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Escape Chaotic Dimension using Iterated Function System that are not Based on Code Space Method

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Abstract

In this paper some IFS attractors are defined using the Escape Time Algorithm applied to certain dynamical system. This algorithm is used to compare how fast different points in the window $\boldsymbol{\mathcal{W}}$ escape to the region $\boldsymbol{\mathcal{V}}$ under the action of a dynamical system. The chaotic behavior of these attractors have been shown in a new way not based on code space method and a new approach for computing fractal dimension of these chaotic attractors are given which we called escape chaotic dimension. Two algorithms are proposed to find the attractors based on Escape Time Algorithm, and to find its fractal dimensions, they are carried out using Delphi Programming language.

Keywords:Fractal dimension, Iterated Function System (IFS), dynamical system, chaotic dynamical system, Escape Time Algorithm (ETA).

Introduction (Brief history of chaos and fractals)

The story began three hundred years ago with Isaac Newton, when he discovered the low of gravity and invented differential equations. Equipped with calculus and laws of motion of the earth around the sun, he was solved the two-body problem, afterwards, people started to ask a natural question: how about three-body problem? For instance, the sun-earth-moon-system?

It was quickly realized that the three-body-problem was much harder to solve. In fact, it was impossible to obtain the solutions in explicit formulas. The situation seemed hopeless. Then two hundred years passed around 1900 when Henri Poincare came along. Instead of attempting to give explicit solution formulas, he focused on qualitative behavior of the problem. Specifically, he asked the following question: what happen to the solutions as time becomes large? He realized that the long-time solution behavior was extremely complicated. In his words, "One will be struck by the complexity of this figure which I do not even attempt to draw. Nothing more properly gives us an idea of complication of the problem of three bodies and, in general, of all the problems in dynamics where there is no uniform integral" [1]. This is the first encounter with chaos. Poincare's work did not receive much attention until 1960's, when Edward Lorenz (1963) published his celebrated paper on the chaotic motion in a simplified model for weather forecast [2]. Lorenz found in his model that small errors in the initial conditions can lead to huge differences in the later motion. This is now called sensitive dependence on initial conditions and is trademark of chaos. This behavior practically makes long-time weather forecasting impossible. But Lorenz also showed that there was order in chaos. When he plotted the solution in three dimensions, he obtained a beautiful butterfly pattern. He further argued that this butterfly had to have infinite layers, a structure now called a fractal [3]. At about the same time, the fundamental work by Stephen Smale shed much light on the understanding of chaotic motion in dynamical systems [4]. Finally, a chaotic dynamical system has been recognized by Sharkovsky [5], Li and Yorke[6], and many others that there is a hidden, self- organizing order in chaotic system.

In 1980's, the chaos theory and fractal geometry was applied to practically many fields of science such as physics, chemistry, biology and so on. An important feature of fractal geometry is that it enables a characterization of irregularity at different scales that the classical Euclidean geometry does not allow for. As a result, many fractal features have been identified, among which the fractal dimension is one of the most important[7]. We will build the theory of fractal dimension from the basic Euclidean definition to the more mathematically exhaustive definitions of Hausdorff and Box-counting dimensions[8]. One might ask why there are several different definitions of dimension. This is simply because a certain definition might be useful for one purpose, but not for another. In many cases the definitions are equivalent, but when they are not, that because of their particular properties that make them more suitable for the task at hand [9].

The paper is organized as follows. In the next section, some background material is included to assist readers less familiar with the detailed to consider. Section 2 is devoted to a contrasting dynamical action-point whose iterates separate from one another. This kind of behavior is symptomatic of what we call chaotic dynamics, or just plain chaos. In Section 3 the proposed method to compute the escape chaotic dimension of some attractors constructed by IFS are presented, with the algorithm for compute escape chaotic dimension based on Box Counting Theorem. Finally Conclusions are drawn in Section4.

1. Theoretical Background

This section is presented an overview of the major concepts and results of fractals and the Iterated Function System (IFS). The theory of fractal sets is a modern domain of research. Iterated function systems have been used to define fractals. Such systems consist of sets of equations, which represent a rotation, a translation, and a scaling. These equations can generate complicated fractal images [10].

