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Specific Approach for Computing Hubble's Constant

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Abstract

Gravitational lensing is the phenomena of light deflection by massive bodies. In this work the gravitational lensing phenomena has been employed to study the cosmological application and calculate Hubble constant H_o which gives an idea about the age of the universe.

There is no general mathematical model applicable for various lensing systems since there are many physical variables included and there is correlation complicity .From here the idea of the study comes to find a suitable procedure to exploit the data in calculating time delay (Δt) then determine Hubble constant H_o .

This study includes processing the intensity data of images A & B of quasar microlensing for two lensing systems Q0957+561 and Q2237+0305 by using MATLAB software to find (Δt) , two techniques were applied :-

1- Discrete Fourier Transform
2- Cross-Correlation Function.

Results have been obtained for Hubble constant H_o after a suitable model was adopted for quasars Q0957+561 and Q2237+0305 and the value of Δt used to obtain H_o , from the value of (Δt) of quasar Q0957+561 ($H_o \approx 66.29$ $km.sec^{-1}.Mpc^{-1}$) have been found and from (Δt) of Q2237+0305 ($H_o \approx 77$ $km.sec^{-1}.Mpc^{-1}$) .

Introduction

One of the interesting facts about the history of related work since 1964 when Refsdal proposed to estimate Hubble constant H_o from multiply imaged quasars by the measurements of the delays in the arrive time between light rays coming from the different images [1] .

For many years astronomers have been concerned with the modeling of lens systems and measurement of the time delay. Gravitational lenses can be used to study the cosmological parameters of the universe. By exploreing a particular lens system in great detail determine all possible observational parameters and model for both lens and source in detail. This way one can determine the amount of dark matter in the lens and also the value of the Hubble constant. Up to now, there is no formal and unique study that mathematically describes the best model of the lensing galaxy, (e.g. modeling the lensing galaxy in Q0957+561). Cosmology is fundamentally concerned with the distribution and dynamics of the material which makes up the universe as a whole extending to a huge distances of about billions of light years [2].

There are many techniques available for getting many cosmological parameters for example (the cosmic expansion scale factor, the dimensionless density parameter, the Hubble constant, the deceleration parameter and perhaps the cosmological constant) through studying the different cosmic phenomena (like gravitational lensing techniques in our work) over the entire universe.

The Standard Cosmological Model:

Due to standard cosmological model, the universe began about 15 billion years ago in spatial event called the Big bang, and it has been expanding since. The universe underwent a period of expansion and cooling into stars and galaxies, the highly energetic photons formed in the first instances cooled also and filling the universe with the sea of the microwave photons background seen today. In this model, the space-time of the universe is governed by the field equations of general relativity. Where, if the universe assumed to be homogeneous and isotropic, it has to have a time dependent but spatially uniform curvature and the field equations dictate the dynamics. Within this framework, the metric of the expanding space-time is given by [3]:

$$ds^2 = c^2dt^2 - R^2(t)\left\{\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta\, d\phi\right)\right\} \quad ...\ 1$$

where, $(t,\ r,\ \theta,\ \Phi)$ are the spherical coordinate co-moving with the universe expansion. The time coordinate t is called *cosmological proper time* and it is singled out as a preferred time coordinate by the property of the spatial homogeneity. The quantity $R(t)$ is the time dependent curvature radius of the finite universe which scales the spatial part of the metric with dimension length, also it is called the *cosmic* *expansion scale factor* which describes the overall expansion of the universe as a function of its age. C is the speed of light and k is the 3 space-time curvature index.

