a^2} \quad \textbf{..... (8)}$$

$$S(\phi_1,\theta_1) = \sum_{t=-\infty}^{n} (a_t \mid \phi_1,\theta_1,w)^2 \quad \textbf{............ (9)}$$

where $S(\phi_1,\theta_1)$ is the sum squares errors, n is the sample size, and $[a_t|\phi_1,\theta_1,w] = E[a_t|\phi_1,\theta_1,w]$,denotes the expectation of a_t conditional on ϕ_1,θ_1 and w.With assumption $E(w_t)=0$, $\{a_t\}$ has the normal distribution with zero mean and constant variance equal to σ_a^2 [9].

2.3 B.J Forecasting Function

The B.j forecast function of z_{n+l} at the forecast origin n is given by the conditional expectation [1]

$$\hat{z}_n(l) = E(z_{n+l} \mid z_n, z_{n-1},...) \quad \textbf{.................... (10)}$$

We can easily obtain the actual forecasts of observed time series with ARMA(1,1) model as follows ,

$$z_{n+l} = \phi_1 z_{n+l-1} + a_{n+l} - \theta_1 a_{n+l-1} \quad \textbf{........ (11)}$$

$$\hat{z}_n(l) = \phi_1 \hat{z}_n(l-1)$$

$$= \phi_1^l z_n - \phi_1^{l-1}\theta_1 a_n, \text{for } l \ge 2 \quad \textbf{.... (12)}$$

$$\hat{z}_n(j) = z_{n+j}, \text{for } j < 0 \quad \textbf{........................... (13)}$$

$$\hat{a}_n(j) = 0, \text{for } j \ge 1, \quad \textbf{...................... (14)}$$

$$\hat{a}_n(j) = z_{n+j} - \hat{z}_{n+j-1}(1) = a_{n+j}, j \le 0 \quad \textbf{.......... (15)}$$

The error of forecasting equation is[1],

$$e_n(l) = z_{n+l} - \hat{z}_n(l) \quad \textbf{.............................. (16)}$$

we can see that $e_n(l)$ is a linear combination of the future random shocks entering the system after n.

Specifically, the one step ahead forecast error is $e_n(1) = z_{n+l} - \hat{z}_n(l) = a_{n+1}$, in general, [1]:

$$E(a_{n+j}) = \begin{cases} 0 & j > 0 \\ a_{n+j} & j \le 0 \end{cases} \quad \textbf{..................... (17)}$$

$$E(z_{t+j}) = \begin{cases} \hat{Z}_t(j) & J \ge 1 \\ Z_{t+j} & j \le 0 \end{cases} \quad \textbf{..................... (18)}$$

Forecast Mean Square Error (FMSE) is

$$FMSE = var(e_t(l)) = E[z_{t+l} - \hat{z}_t(l)]^2 \quad \textbf{.(19)}$$

$$FMSE(\hat{z}_n(l)) = (1 + \sum_{j=1}^{l-1}(\phi^{j-1}(\phi_1 - \theta_1))^2)\sigma_a^2 \quad \textbf{(20)}$$

for L=1,2,3 …..

3. Evolution Strategies ES $(\mu \overset{+}{,} \lambda)$

An individual $\vec{a} = (\vec{x}, \vec{\sigma}) \in I$ in $(\mu \overset{+}{,} \lambda)$-ES consists of the following components [4] [5]:

- $\vec{x} \in R^n$: The vector of object variables.
- $\vec{\sigma} \in R_+^{n\sigma}$: A vector of step length or standard deviations $(1 \le n_\sigma \le n)$ of the normal distribution. The strategy parameter $\vec{\sigma}$ (also called the internal model) determines the variances of the n-dimensional normal distribution, which is used for exploring the search space. The user of an evolution strategy, depending on his feeling about the degree of freedom required, can vary the amount of strategy parameters attached to an individual. As a rule of thumb, the global search reliability increased at the cost of computing time when the number of strategy parameters is increased. The only part of $\vec{a}$ entering the objective function evaluation is $\vec{x}$, and the fitness of an individual $\phi(\vec{a})$ is identical to its objective function value $f(\vec{x})$, i.e. $\phi(\vec{a}) = f(\vec{x})$ .Three ES operators are adopted in this work[7]. These are :