An IFS is defined through a finite set of contractive functions $f_i : X \to X$, or short: IFS$\{X; f_1, f_2,..., f_N\}$, where (X, d) is a complete metric space and $d : X \times X \to R$ is a distance function. The functions f_i are called contractive if and only if there exist a so-called contractivity factor $s_i \in [0,1)$ with $d(f_i(x), f_i(y)) \le s_i.d(x, y)$ for all $x, y \in X$ [10]. Then a complete metric space (H(X),h) based on (X, d) is defined, where H(X) consist of nonempty compact subset of X and h(X) denotes the Hausdorff metric which describe the 'similarity' of two sets, given by: $h(A,B) = \max(\max_{x\in A} \min_{y\in B} d(x, y), \max_{y\in B} \min_{x\in A} d(x, y))$ [9].

The functions $f_i : X \to X$ can be extended to functions $F : H(X) \to H(X)$, defined by $F(A) = \bigcup_{i=1}^{N} f_i(A), A \in H(X)$, being the so-called Hutchinson Operator. The contractivity factor s of the Hutchinson Operator is given by $s = \max\{s_1,..., s_N\}$. Due to the contractive property there exists a unique set A_o with $A_\circ = F(A_\circ)$, A_o is called the attractor of the IFS, and it satisfies $A_\circ = \lim_{n\to\infty} F^{\circ n}(A), \forall A \in H(X)$, where $F^{\circ n}$ denotes the n-fold composition of F (or forward iterates of F). This attractor set A_o is what we call a fractal. The fractal itself is the limit as the number of iterations approaches infinity [10].

A dynamical system is sources of deterministic fractals, the reasons for this are deeply intertwined with IFS theory. In the following, the idea of a dynamical system and some of the associated terminology are introduced [11].

Definition1, [12]:

A *dynamical system* is a transformation *f: X* $\longrightarrow$ *X* on a metric space *(X, d)*. It is denoted by *{X; f}*.The *orbit* of a point $x \in X$ is the sequence $\left\{f^{\circ n}(x)\right\}_{n=0}^{\infty}$.

Theorem1, [11]:

Let $\{X; f_1, f_2,..., f_N\}$ be an IFS. The dynamical system *{H(X); W}*, possesses a unique fixed point $A \in H(X)$, where $W(A) = \bigcup_{n=1}^{N} f_n(A)$, for all $A \in H(X)$, is a contraction mapping on the complete metric space *(H(X), h(d))* with contractivity factor $0 \le s < 1$, such that

$h(W(A), W(B)) \le s.\, h(A, B)$, for all $A, B \in H(X)$. The attractor of IFS is an attractor fixed point of the dynamical system *{H(X); W}*.

Theorem2, [11]:

Let $\{X, \omega_1, \omega_2, \cdots, \omega_n\}$ be an IFS with attractor *A*. The IFS is totally disconnected if and only if $\omega_i(A) \cap \omega_j(A) = \phi$, for all $i, j \in \{1,2,..., N\}$ with $i \ne j$.

Definition2, [11]:

Let *{X; f_n, n = 1, 2,...,N}* be totally disconnected IFS with attractor *A*. The associated shift transformation on A is the transformation *S: A* $\longrightarrow$ *A*, defined by, $S(a) = f_n^{-1}(a)$ for $a \in f_n(A)$, Where f_n is viewed as transformation on A. The dynamical system $\{A; S\}$ is called the shift dynamical system associated with the IFS.

Theorem3, [14]:

(The Box-Counting Theorem**)** Let $A \in H(R^m)$, where the Euclidean metric is used. Cover R^m by closed square boxes of side length $(1/2^n)$. Let $N_n(A)$ denote the number of boxes of side length $(1/2^n)$ which intersect the attractor. If $D = \lim_{n\to\infty}\left\{\frac{\ln(N_n(A))}{\ln(2^n)}\right\}$, then A has fractal dimension D.

2. The Chaotic Dynamical system based on Escape Time Algorithm.

In this section we introduce two methods of describing the way in which iterates of neighboring points separate from one another: sensitive dependence on initial condition, and the Lyapunov exponent. These notions are fundamental to the concept of chaos [15].

