Local Existence and Uniqueness of Sobolev Type Semilinear Initial Value Problems in Banach Spaces

By

Dr. Akram M. Al Abood & Manaf A. Salah

Department of Mathematics and computer applications, College of Science, Al-Nahrain University, Baghdad, Iraq.

Abstract

The aim of this paper is to prove the local existence and uniqueness of the mild solution of semilinear initial value control problems in suitable Banach spaces using resolvent operator and Schauder fixed point theorem.

Keywords: Local existence, uniqueness, mild solution, fixed point theorem and resolvent operator of control problems.

Introduction

Corduneanu [1] and Gripenberg et. al [2] studied the problem of existence solutions for Volterra integral equations of various types. Grimmer [3] introduced the resolvent generators for integral equations in Banach spaces. Liu [4] studied the weak solutions of integrodifferential equations by using resolvent operators and semigroup theory. Fitzgibbon [5] investigated the existence problem for semilinear integrodifferenmtial equations in Banach spaces using the method of semigroups and Banach's fixed point theorem. Byszewski [6] proved the existence and uniqueness of mild solutions of nonlocal Cauchy problem. Lin and Liu [7] investigated the nonlocal Cauchy problem of semilinear integrodifferential equations by using resolvent operators and discussed the existence problem for semilinear Sobolev type equations in Banach spaces. Balachandran et. al [8] established the existence of solutions for Sobolev type integrodifferential equations in Banach spaces. Recently, Balachandran et. al [9] investigated the same problem for Sobolev type delay integrodifferential equations. Several authors have studied the problem of existence of solutions of semilinear differential equations and Sobolev type equations [3, 7, 10, 11, 12]. Bahuguna D. in 1997 [13], has studied the local existence without uniqueness the mild solution of the semilinear initial value problem:

$$\left.\begin{aligned} &\frac{d}{dt}x(t)=Ax(t)+f(t,x(t))+\int_{s=0}^{t}h(t-s)g(s,x(s))ds,\ t>0 \\ &x(0)=x_0 \end{aligned}\right\}$$

Manaf in 2005 [14], has studied the local existence and uniqueness of the mild solution to the semilinear initial value control problem:

$$\left.\begin{aligned} &\frac{d}{dt}x(t)=Ax(t)+f(t,x(t))+\int_{s=0}^{t}h(t-s)g(s,x(s))ds-Bu(t),\ t>0 \\ &x(0)=x_0 \end{aligned}\right\}$$

Krishnan Balachandran in 2003 [15], has studied the existence and uniqueness of themild solution to the semilinear initial value problem:

$$\left.\begin{aligned} &\frac{d}{dt}[Ex(t)]=A\left[x(t)+\int_{s=0}^{t}F(t-s)x(s)ds\right]+f(t,x(t)) \\ &x(0)=x_0 \end{aligned}\right\}$$

Our work is concerned with the semilinear initial value control problem:

$$\left.\begin{aligned} &\frac{d}{dt}[Ex(t)]=A\left[x(t)+\int_{s=0}^{t}F(t-s)x(s)ds\right]+f(t,x(t))+ \\ &\qquad\int_{s=0}^{t}h(t-s)g(s,x(s))ds-Bu(t),\ t>0 \\ &x(0)=x_0,\ t\in J=[0,r) \end{aligned}\right\}$$

where A and E are closed linear operators with domain contained in a suitable Banach space X, F(t) is a bounded operator for $t \in J$ and f, g are nonlinear maps defined from $[0, r)\times X$ into X, h is the real valued continuous function defined from $[0, r)$ into R, where R is the set of real numbers and B is a bounded linear operator defined from O into X, where O is a Banach space and $\omega(.)$ be the arbitrary control function is given in $L^p([0,r),O)$, a Banach space of control functions with $\|\omega(t)\|_O \le K_1$, for $0 \le t < r$, $F(t) \in B(X)$, $t \in J$ and $F(t) : Y \longrightarrow Y$ and for x(.) continuous in Y, $AF(.)x(.) \in L^1(J, X)$. For $x \in X$, $F'(t)x$ is continuous in $t \in J$ and Y is the Banach space formed from D(A), the domain of X, endowed with the graph norm.The local existence and uniqueness of the mild solution to the semilinear initial value control problem given (3) have been developed by using semigroup theory and Schauder fixed point theorem.