The metric in equation (1) known as the Friedman – Robertson - Walker (F-R-W) metric and predicts that the universe non-static and showing a general expansion with time. Therefore, observationally, this expansion induces an apparent "Doppler shift" in the radiation of a sources expanding with the universe and can be expressed in terms of redshift *(z)*. If the light emitted from source at time t_e and received by an observer at time t_o then the redshift z of the source is given by:

$$\frac{R(t_o)}{R(t_e)} = \frac{\lambda_o}{\lambda_e} = 1 + z \Rightarrow z = \frac{\Delta\lambda}{\lambda_e}$$

where t_e is the epoch of emission of the observed radiation from a source, t_o is the present epoch. The emission wavelength is given by λ_e while λ_o is the same wavelength at the observer (the full derivation is shown in appendix A2). According to the field equations, the cosmic expansion scale factor of the universe *R(t)* also governed by:

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G\rho_o}{3}\frac{R_o}{R} - \frac{kc^2}{R^2} + \frac{1}{3}\Lambda c^2 \quad \ldots\ldots 2$$

$$2\frac{\ddot{R}}{R} + \frac{\dot{R}^2 + kc^2}{R^2} - \Lambda c^2 = 0$$

where R is given to be time dependent *(R(t))*, R_o is the cosmic expansion scale factor at the present epoch t_o. $\dot{R}$, $\ddot{R}$ is the first and second derivative with respect to time. G is the gravitational constant. ρ_o is the average mass density of the universe and Λ is the Einstein's cosmological constant, which manifests itself as a repulsion power opposing the gravity of expansion [4]. The ratio $(\dot{R}/R)$ in equation (2) also gives the cosmological definition for the Hubble low , where distance between any pair of fundamental particles expanding with the universe given by *R(t)r*, so that their mutual recession velocity is then proportional to the first derivative of *R(t)*, then Hubble's law given by (the full derivation is shown in appendix A3) [5]:

$$H(t) = \frac{\dot{R}(t)}{R(t)} \quad \ldots\ldots 3$$

The proportional constant in equation (3) is the so called (*Hubble's constant*) which introduced firstly by the American astronomer Edwin Hubble in his 1929's paper.

H(t) measures how fast the universe is expanding per unit distance. Therefore, it is given by units ($km.s^{-1}.Mpc^{-1}$). Also its inverse is proportional to the age of universe [6].

$$H_o^{-1} \propto t_o$$

The sub script (o) in the cosmological parameters like (H_o , R_o , Ω_o , q_o) represents them values at the present epoch ($t = t_o$).

In many calculations of observational and physical cosmology, H_o is also given by the below simplified formula:

$$H_o = 100 h_o \qquad (km\,s^{-1}Mpc^{-1})$$

The present estimate of h_o lies in the range $(0.5 \le h_o \le 1)$. And for t_o is in the range of approximately 9 to 18 billion years [6].

Gravitational Lensing Theory

One of the consequences of the theory of General Relativity is that light rays are deflected in gravitational fields. Although this prediction was made in the twentieth century, speculations that light rays might be bent by gravitation had been proposed much earlier. Issac Newton suggested as early as (1704) that the gravitational field of a massive object could possibly bend light rays, just as it would alter the trajectory of material particles. A century later, Laplace independently made the same suggestion. Furthermore the astronomer Johann Von Soldner (1804) at the Munich observatory found that in the framework of Newtonian mechanics a light ray passing near the limb of the sun should undergo an angular deflection of 0.875". In 1915 Einstein predicted that a light ray passing near the solar limb should be deflected by an angle given by:

$$\tilde{\alpha} = \frac{4GM}{C^2 R} \ll 1$$

where G is the gravitational constant, C is the velocity of light and M and R are the mass and radius of the sun (or any other compact lens) respectively [7].

Principle of Lensing

A gravitational lens is an extraordinary astronomical subject that is really made up of two separate objects.

The necessary parts of a gravitational lens are:

1. A luminous object called the source (S).
2. A massive object called the lens (L).

When light from the background source passes by the foreground lens, it will be deflected and sometimes magnified . These effects result in an object which appears to be a different shape or brighter than it would ordinarily appear.

Fig. (1) illustrates hat the observer seen an odd number of images which appear in the normal directions of local wave front, and that the light of these images travel for different times, giving rise to a relative time – delay between them [8].