2.1 Chaotic Dynamical systems

The word "chaos" is familiar in every day speech. It is normally means a lack of order or predictability. Thus one says the weather is chaotic, or that rising particles of smoke are chaotic, or that the stock market is chaotic. Both sensitive dependence on initial condition and the Lyapunove exponent qualify as measure of unpredictability [16].

Before giving a definition of chaos we first present some background material.

Definition3, [12]:

The dynamical system $\{X, f\}$ is sensitive to initial conditions if there exists $\delta > 0$ such that, for any $x \in X$ and any ball $B(x,\varepsilon)$ with radius $\varepsilon > 0$, there is $y \in B(x,\varepsilon)$ and an integer $n \geq 0$ such that $d(f^{\circ n}(x), f^{\circ n}(y)) > \delta$. Roughly, orbits that begin close together get pushed apart by the action of the dynamical system.

Now, we define the Lyapunov exponents for initial condition x_o, as the following:

Definition4, [15]:

Let f be continuously differentiable map on R^m, and the Jacobian of the n-th iterate of f at x_o denoted by: $J_n = Df^{\circ n}(x_o)$ and let $j_1(n) \geq j_2(n) \geq \ldots \geq j_m(n)$, be the magnitudes of the eigenvalues of J_n. The m-th Lyapunov number of x_o can be defined as:

$\lambda_i = exp(h_i) = \lim_{n\to\infty} [j_i(n)]^{1/n}$, i = 1, 2, …, m.

Where λ_i are the Lyapunov exponents of f.

A certain degree of order in chaotic system has led to various definition of chaos in literature. In this paper, we are going to adopt Gulick definition of chaos.

Definition5, [15]:

A dynamical system $\{X, f\}$ is chaotic if it satisfies at least one of the following conditions:

(1)It is sensitive to initial conditions;

(2) It has positive Lyapunov exponent at each point in its domain that is not eventually periodic.

2.2 Escape Time Algorithm of a shift dynamical system associated with IFS

The Escape Time Algorithm (ETA) can be applied to any dynamical system of the form $\{R^2; f\}$, $\{C; f\}$. We need only to specify a viewing window $\mathcal{W}$ and a region $\mathcal{V}$, to which orbits of points in $\mathcal{W}$ might escape. The result will be the "attractor" A of $\mathcal{W}$, wherein the pixel corresponding to the point z is colored according to the smallest value of the positive integer n, such that $f^{\circ n}(z) \in \mathcal{V}$. A special color, such as black, may be used to represent points whose orbits do not reach $\mathcal{V}$ before (n+1) iterations [11].

The relationship between the dynamical system $\{R^2, f\}$ and the IFS $\{R^2; \omega_1, \omega_2, ..., \omega_n\}$ is that $\{A,f\}$ is a shift dynamical system associated with IFS.

In the following is an algorithm for computing attractors of some IFS on R^2.

(i) Find a dynamical system $\{R^2, f\}$ which is an extension of a shift dynamical system associated with the IFS.

(ii) Apply the Escape Time Algorithm, with $\mathcal{V}$ and $\mathcal{W}$ chosen appropriately, but plot only those points whose numerical orbits require sufficiently much iteration before they reach $\mathcal{V}$ white color and plotting those points whose orbit does not reach $\mathcal{V}$ by black color.

The Escape Time Algorithm[11]

1- Given $\mathcal{W} \subseteq R^2$, such that $\mathcal{W}=\{(x,y)\in R^2 : a\le x\le c,\ b\le y\le d\}$, the array of points in $\mathcal{W}$ is defined by, $x_{p,q}=(a+p(c-a)/M\ ,\ b+q(d-b)/M)$, $p,q=1,2,\ldots,M$, for any +ve integer M.

2- Let $\mathcal{C}$ be a circle centered at the origin, the set $\mathcal{V}$ is defined such that, $\mathcal{V} = \{(x, y) \in R^2: x^2 + y^2 > r\}$,
where r is sufficiently large number.