1- Arithmetic crossover: Each gen in the offspring will be a linear combination of the values in the ancestors' chromosomes, in the same positions. If a_i and b_i are the offspring's genes, and z_i and w_i the ancestors' ones ,at the position i then

$a_i = \lambda . z_i + (1 - \lambda) . w_i$

and

$b_i = \lambda . w_i + (1 - \lambda) . z_i$,

where λ is a random number with U(0,1),[7].

2-Gaussian perturbation: A mutation operator that adds ,to a given gene ,a value taken from a Gaussian distribution ,with zero mean small perturbations will be preferred over larger ones[7]. Mutation operator $m\{\tau_o, \tau\} : I \longrightarrow I$, is defined as follows

$$m_{\{\tau_0, \tau\}}(\vec{a}) = m_x(\vec{x}) \circ m_\sigma(\vec{\sigma}) = (\vec{x}', \vec{\sigma}') \quad \textbf{....(21)}$$

Which proceeds by:

- First, mutating the strategy parameters $\vec{\sigma}$:

$$m_\sigma : R_+^{n\sigma} \rightarrow R_+^{n\sigma}$$

$$m_\sigma(\vec{\sigma}) = \vec{\sigma}' = (\sigma_1 \exp(z_1 + z_0), \ldots, \sigma_{n_\sigma} \exp(z_{n_\sigma} + z_0)) \quad \textbf{...(22)}$$

where

$z_0 \sim N(0, \tau_0^2)$, $z_i \sim N(0, \tau^2)$ $\forall i \in \{1, \ldots, n_\sigma\}$

- Secondly, modifying $\vec{x}$ according to the new set of strategy parameters obtained from mutating $\vec{\sigma}$: $m_x : R^n \rightarrow R^n$

$$m_x(\vec{x}) = \vec{x}' = (x_1 + z_1, \ldots, x_n + z_n) \quad \textbf{.............(23)}$$

3- Selection operator: The selection operators on converting the fitness value into its ranking in the population .After the selecting scheme below is applied:

$S : I^{\mu+\lambda} \rightarrow I^\mu$

$S(P) = P'$,where $|P| = \mu + \lambda$, $|P'| = \mu$, and

$\forall \vec{a}' \in P' : \nexists \vec{a} \in P - P' : f(\vec{x}) \le f(\vec{x}')$

3. Conceptual Algorithm

The ES used in this work is given by following pseudo-code:

BEGIN

Population initialization and evolution
While (termination criteria is not met)
Creat new individuals using ES operators
Evaluate the new individuals (offspring)