Preliminaries

Consider the Sobolev type semilinear initial value control problem:

$$\frac{d}{dt}[Ex(t)] = A\left[x(t) + \int_0^t F(t-s)x(s)ds\right] + f(t,x(t)) + \int_{0}^{t} h(t-s)g(s,x(s))ds + B\omega(t), t > 0;$$

$$x(0) = x_0, t \in J = [0, r)$$

Definition (1):

A family of bounded linear operator $R(t) \in B(X)$ for $t \in [0, r]$ is called the resolvent operator for:

$$\frac{d}{dt}x(t) = A\left[x(t) + \int_0^t F(t-s)x(s)ds\right]$$

if:

(i) $R(0) = I$, where I is the identity operator.

(ii) For all $x \in X$, $R(t)x$ is continuous for $t \in J$.

(iii) $R(t) \in B(Y)$, $t \in J$, for $y \in Y$, $R(.)y \in C^1([0, r], X) \cap C([0, r], Y)$ and

$$\frac{d}{dt}R(t)y - AE^{-1}\left[R(t)y + \int_0^t F(t-s)R(s)y\,ds\right] =$$

$$R(t)AE^{-1}y + \int_0^t R(t-s)AE^{-1}F(s)y\,ds, t \in J$$

Definition (2):

A function $x(.) \in C([0, r), X)$ is called a mild solution of equation (3) if it satisfies the integral equation:

$$x(t) = E^{-1}R(t)Ex_0 + E^{-1}\int_0^t R(t-s)\left[f(s, x_u(s)) + \int_0^s h(s-\tau)g(\tau, x_u(\tau))d\tau + B\omega(s)\right]ds$$

The local existence and uniqueness of a mild solution of problem (3) have been developed, by assuming the following assumptions:

A_1 : The operator $A : D(A) \subset X \longrightarrow X$ and $E : D(E) \subset X \longrightarrow X$ are closed linear operators.

A_2 : $D(E) \subset D(A)$ and E is bijective.

A_3 : $E^{-1} : X \longrightarrow D(E)$ is bounded operator and $E^{-1}F = FE^{-1}$.

A_4 : AE^{-1} generates a strongly continuous semigroup of bounded operators in X.

A_5 : The resolvent operator $R(t)$ is compact in X.

A_6 : Let $\rho > 0$, such that $\mathcal{B}_\rho(x_0) = \{x \in X : \|x - x_0\|_X \le \rho\}$, where $x_0 \in U$ {open subset of X}, the nonlinear maps f, g define from $[0, r) \times U$ into X, satisfy the locally Lipschitz condition with respect to second argument, i.e.,

$$\|f(t, v_1) - f(t, v_2)\|_X \le L_0\|v_1 - v_2\|_X \text{ and},$$

$$\|g(t, v_1) - g(t, v_2)\|_X \le L_1\|v_1 - v_2\|_X$$

For $0 \le t < r$ and $v_1, v_2 \in \mathcal{B}_\rho(x_0)$ and L_0, L_1 are Lipschitz constant.

A_7 : h is continuous function, $h \in L^1([0, r[, R)$, where R is the set of real numbers.

A_8 : $\omega(.)$ be the arbitrary control function is given in $L^P([0, r], O)$, a Banach space of control functions with O as a Banach space and here B is a bounded linear operator from O into X with $\|\omega(t)\|_O \le K_1$, for $0 \le t < r$.

A_9 : Let $t' > 0$, such that $\|f(t, v)\|_X \le N_1$, $\|g(t, v)\|_X \le N_2$, for $0 \le t \le t'$ and $v \in \mathcal{B}_\rho(x_0)$, also let $t'' > 0$, such that $\|R(t)E^{-1}R(t)Ex_0 - x_0\|_X \le \rho'$, for $0 \le t \le t''$ and $x_0 \in U$, where ρ' is a positive constant such that $\rho' < \rho$.