3- Let f be a function with the orbit of point $\{f^{\circ n}(x_{p,q})\}_{n=0}^{\infty}$, where $x_{p,q}\in\mathcal{W}$.

4-Repeat, $\forall\ x_{p,q}\in\mathcal{W}$.

IF $\{f^{\circ n}(x_{p,q})\}_{n=0}^{\infty}\subseteq\mathcal{V}$, then $x_{p,q}$ is colored with color indexed by n.

Else it is colored **Black**.

End IF.

5-Change all colors except black color to be white, such that:

$$x_{p,q}=\begin{cases} White & if\ \ f^{\circ n}(x_{p,q})\in V\ \ for\, some\, n\le N \\ Black & if\ \ f^{\circ n}(x_{p,q})\notin V\ \ for\, all\, n\le N \end{cases}$$

6-Then the set of escape time point A in $\mathcal{W}$ is defined as follows,

A= $\{x_{p,q} \in \mathcal{W}: f^{\circ n}(x_{p,q}) \notin \mathcal{V}$, for all $n \le N\}$
$= \{x_{p,q} \in \mathcal{W}$, such that $x_{p,q}$ is black point$\}$.

The set A is called the attractor constructed by Escape Time Algorithm.
Now we will introduce the generalization Escape Time algorithm written in Delphi for computing attractors of IFS $\{R^2;\omega_1,\omega_2,...,\omega_n\}$.

General Escape Time Algorithm for Computing aAttractors of IFS in R^2

INPUT: **READ** m, v{ m = number of regions, v=value }
FOR i=1 **TO** m
READ ω_i^{-1} (x,y) = (g_i(x),h_i(y))
ENDFOR {i}
READ a, b, c, d, R, M, numits{numits=number of iterations}
PROCESS: **FOR** p=1 **TO** M
FOR q=1 **TO** M
x = a + (c-a)*p/M
y = b + (d-b)*q/M
FOR n=1 **TO** numits
IF (x {≤,≥,<,>}v) and (y{≤,≥,<,>}v) **THEN**
x = g_i(x), y = h_i(y)
ENDIF
IF x*x + y*y > R **THEN**
Graph (p, q), Color (n), n = numits
ENDFOR {n}
ENDFOR {q}
ENDFOR {p}
OUTPUT: GRAPH of m Regions
END.

2.3 The Escape Chaotic Dynamical System no Based on Code Space.

The following theorem was proved before based on code space method in [3, 11, 12]. Using ETA, a new proves for this theorem is proposed as follows.

Theorem 4:

The shift dynamical system associated with a totally disconnected IFS of two or more transformations is chaotic.

Proof:

Let $\{R^2,\omega_1,\omega_2,\cdots,\omega_n\}$ be a totally disconnected IFS in R^2, and A be the attractor of the IFS. Consider the dynamical system $\{R^2, f\}$ where $f: R^2\to R^2$ defined by,

$$f(x,y)=\begin{Bmatrix}\omega_1^{-1}(x,y)\\ \omega_2^{-1}(x,y)\\ \\ \omega_n^{-1}(x,y)\end{Bmatrix}$$

This dynamical system is related to the IFS $\{R^2,\omega_1,\omega_2,\cdots,\omega_n\}$.

The relationship between the dynamical system $\{R^2, f\}$ and the IFS is that $\{A, f\}$ is a shift dynamical system associated with IFS. $f: A\to A$. The dynamical system $\{R^2, f\}$ is an extension of the shift dynamical system$\{A, f\}$. Now we apply the Escape Time Algorithm with $\mathcal{V}$ and $\mathcal{W}$ chosen appropriately as in section 2.2. The points whose numerical orbits require sufficiently many iterations before they reach $\mathcal{V}$ are plotted with white color, and the points whose orbit does not reach $\mathcal{V}$ are plotted black color.
Then the set of escape time point A in $\mathcal{W}$ is defined as follows,

A= $\{(x,y) \in \mathcal{W}: f^{\circ n}(x,y) \notin \mathcal{V}$, for all $n \le N\}$
$= \{(x,y) \in \mathcal{W}$, such that (x,y) is black point$\}$.