Select the survivors and add to the next generation
END

Figure (2): ***Domnstrates forecasting ARMA model based on ES strategy.***

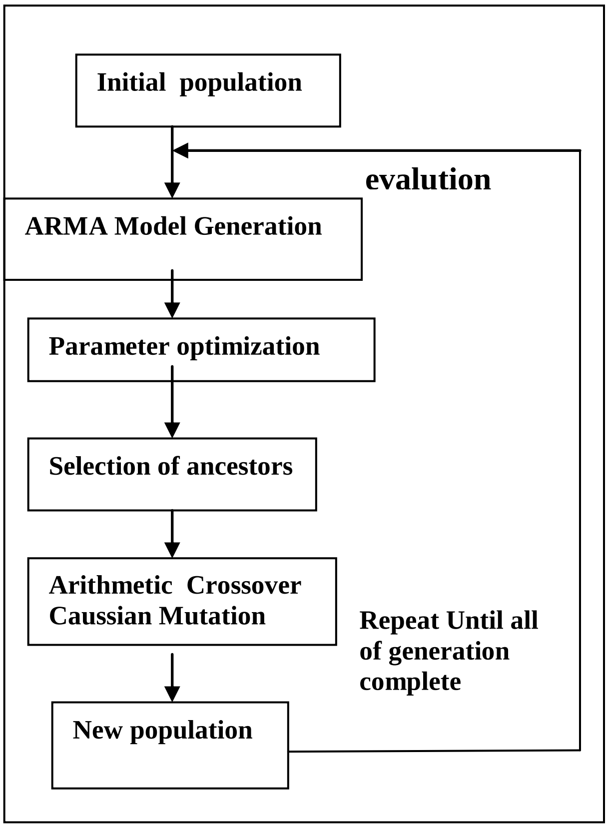

Figure 3: flowchart schematic representation of ES methodology for forecasting ARMA model

4. Experimental Results

This section explain the result that come from simulating ($\mu+\lambda$)-ES, B.J methodology to forecast an observation time series with ARMA (1,1) model. Experiment designing follows the steps of time series analysis which is given in figure (1) these steps passes the following three phases:

First phase: Adopted to generate different simples with different (simple size n) in this study take $n = 25,50,100$,and different values of model parameters ϕ set to $(\pm 0.05, \pm 0.3, \pm 75)$ and θ set to $(\pm 0.1, \pm 0.4, \pm .9)$ samples draw one with respect to eq(5). The random samples are generated by using Box-Muller formula.

Phase Two: Estimation of parameters get from optimizing likelihood function of model given by Eq.(8) by applying ES-algorithm for optimizing likelihood function for ϕ ,θ in search space [-1,1] to use it in next phase.

Phase Three: Here we applied our interested algorithm (ES) to forecast future value of time series observations for lead time(l =1,2,3) values of μ and λ are set to 15 and 100 respectively with the initial values for strategy parameters $\vec{\sigma}$, are set to 3.0 .

All results were obtained by running each one of these experiments 5 different runs and each iterates with 75 generations and averaging the resulting data. Further, the results of ($\mu+\lambda$)-ES are compared with those obtained by result of simulating experiments of BJ methodology (which designed for the same value of sample size (n) and model parameters (ϕ_1, θ_1) with 1000 runs. All test reported in this work conducted using programming environments developed in Matlab6.5. The comparisons make based on

1- Forecast Mean square error (FMSE):

In general from eq(20)

$$FMSE = var(e_t(l)) = E[z_{t+l} - \hat{z}_t(l)]^2$$

$$e_t = y_t - \hat{y}_t \quad \text{................(24)}$$

so

$$SSE = \sum_{i=t+1}^{t+l} e_i^2 \qquad FMSE = \frac{SSE}{L} \quad \text{......................(25)}$$

2- Theil's U-statistic

One measure that has these characteristics is the U-statistic developed by theil (1966) this statistic allows a relative comparison of formal forecasting methods The positive characteristic that given much more weight than small errors the positive characteristic that is given up to theil's –

statistic as measure of accuracy is that of intuitive interpreting .

The mathematical definition of this statistic is

$$Theil's\,U = \sqrt{\frac{\sum_{i=1}^{l}(FPE_{t+i} - APE_{t+i})}{\sum_{i=1}^{l}APE_{t+i}}} \quad \textbf{....(26)}$$

where

$$FPE_{t+i} = \frac{F_{t+it} - y_t}{y_t} \quad \text{(forecast relative change)}$$

$$APE_{t+i} = \frac{y_{t+i} - y_t}{y_t} \quad \text{(actual relative change)}$$

result of experiments cleared in table(1) which is used to show the comparison among B.J mythology and ES algorithm based on values of FMSE and u-statistic besides this tables (2-7) represent result of FMSE ,U-statistic and number of generation for different values of sample size and model parameters, in addition figures (4 -9)used to describe the relation for FMSE, sample size and number of generation. The experiments on a set of data give some impressions of the behaviors of both ES and BJ methods. As one can see that the MSEs ,u-statistic of ES are smaller than those of BJ. This indicates that ES is more reliable than BJ to give prediction of future values of observed time series. Moreover, one can see that values of FMSE ,U-statistic are decreased as the sample size increased. For ES one can also see that the values of FMSE, Thiels' (U- statistic) decreases when increasing the number of generation and sample size. Also the behavior of our test statistic decreases when decreases values of parameters (ϕ_1, θ_1) .