Gravitational lenses come in several different shapes and sizes depending on what types of objects are involved, their distances from the observer and each other, and how close the light from a distant quasar (source) is deflected by close passage to a single galaxy (lens). In this case, the result appears to be several identical quasars located very close together; these usually occur as double, triple and quadruple images of the same source. In addition to these multiply imaged objects; gravitational lenses can also appear as arcs and sometimes even complete rings of light [9].

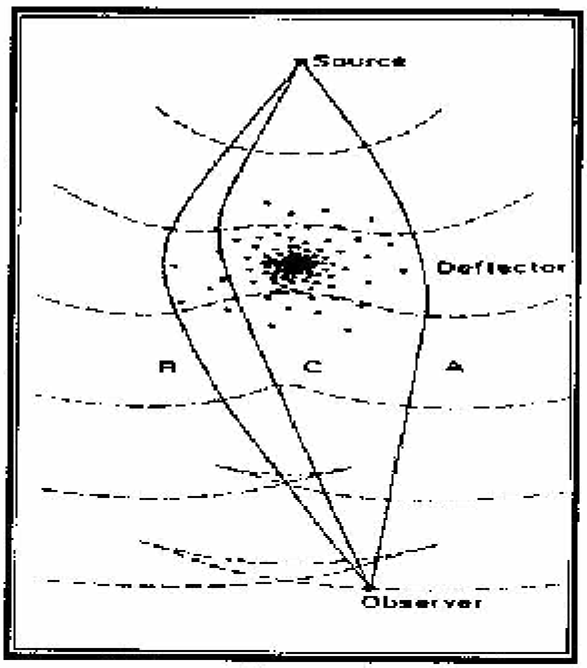

Figure (1)
Deformation of the wave front of a light signal propagating from a source to the observer

Classification of Lensing

In general, gravitational lensing can be classified as follows;

Multiply Imaged Quasars

Multiply imaged quasars were the first category of lensed systems to be discovered. By now more than 60 multiply imaged quasar systems have been found, most of them doubles or quadruplets, recently even a six images configuration was discovered. In 1979, gravitational lensing became an observational science when the double quasar Q0957+561 was discovered.

This quasar discovered by the discoverer Dennis Walsh. Up–to–date tables of multiply imaged quasars and gravitational lens candidates are provided by the CASTLES[1] group [10].

Microlensing

In another extreme, image splitting at the microarcsecond level from the lensing effect of stars or other compact objects can not be resolved by current telescopes yet . This so – called microlensing and can be observed as a flux variation of the background object because the lensing mass also magnifies the background object.

Microlensing comes in three varieties; -

1. Star-star lensing, or " local " microlensing, where stars in the Galactic disk or halo deflect the light of background stars in nearby galaxies (Large Magelanic Cloud (LMC), Small Magellanic Cloud (SMC)).
2. The second variant is star- quasar lensing, where stars in a distant(Lensing) galaxy act as microlenses on a quasar at cosmological distances.
3. Self – lensing, in which both the lenses and the sources are part of the LMC.

Strong and Weak Lensing

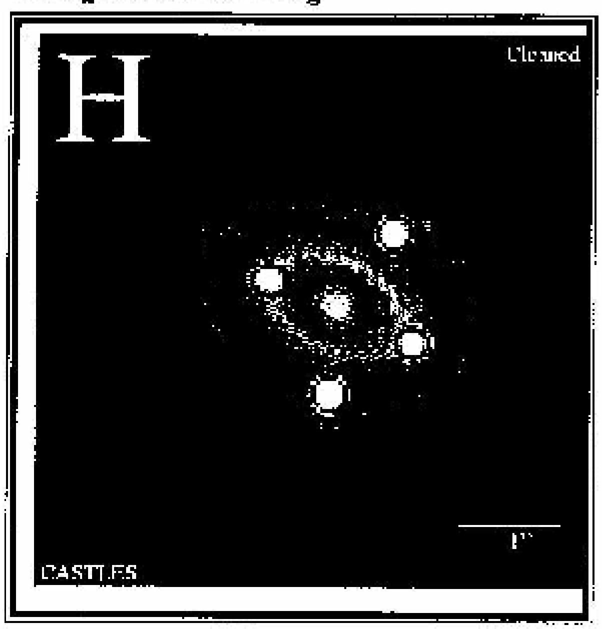

Figure (2)
The gravitational lens system Q2237+0305

Gravitational lensing phenomena due to galaxy clusters can naturally be split into two categories, strong and weak. Strong lensing was detected in 1986, when highly elongated curved long features of low surface brightness were found in two