Now, to show that the attractor A which is constructed by ETA is chaotic, we have to prove that the shift dynamical system $\{A, f\}$ is sensitive to initial conditions.

Since the attractor are dense set therefore in every neighborhood of each black point in the attractor set A there are white points not belong to A (belong to $\mathcal{V}$) whose orbits escape to infinity. So consider two nearby initial points $(x_\circ, y_\circ)$ and (x_1, y_1) one in attractor A and the other one outside of A and, then as the iteration proceeds, when we take $\varepsilon > 0$ and N is large enough one orbit remains in $\mathcal{W}$ (don't belong to $\mathcal{V}$) forever, while the other one escape to $\mathcal{V}$.

That is,

$d(f^{\circ n}(x_o, y_o), f^{\circ n}(x_1, y_1)) > \delta$, where one point is black, while the other one is white for some $n \le N$. In other words, small differences in the initial conditions will lead to vastly different orbits later on. This is the so –called sensitive dependence on initial conditions. So small initial distance between the two orbits is amplified rapidly by the iteration. Therefore the shift dynamical system $\{A, f\}$ is chaotic.

3. Escape Chaotic Dimension using IFS Attractors.

Fractal geometry has various approaches to compute the fractal dimension of an object. These approaches can be classified as belonging to the Hausdorff-Besicovitch Dimension (like the Box-Counting and Dividers methods) or to the Bouligand-Minkowski Dimension (Minkowski fractal dimension method). Fractal dimension D is a key quantity in fractal geometry. The D value can be a non-integer and can be used as an indicator of the complexity of the curves and surfaces [16].

3.1 Escape Chaotic Dimension Algorithm

A new Algorithm carried out using Delphi for computing the fractal dimension of chaotic attractors constructed by IFS is based on finding the shift dynamical system associated with IFS $\{R^2; \omega_1, \omega_2, ..., \omega_n\}$.

We call this fractal dimension by escape chaotic dimension. A new method is based on Box-Counting theorem 3. A new technique that counts the fixed black points in the window $\mathcal{W}$ that not escapes to the region $\mathcal{V}$ is applied. It is different from the original technique that is based on counting of boxes of side length $(1/2^n)$ which intersect the attractor.

<u>Escape chaotic Dimension Algorithm</u>

```
INPUT:          READ m , v
{ m = number of regions, v=value }
 FOR i=1 TO m
 READ ωi^-1 (x,y) = (gi(x),hi(y))
ENDFOR {i}
READ a , b , c , d , R, M , numits   {numits=number of iterations}
A = 0              {the black points}
PROCESS: FOR p=1 TO M
 FOR q=1 TO M
 x = a + (c-a)*p/M
y = b + (d-b)*q/M
 FOR n=1 TO numits
IF (x {≤,≥,<,>}v) and (y{≤,≥,<,>}v)
THEN   x = gi(x) , y = hi(y)
 ENDIF
 IF SQRT(x^2+ y^2) < R THEN A=A+1
ENDFOR {n}
ENDFOR {q}
ENDFOR {p}
D = ln (A) / (numits*ln (2))
OUTPUT: Dimension is: D
END.
```

<u>3.2 Examples</u>

In the following examples the Escape chaotic dimension of some IFS attractors are applied to certain chaotic dynamical system. Calculated using the Escape Time methode.

Example1.

Consider the IFS $\{R^2, \omega_1, \omega_2\}$, where

$\omega_1(x, y)$=1/3(x, y) and

$\omega_2(x, y)$=1/3(x, y) + 2/3(100, 0).

The attractor of these IFS is Cantor set C. We find the dynamical system that is related to $\{R^2; \omega_1, \omega_2\}$. The inverse transformations are

$\omega_1^{-1}(x, y) = (3x, 3y)$ if $x \le ½$,

$\omega_2^{-1}(x, y) = (3x - 200, 3y)$ if $x > 1/2$.