5.Conclusion

The surge of new bio-inspired optimization techniques such as ES, has created new exciting possibilities to the field of forecasting. Following such a trend, it is presented in this work a constructive approach to build time series forecasting models, assuming no prior knowledge about the behavior of evolving time f ARMA models . Furthermore, the systems that are generated work autonomously and do not require any kind of statistical data analysis. The main handicap is the computational complexity of the proposed approach. Nevertheless, time complexity could be reduced if a subset of promising models were incorporated into the ES's initial population, although this would require the use of a priori information. Since most of the real-world time series use daily or monthly data, this is not considered a major concern. In future work it is intended to enrich the GA forecasting models with the integration of nonlinear functions (e.g., logarithmic or trigonometric). Another area of interest may rely on the application of similar techniques to long term and multivariate forecasting. Once the ESs revealed good results in parameter optimization and model selection and forecasting .

6. References

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Table 1: Values of FMSE and Thiels' (U-statistic) for comparison between ES and BJ methodology averaging for 1000 run of BJ methodology experiments ,and (averaging Es experiments for 75 generation).

n	ϕ_1,θ_1	ES-forecasting		BJ. forecasting	
		FMSE	u-statistic	FMSE	u-statistic
25	-.05,-.1	0.0059	0.039	1.1898	0.6428
	-.3,-.4	0.01509	0.05431	2.6585	0.7247
	-.75,-.9	0.02401	0.12491	10.907	3.2776
	.05,.1	0.002403	0.03018	0.8828	0.6028
	.3,.4	0.06196	0.16543	0.4674	0.3862
	.75,.9	1.01948	0.626103	0.0245	0.0839
50	-.05,-.1	0.0058	0.04	1.241	0.6253
	-.3,-.4	0.01757	0.10778	2.4535	0.7163
	-.75,-.9	0.03057	0.12027	5.2.104	2.3942
	.05,.1	0.00152	0.01620	0.8356	0.5498
	.3,.4	0.04518	0.0917	0.3941	0.378
	.75,.9	0.736786	0.37326	0.0279	0.0748
100	-.05,-.1	0.0017	0.019	1.1877	0.6072
	-.3,-.4	0.008431	0.06312	2.3676	0.715
	-.75,-.9	0.05512	0.15500	3.7134	1.489
	.05,.1	0.00075	0.03853	0.7794	0.46062
	.3,.4	0.02002	0.05286	0.3597	0.35078
	.75,.9	0.81287	0.627722	0.0359	0.7106

Table (2): Values of FMSE and Thiels'(U-statistic) based on sample size and no.of generation. $\phi_1 = .3, \theta_1 = .4$

No. of gen	n=25		n=50		n=100	
	FMSE	u-statistic	FMSE	u-statistic	FMSE	u-statistic
10	0.09228	0.21504	0.06278	0.09946	0.02474	0.06134
20	0.08982	0.21242	0.0421	0.09894	0.0153	0.0458
30	0.06166	0.17372	0.042	0.09696	0.01254	0.03444
40	0.06028	0.17336	0.03838	0.09382	0.0121	0.0303
50	0.06002	0.16756	0.0325	0.09272	0.0112	0.02274
60	0.04848	0.10704	0.0312	0.0925	0.0038	0.02264
70	0.04816	0.107042	0.031194	0.0875	0.00298	0.00728