So to find Δt for these systems, two techniques are used:-

1. Δt from Discrete Fourier Transform.

Theoretically, correlation between irregularly sampled data is better studied by discrete mathematical techniques. In the present work a method developed by Garido and described in Moles et al. (1986) [12] is used. Let $F_A(u)$ and $F_B(u)$ are the discrete Fourier transforms of the data for image A and B respectively with n data points for each, the cross-periodogram will be :

$$F_{A,B}(u) = \frac{n}{2\Pi} F_A(u).F^*_B(u)$$

and the inverse Fourier transform of $F_{A,B}(u)$ is the cross-covariance function. The calculation is thus made only on actual data points. This method has been applied for Q0957+561 and Q2237+0305. Fig. (3) illustrates the application of this method for Q0957+561. The peak observed at Δt ≈ 350 days. Fig. (4) represents the same application for Q2237+0305, and then Δt have been found ≈ 441 days.

2. Δt from Cross-Correlation Function.

In this technique the light curved obtained by applying the standard cross-correlation on the reduced function $A'(t)=A(t)-\langle A\rangle$ and $B'(t)=B(t)-\langle B\rangle$ gives the correlation coefficient c(Δt) .

Figure (5) shows the obtained light curve from this technique for Q0957−561 lensing system. For the whole data set, a maximum of 0.57 is found at Δt ≈450 days. The same technique is applied to Q2237+0305, Figure (6) shows that at maximum of 0.52 Δt ≈441 days.