Define f: $R^2 \longrightarrow R^2$, by

f (x,y) =

$$\begin{cases} \omega_1^{-1}(x, y) = (3x, 3y), & if \;\; x \le 1/2 \\ \omega_2^{-1}(x, y) = (3x - 200, 3y), & if \;\; x > 1/2. \end{cases}$$

We observe that the IFS is totally disconnected by theorem (2). $\{C, f\}$ is a shift dynamical system associated with IFS. The dynamical system $\{R^2; f\}$ is related to the IFS $\{R^2; \omega_1, \omega_2\}$ and it is an extension of the shift dynamical system {C; f}. By applying the general Escape Time Algorithm to this dynamical system $\{R^2; f\}$ with

$\mathcal{W}$={ $(x, y) \in R^2$ *:a≤x≤c, b≤y≤d*}, where(a,b)=(0,0) and (c,d)=(100,100) and $\mathcal{V}$ = {(*x, y*) ∈ R^2: $x^2 + y^2 > 200$}, f : $R^2 \longrightarrow R^2$, and *N* is large enough we get the fractal set C constructed by this algorithm defined by:

C = { (x, y) ∈ $\mathcal{W}$: $f^{\circ n}(x,y) \notin \mathcal{V}$, for all n ≤ N}. That is, the black point represents the Cantor set C.

By theorem (4) the shift dynamical system {C; f} which associated with totally disconnected IFS {R^2; ω_1, ω_2} is chaotic. In other word we can say in the neighborhood of each black point in C, there are white points does not belong to C (belong to $\mathcal{V}$) whose orbits escape to infinity.

By applying the Escape chaotic dimension algorithm we have D(C) =0.63.

Example2.

Consider the IFS {R^2; ω_1, ω_2, ω_3}, Where

$\omega_1(x,y) = 1/2(x,y) + 1/2(0,100)$,

$\omega_2(x,y) = 1/2(x,y) + 1/2(100,0)$ and

$\omega_3(x,y) = 1/2(x, y)$.

The attractor of these IFS is a Sierpinski triangle S.

The inverse transformation of ω_1, ω_2, and ω_3 are:

$\omega_1^{-1}(x,y) = (2x, 2y-100)$ *if* y > 1/2,

$\omega_2^{-1}(x,y) = (2x-100, 2y)$ *if* x > ½ and $y \le 1/2$

$\omega_3^{-1}(x,y) = (2x, 2y)$ *,otherwise*

Define f(x, y) =

$$\begin{cases} \omega_1^{-1}(x,y) = (2x,2y-100) \ \ if \ \ y>1/2, \\ \omega_2^{-1}(x,y) = (2x-100,2y) \ \ if \ \ x>½ \text{ and } y \le 1/2 \\ \omega_3^{-1}(x,y) = (2x,2y) \ \ ,otherwise \end{cases}$$

We observe that the IFS are totally disconnected by theorem (2). $\{S, f\}$ is a shift dynamical system associated with IFS. The dynamical system {R^2; f} where f : $R^2 \longrightarrow R^2$ is related to the IFS {R^2; $\omega_1, \omega_2, \omega_3$} is an extension of the shift dynamical system {S; f}.

By applying the general ETA to this dynamical system {R^2; f} with $\mathcal{W}$={ $(x, y) \in R^2$ *:a≤x≤c, b≤y≤d*}, where (a,b)=(0,0) and (c,d)=(100,100) and $\mathcal{V}$ = {(*x, y*) ∈ R^2: $x^2 + y^2 > 200$}, f : $R^2 \longrightarrow R^2$, and by taking *N* large enough we get the fractal set S constructed by this algorithm, defined by:

S = { $(x, y) \in \mathcal{W}$: $f^{\circ n}(x,y) \notin \mathcal{V}$, for all n ≤ N}.That is, the black point represents Sierpinski triangle.

The IFS {$R^2; \omega_1, \omega_2, \omega_3$} is chaotic by theorem (4), and by applying the Escape chaotic dimension algorithm we find D(S) =1.58.

Example3.