المستخلص

اولى الباحثون اهمية خاصة لدراسة وتحليل السلاسل الزمنية مستخدمين كافة الاساليب الرياضية والاحتمالية والبيانية والحاسوبية المتاحة لهذا الغرض وذلك لكون السلسلة الزمنية والتي تعرف بانها مجموعة من المشاهدات المعتمدة والمولدة بالتعاقب عل وفق ترتيب (تسلسل) زمني معين تدخل في دراسة وتحليل العديد من الظواهر الاقتصادية والاجتماعية والهندسية. تهدف هذه الدراسة لمحاكاة خوارزمية التطور $ES^{(\mu+\lambda)}$ لحساب التكهن لسلسة الزمنية تتبع مشاهداتها النموذج العشوائي ARMA(1,1) . حساب التنبوء المستقبلي لبيانات السلسلة الزمنية باعتماد خوارزمية $ES^{(\mu+\lambda)}$ يمر بمراحلها الثلاثة (crossover, mutation, selction) محاكاة خوارزمية ES تمت من خلال تصميم عدد من التجارب وبالاعتماد على استخدام قيم مختلفة لكل من حجم العينة (n=25,50,10) و معلمتي النموذج $\theta_1 = \pm 0.05, \pm 0.3, \pm 75$),($\phi_1 = \pm 0.1, \pm 0.4, \pm .9$) بالاضافة (Lead time l =1,2,3) وقيم $(\mu,\lambda = 100,15)$. ان نتائج هذه التجارب تمت مقارنتها مع النتائج التي تم الحصول عليها من تصميم تجربة اخرى وذلك لدراسة التكهن المستقبلي للسلسلة الزمنية بالاعتماد على واحدة من الاساليب التقليدية والتي تعرف باسم (بوكس جنكنز) حيث استخدمت نفس حجوم العينة وقيم المعلمات بالاضافة الى (عدد مرات تكرار التجربة مساوي (RR=1000) .لخصت النتائج في عدد من الجداول والتي تمثلت في الجدول رقم (1) والذي يوضح المقارنة بين خوارزمية $ES^{(\mu+\lambda)}$ وطريقة Box –Jenkins بالاعتماد على كل من اصائية ثايل Thiels' (U- statistic)ومتوسط مربعات خطا التكهن FMSE اثبتت النتائج قدرة خوارزميةُ 'ES في حساب التكهن المستقبلي وذلك من خلال استحصالها القيم الجيدة لكل من احصائية FMSE, Thiels' . (FMSE) اما الجداول المرقمة (2-7) فتمثل العلاقة بين كل من القيم الاحصائية لكل من (FMSE) واحصائية ثايل وحجم العينة وقيم المعلمات بالاضافة الى رقم الولادة (generation) حيث ان عدد الولادات المستخدم يساوي (gen=70) بالاضافة الى الاشكال التوضيحية(4-9) والتي استخدمت لتوضيح قيم FMSEمع حجم العينة ورقم الولادة . اثبتت خوارزمية (ES) قدرتها في حساب المخمنات لمعلمتي النموذج وذلك من خلال ايجاد الحل الامثل لدالة الامكان الاعظم للنموذج المستخدم [9].

Table (3): Values of MSE and Thiels' (U-statistic) based on sample size and no.of generation for $\phi_1 = -.3, \theta_1 = -.4$

No. of gen	n=25		n=50		n=100	
	FMSE	u-statistic	FMSE	u-statistic	FMSE	u-statistic
10	0.02756	0.06552	0.018227	0.0592	0.0123	0.0585
20	0.01582	0.05838	0.013776	0.0522	0.00992	0.0401
30	0.01494	0.0475	0.013634	0.0442	0.00658	0.03518
40	0.0141	0.04742	0.01358	0.0366	0.00632	0.0236
50	0.01406	0.04008	0.011575	0.010162	0.0062	0.010312
60	0.01401	0.0382	0.01051	0.010082	0.00619	0.010298
70	0.01208	0.03595	0.01028	0.010078	0.006094	0.010238

Table (4): Values of MSE and Thiels' (U-statistic) based on sample size and no.of generation for $\phi_1 = .75, \theta_1 = .9$.