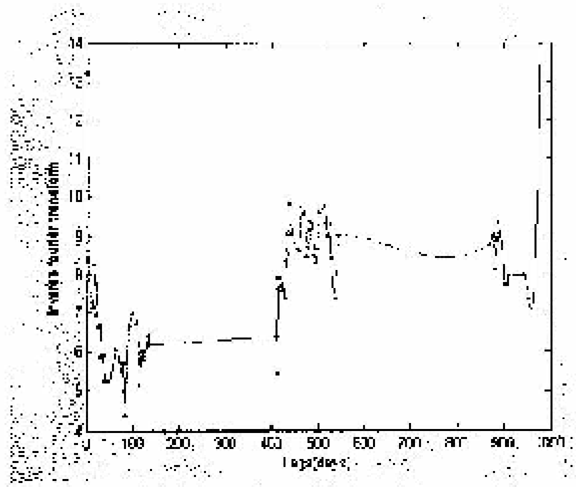

Figure (4)
Inverse Fourier Transform of Q2237+0305

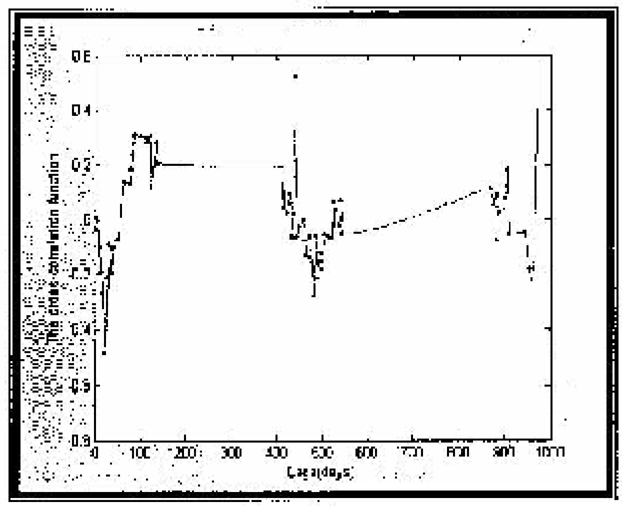

Figure (5)
The cross-correlation function of Q2237+0305

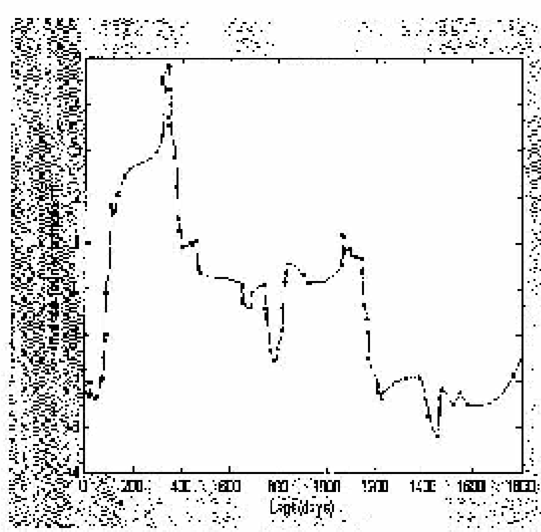

Figure (3)
Inverse Fourier transform of Q0957+561

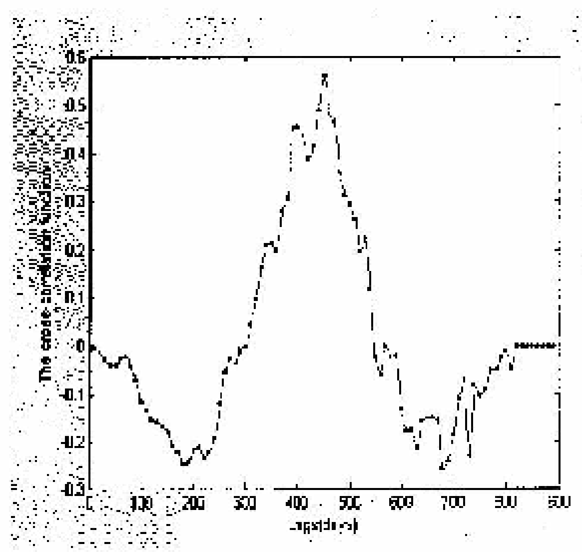

Figure (6)
The cross-correlation function of Q0957+561

Modeling the Gravitational Lens Q0957+561and Q2237+0305 .

In addition to a measurement of the time delay, it is also necessary to develop a reliable model to calculate the value of H_o. QSO.0957+561 has been studied by a number of groups over the years. The lensing galaxy in the gravitationally lensed quasar 0957 561 have been modeled by Grogin and Narayan (1996) to estimate Hubble constant as : -

$$H_o = (82 \pm 6)(1-k)\left(\frac{\Delta t}{1.14yr}\right)^{-1} km.\sec^{-1} .Mpc^{-1} .....4$$

Where k refers to the unknown convergence due to the cluster surrounding the lensing galaxy. Because the convergence (k) is unknown, so it was assumed to be zero (it can have the smallest value since it cannot be negative) , the modeling is used for the two cases :

1. For Q0957+561, Δt_{ave} had been found from the two techniques to be 430 days. Δt_{ave} it will be substituted in equation (4),then H_o=85.37 $Km.sec^{-1}.Mpc^{-1}$.

   If the one-dimensional velocity dispersion σ of the lensing galaxy was taken into account [8] then: -

$$H_o = (82.5)\left(\frac{\sigma}{322kms^{-1}}\right)^{2}\left(\frac{\Delta t}{1.1yr}\right)^{-1} km.\sec^{-1} .Mpc...5$$

Where σ=288 $km.sec^{-1}.Mpc^{-1}$ for Q0957+561, which was obtained from CASTLE data observations , so that H_o= 66.29 $Km.sec^{-1}.Mpc^{-1}$

2. For Q2237+0305, after Δt_{ave}=441 days was found, it will be substituted in equation (4), then H_o = 77 $Km.sec^{-1}.Mpc^{-1}$.