Consider the IFS {R^2; ω_1, ω_2, ω_3, ω_4}, Where

$\omega_1(x,y)$ =1/2*(x,y)*

$\omega_2(x,y) = 1/2(x,y) + 1/2(100,0)$,

$\omega_3(x,y) = 1/3(x,y) + 2/3(0,100)$ and

$\omega_4(x,y) = 1/4(x,y) + 3/4(100,100)$

The attractor of these IFS is attractor fractal tree *F*. The inverse transformation of ω_1, ω_2, ω_3 and ω_4 are:

$\omega_1^{-1}(x,y) = (2x, 2y)$ *if* $x \le 1/2, y \le 1/2$,

$\omega_2^{-1}(x,y) = (2x-100, 2y)$ *if* x > ½ and $y < 1/2$,

$\omega_3^{-1}(x,y) = (2x, 2y-100)$ *if* $x < 1/2$, y > 1/2,

$\omega_4^{-1}(x,y) = (4x-300, 4y-300)$ *if* x > ½, y > ½

Define by *f: $R^2 \longrightarrow R^2$ by f(x, y* = $\begin{Bmatrix} \omega_1^{-1}(x,y) \\ \omega_2^{-1}(x,y) \\ \omega_3^{-1}(x,y) \\ \omega_4^{-1}(x,y) \end{Bmatrix}$

The dynamical system *{R^2; f}* is related to the IFS {$R^2; \omega_1, \omega_2, \omega_3, \omega_4$} and it is an extension of the shift dynamical system *{F; f}* associated with IFS{$R^2; \omega_1, \omega_2, \omega_3, \omega_4$} .

By applying the general ETA to the dynamical system {R^2; f} with $\mathcal{W}$={ $(x, y) \in R^2$ *:a≤x≤c, b≤y≤d*}, where(a,b) (0,0) and (c,d)=(100,100) and $\mathcal{V}$ = {(*x, y*) ∈ R^2: $x^2 + y^2 > 200$}, *f : $R^2 \longrightarrow R^2$*, and by taking *N* large enough, we get the fractal set F constructed by this algorithm defined by:

F = { $(x, y) \in \mathcal{W}$: $f^{\circ n}(x,y) \notin \mathcal{V}$, for all n ≤ N}.That is, the black point represents the fern fractal *F*.

We observe that the IFS are totally disconnected by theorem (2). Therefore {$R^2; \omega_1, \omega_2, \omega_3, \omega_4$} is chaotic by theorem (4).

And by using the Escape chaotic dimension algorithm for the dynamical system {F; f} we found that D (f) =1.75829.

We show in above examples the chaotic behavior of the attractors constructed by ETA and counting the escape chaotic dimension of these attractors on R^2 , this method for finding dimension use only for attractors constructed by ETA and its faster than other classical method moreover its more precise.

4. Conclusions

Fractal is defined as the attractor of mutually recursive function called IFS. These attractors are defined using the Escape Time Algorithm. This proposed Algorithm is a modification and generalization of the Escape Time Algorithm. It is carried out using Delphi and implements on some known fractals sets which considered as an invariant sets in R^2, and generated using the IFS $\{R^2, f_1, f_2,..., f_n\}$.The chaotic behaviors of these attractors have been proved without using the code space method. Also a new method is proposed to find the Escape dimension of some chaotic systems based on Box-Counting theorem. A new technique that counts the fixed black point in the window w that not escapes to the regain v is applied. It is different from the original technique that is based on counting of boxes $N_n(A)$.

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الخلاصة

في هذا البحث الجواذب لنظام الدوال التكرارية قد عرف باستخدام خوارزمية زمن الهروب والذي طبق باستخدام نظام ديناميكي معين. هذه الخوارزمية استخدمت للمقارنة بين النقاط المختلفة الهاربة إلى المنطقة من النقاط الثابتة في النافذة بفعل ذلك النظام الديناميكي.السلوك الفوضوي لتلك الجواذب قد اثبت باستخدام طريقة جديدة تختلف عن الطريقة المبرهنة سابقا.كذلك قدمنا طريقة جديدة لحساب الإبعاد للجوا ذب المتكونة بواسطة خوارزمية زمن الهروب وخوارزميتين بلغة دلفي احدهما أعمام لخوارزمية بعد زمن الهروب والأخرى لحساب البعد ألكسوري للجوا ذب الفوضوية والذي سمي بعد فوضى الهروب.