No. of gen	n=25		n=50		n=100	
	FMSE	u-statistic	FMSE	u-statistic	FMSE	u-statistic
10	1.2625	0.71054	1.25268	0.51578	0.91014	0.57292
20	1.05376	0.65296	0.94606	0.4385	0.8225	0.51856
30	1.06.96	0.63244	0.92468	0.41988	0.78756	0.5128
40	1.0546	0.63004	08.54548	0.32106	0.78092	0.30432
50	1.00404	0.59538	0.853586	0.3101	0.73495	0.20431
60	0.94426	0.50624	0.7766	0.28958	0.71244	0.16276
70	0.88028	0.50152	0.7508	0.28792	0.71182	0.11284

Table (5): Values of MSE and Thiels' (U-statistic) based on sample size and no.of generation for $\phi_1 = -.75, \theta_1 = -.9$.

No. of gen	n=25		n=50		n=100	
	FMSE	u-statistic	FMSE	u-statistic	FMSE	u-statistic
10	0.03142	0.1348	0.03072	0.13136	0.011584	0.121912
20	0.02254	0.11958	0.02034	0.12044	0.0106	0.10528
30	0.0223	0.11448	0.02011	0.11404	0.010126	0.10411
40	0.01898	0.11272	0.01562	0.1136	0.01102	0.10218
50	0.01246	0.112336	0.01026	0.1120	0.001088	0.013864
60	0.00858	0.1103	0.002048	0.1099	0.001022	0.034926
70	0.00318	0.10536	0.00202	0.10736	0.001012	0013424

Table (6) : Values of MSE and Thiels' (U-statistic) based on sample size and no.of generation for $\phi_1 = 0.5, \theta_1 = 0.1$.

No. of gen	n=25		n=50		n=100	
	FMSE	u-statistic	FMSE	u-statistic	FMSE	u-statistic
10	.00892	0.05932	0.007972	0.028461	0.00182	0.01942
20	0.00512	0.03874	0.003346	0.025986	0.00032	0.0102
30	0.0046	0.02862	0.002733	0.023767	0.0004	0.00884
40	0.0044	0.02462	0.001644	0.016895	0.0004	0.00884
50	0.0042	0.02158	0.00135	0.00629	0.0004	0.00066
60	0.00414	0.01924	0.001225	0.0006	0.000021	0.00019
70	0.00272	0.01918	0.00025	0.0059	0.0002	0.00012

Table(7): Values of MSE and Thiels' (U-statistic) based $\phi_1 = -05, \theta_1 = -0.1$ on sample size and no.of generation for

No. of gen	n=25		n=50		n=100	
	FMSE	u-statistic	FMSE	u-statistic	FMSE	u-statistic
10	0.0075	0.048	0.0065	0.043	0.0009	0.0148
20	0.0068	0.035	0.0061	0.038	0.0008	0.0145
30	0.0063	0.032	0.0055	0.038	0.0005	0.0122
40	0.0064	0.03	0.0034	0.038	0.00029	0.0128
50	0.0052	0.028	0.003018	0.032	0.00024	0.01166
60	0.0052	0.026	0.002566	0.029	0.000229	0.0109
70	0.0046	0.021	0.00228	0.024	0.000102	0.01012

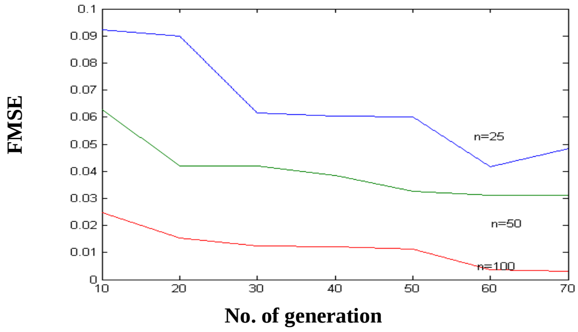

Figure 4: Relation among FMSE , sample size and no.of generation for $\phi_1 = .3, \theta_1 = .4$