If the one-dimensional velocity dispersion σ of the lensing galaxy in Q2237+0305 was taken to be account (σ=215 $Km.sec^{-1}$ obtained from castle data observation and substituted in equation (5) then H_o=33.44 $Km.sec^{-1}.Mpc^{-1}$).

Results and Discussion

The discussion of the results obtained out of this paper lead to the following conclusions

1- Predicting the value of Hubble's Constant with good agreement comparing with other methods.

2- The measurement of Δt is definitive proof of A and B are really images of a single distant quasar.

When equation (4) had been applied for two lensing systems , the value of H_o for the Q2237+030 equal to 33.44 $km.Mpc^{-1}.sec^{-1}$ but for Q0957+561 , H_o=66.29 $km.Mpc^{-1}.sec^{-1}$, this is because of the velocity dispersion σ . The velocity dispersion refers only to the stars in the lensing galaxy , where as the gravitational lensing is done by the total mass .

The lensing galaxy in the gravitationally lensed quasar Q0957+561 is a bright cluster elliptical residing in a cluster of galaxies which contribute to the lensing and that the clusters of galaxies contain large amount (90%) of the dark matter and between galaxies within a cluster , while the lensing galaxy in Q2237+0305 is a spiral galaxy which contains interstellar gas and dust .

The straight forward approach is to assume that the velocity dispersion of the stars and that of the dark matter partcles are equal . However , Turner et al. (1984)[13] argued that in many circumstances of the σ of the stars would be lower than that of the total mass by a factor of $(2/3)^{1/2}$. This makes quite an important difference to the results , for instance , Narayan (1991)[14] derived on the basis of specific model , assuming $\Delta\tau_{BA}$=536 days and σ=303 $Km.s^{-1}$ that H_o=37 $Km.s^{-1}$. Mpc^{-1} if the dark matter has the same dispersion as the stars and H_o=56 $Km.s^{-1}.Mpc^{-1}$ if the correction factor of $(2/3)^{1/2}$ is applied.