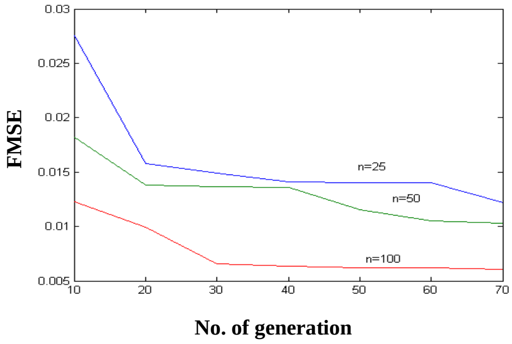

Figure 5: Relation among FMSE , sample size and no.of generation for $\phi_1 = -.3, \theta_1 = -.4$

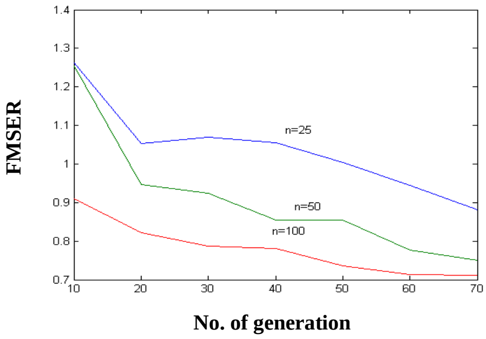

Figure 6: Relation among FMSE , sample size and no.of generation for $\phi_1 = 0.75, \theta_1 = 0.9$

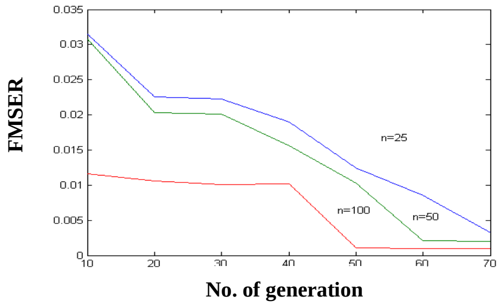

Figure 7: Relation among FMSE , sample size and no.of generation for $\phi_1 = -0.75, \theta_1 = -0.9$

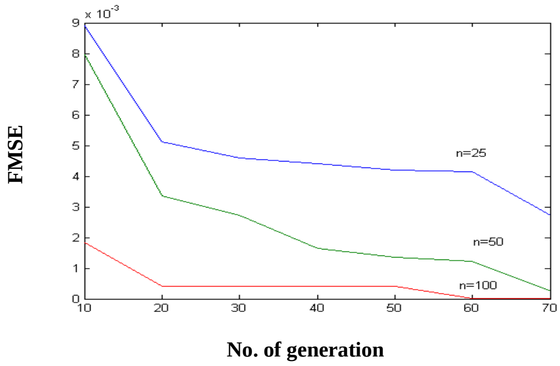

Figure 8: Relation among FMSE , sample size and no.of generation for $\phi_1 = 0.5, \theta_1 = 0.1$

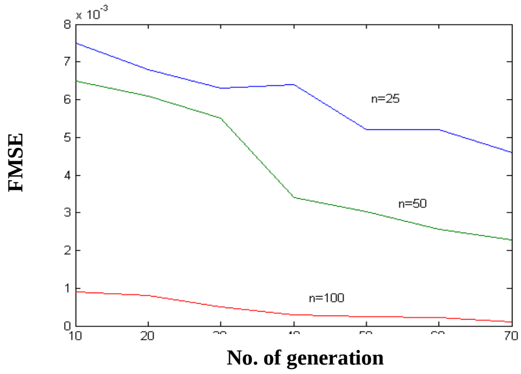

Figure 9: Relation among FMSE , sample size and no.of generation for $\phi_1 = -0.5, \theta_1 = -0.1$