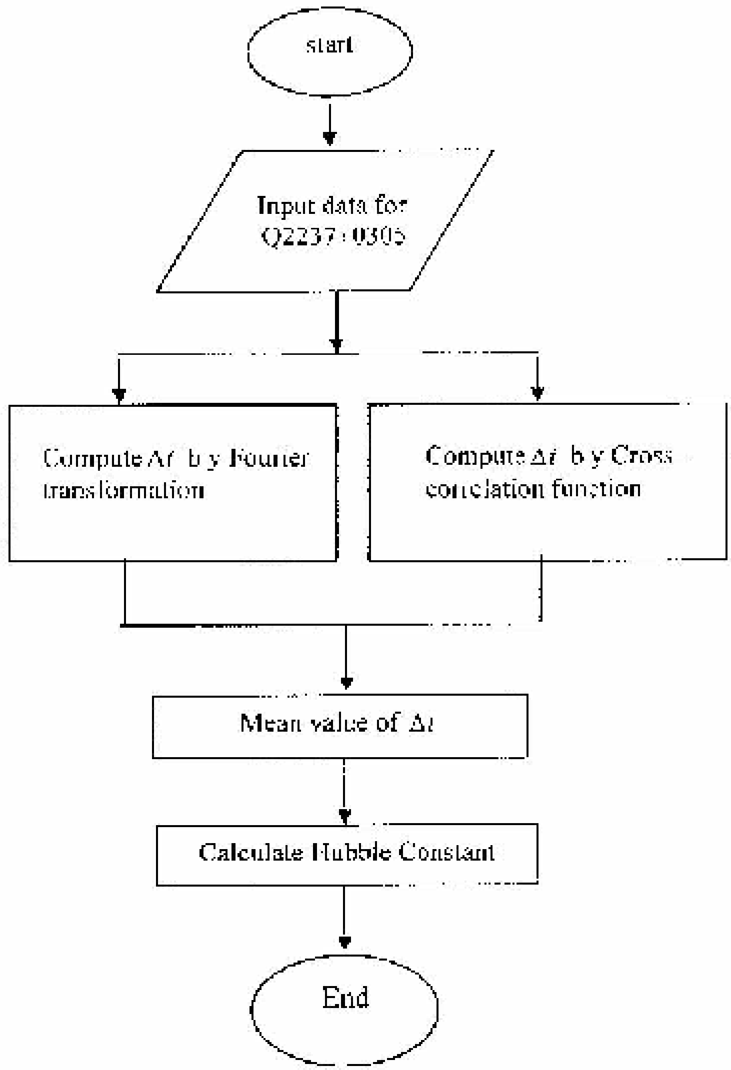

Fig (7) Flow chart of the procedure

Table (1)

Photometric data on Q0957+561 A, B

No.	Julian date	Magnitude A	±	Magnitude B	±
1	4194	17.5	0.05	17.74	0.06
2	4227	17.5	0.05	17.65	0.08
3	4304	17.64	0.04	17.74	0.05
4	4315	17.61	0.04	17.71	0.05
5	4323	17.5	0.05	17.76	0.07
6	4337	17.6	0.04	17.77	0.05
7	4339	17.54	0.03	17.69	0.04
8	4344	17.56	0.03	17.66	0.04
9	4378	17.55	0.03	17.7	0.04
10	4406	17.55	0.03	17.76	0.04
11	4554	17.64	0.03	17.61	0.04
12	4556	17.66	0.04	17.6	0.05
13	4607	17.57	0.02	17.77	0.03
14	4632	17.59	0.04	17.7	0.05
15	4636	17.54	0.02	17.65	0.02
16	4662	17.57	0.03	17.61	0.04
17	4637	17.51	0.04	17.6	0.05
18	4594	17.52	0.03	17.67	0.04
19	4720	17.5	0.03	17.66	0.04
20	4721	17.53	0.04	17.61	0.05
21	4723	17.56	0.03	17.64	0.04
22	4724	17.55	0.03	17.61	0.04
23	4724	17.51	0.03	17.64	0.04
24	4730	17.51	0.04	17.55	0.05
25	4731	17.5	0.03	17.61	0.04
26	4736	17.5	0.03	17.56	0.04
27	4965	17.62	0.03	17.69	0.04
28	4974	17.41	0.03	17.7	0.04
29	4993	17.48	0.03	17.6	0.04
30	5015	17.52	0.03	17.67	0.04
31	5023	17.51	0.03	17.62	0.04
32	5045	17.51	0.03	17.51	0.04
33	5055	17.52	0.03	17.58	0.04
34	5073	17.41	0.03	17.51	0.04
35	5085	17.43	0.03	17.6	0.04
36	5085	17.4	0.03	17.64	0.04
37	5087	17.44	0.03	17.55	0.04
38	5102	17.5	0.03	17.57	0.04
39	5108	17.5	0.03	17.64	0.04
40	5118	17.47	0.03	17.6	0.04
41	5134	17.42	0.04	17.6	0.05
42	5285	17.32	0.03	17.54	0.04
43	5293	17.34	0.03	17.55	0.04
44	5294	17.35	0.03	17.55	0.04
45	5295	17.42	0.03	17.61	0.05
46	5352	17.39	0.10	17.7	0.04
47	5374	17.4	0.10	17.55	0.04
48	5379	17.4	0.03	17.55	0.04
49	5381	17.41	0.03	17.6	0.04
50	5405	17.31	0.03	17.62	0.04
51	5428	17.36	0.03	17.51	0.04
52	5431	17.36	0.03	17.62	0.04
53	5437	17.37	0.04	17.61	0.05
54	5436	17.36	0.03	17.65	0.04
55	5440	17.18	0.03	17.64	0.04
56	5441	17.31	0.03	17.63	0.05
57	5458	17.3	0.04	17.52	0.05
58	5461	17.4	0.03	17.52	0.04
59	5473	17.4	0.03	17.53	0.04
60	5488	17.32	0.04	17.63	0.05

Table (2)

Photometric data on Q2237+0305 A,B

No.	Julian date	Image A (mag)	Image B (mag)
1	871	0.51	0.56
2	875	0.72	0.66
3	885	0.30	0.5
4	885	[illegible]	0.58
5	874	0.2	0.41
6	894	0.33	0.52
7	897	0.32	0.57
8	901	0.29	0.47
9	903	[illegible]	0.74
10	909	0.19	0.53
11	[illegible]	0.18	0.62
12	[illegible]	0.25	0.68
13	[illegible]	0.20	0.72
14	949	0.25	0.45
15	951	0.13	0.5
16	951	0.24	0.5
17	955	0.26	0.55
18	957	0.24	0.41
19	952	0.24	0.54
20	954	0.02	0.71
21	986	0.04	0.77
22	990	0.16	0.57
23	992	0.25	0.71
24	994	0.17	0.51
25	1005	0.30	0.57
26	1005	0.13	0.52
27	[illegible]	0.35	0.44
28	[illegible]	0.35	0.55
29	1226	0.20	0.90
30	1283	0	0.75
31	1292	-0.08	0.76
32	1300	0.37	0.71
33	1305	-0.05	0.71
34	1306	0.19	0.80
35	1306	-0.51	0.86
36	1310	-0.21	0.65
37	1314	-0.01	0.72
38	1320	0.13	0.56
39	1334	-0.04	0.75
40	1336	-0.45	1.04
41	1342	-0.20	1.04
42	1344	0.5	0.72
43	1348	-0.14	0.84
44	1351	-0.12	0.67
45	1355	-0.16	0.85
46	1360	0.2	1.12
47	1364	-0.04	0.99
48	[illegible]	0.00	[illegible]
49	[illegible]	[illegible]	0.84
50	[illegible]	[illegible]	[illegible]
51	[illegible]	0.33	[illegible]
52	[illegible]	-0.14	[illegible]
53	1391	-0.08	[illegible]
54	1395	-0.02	0.77
55	[illegible]	0.14	0.57

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المستخلص

التعدس التجذبي هو ظاهرة انحراف الضوء عن مساره بسبب مروره قرب الاجسام الثقيلة.في هذا العمل تم توظيف ظاهرة التعدس التجذبي لغرض دراسة الظواهر الكونية و ايجاد ثابت هابل H_0 الذي يعطينا فكرة عن عمر الكون .في الحقيقة لا توجد علاقة رياضية شاملة قابلة للتطبيق لمختلف الانظمة التعدسية بسبب تعدد المعلمات الفيزيائية و تعقيد ترابطها .من هنا جاءت فكرة الدراسة التي تتضمن ايجاد طريقة عملية لتوظيف البيانات لإيجاد زمن التأخر (Δt) والذي بدوره يستخدم لإيجاد ثابت هابل H_0 .تم استخدام بيانات الشدة الضوئية للصورتين (A,B) الخاصة بالكويزرين Q2237-0305 و Q0957-561 لإيجاد زمن التأخر باستخدام برنامج MATLAB بتطبيق تقنيتين هما :

1. Discrete Fourier Transform
2. Cross Correlation Function

وقد تم الحصول على نتائج لقيم ثابت هابل بعد تبني موديل مناسب للكويزرين Q0957-561 و Q2237-0305 وتعويض قيمة Δt لهما المستخرجة بالطريقتين أعلاه ففي حالة الكويزر الاول وبعد ايجاد Δt (H_0 = 66.29 $Km.sec^{-1}.Mpc^{-1}$) ومن قيمة Δt للكويزر Q2237-0305 وجد ان (H_0 = 77 $Km.sec^{-1}.Mpc^{-1}$) .