A_{10} : Let $t_1 > 0$, such that:

$t_1 = \min\{r, t', t''\}$ and satisfy the following conditions:

$$\text{(i)} \quad t_1 \le \frac{\rho - \rho'}{L_0 M(N_1 + K_0 K_1 + h_{t_1} N_2)}$$

$$\text{And} \quad \text{(ii)} \quad t_1 < \frac{1}{(L_0 + h_{t_1} L_1) L_0 M}$$

Main Results (4)

We introduce the following main theorem:

Theorem (1):

Assume the hypotheses $(A_1) - (A_{10})$ hold. Then, for every $x_0 \in U$, there exist a fixed number t_1, $0 < t_1 < r$, such that the semilinear initial value control problem of equation (3) has a unique local mild solution $x_u \in C[0, t_1], X)$ for every control function $\omega(.) \in L^P([0, t_1], O)$.

Proof:

Without loss of generality, we may suppose $\rho <$ ∞, because we are concerned here with the local existence only.

There exist $M > 0$, such that $\|R(t)\| \le M$, $0 \le t \le r$ {since $R(t)$ is a bounded linear operator on X}. Assume:

$$h_r = \int_0^r |h(s)|ds$$

we set $Y = C([0, t_1], X)$, where Y is a Banach space with sup norm defined as follows:

$$\|y\|_Y = \sup_{0 \le t \le t_1} \|y(t)\|_X$$

Define:

$S_\omega = \{x_\omega \in Y : x_\omega(0) = x_0,\ x_\omega(t) \in \mathcal{B}_\rho(x_0)$, for a given $\omega(.) \in L^p([0, t_1], O)\}$

Clearly S_ω is a bounded, convex and closed of Y.

Define a map $F_\omega : S_\omega \longrightarrow Y$, by:

$$(F_\omega x_\omega)(t) = E^{-1}Ex_0 + E^{-1}\int_0^t R(t-s)\Big[f(s, x_\omega(s)) + \int_0^s h(s-\tau)g(\tau, x_\omega(\tau))d\tau + B\omega(s)\Big]ds$$

For arbitrary control function $\omega(.) \in L^p([0, t_1], O)$.

To show that $F_\omega(S_\omega) \subset S_\omega$, let x_ω be an arbitrary element in S_ω, such that $F_\omega x_\omega \in F_\omega(S_\omega)$. To prove $F_\omega x_\omega \in S_\omega$, notice that $F_\omega x_\omega \in Y$ {by the definition of the map F_ω} and $(F_\omega x_\omega)(0) = x_0$ {by equation(6)}, to prove $(F_\omega x_\omega)(t) \in \mathcal{B}_\rho(x_0)$, for any $x_\omega \in S_\omega$, from the definition of the closed ball $\mathcal{B}_\rho(x_0)$, notice that $(F_\omega S_\omega)(t) \subset X$ and

$$\|(F_\omega x_\omega)(t) - x_0\|_X = \|E^{-1}R(t)Ex_0 - x_0 + E^{-1}\int_0^t R(t-s)\Big[f(s, x_\omega(s)) + B\omega(s) + \int_0^s h(s-\tau)g(\tau, x_\omega(\tau))d\tau\Big]ds\|_X$$

$$\le \|E^{-1}R(t)Ex_0 - x_0\| + \|E^{-1}\| K_3 K_1$$

After a series of simplifications and using the conditions A_3, A_4, A_9 and $(A_{10}\ .i)$with equation(5), we get: $\|(F_\omega x_\omega)(t) - x_0\|_X \le \rho$, for $0 \le t \le t_1$, i.e. $(F_\omega x_\omega)(t) \in \mathcal{B}_\rho(x_0)$ for $0 \le t \le t_1$, hence $F_\omega x_\omega \in S_\omega$ for arbitrary $x_\omega \in S_\omega$, which implies that

$F_\omega : S_\omega \longrightarrow S_\omega$, so one can select the time t_1 such that:

$$t_1 = \min\left\{r, r', r'', \frac{\rho - \rho'}{L_E M(N_1 + K_0 K_1 + h_{t_1} N_2)}\right\}$$

To complete the proof, we have to show that $F_\omega : S_\omega \longrightarrow S_\omega$ is a continuous map, given $\|x_\omega^n - x_\omega\|_Y \longrightarrow 0$, as $n \longrightarrow \infty$, to prove $\|F_\omega x_\omega^n - F_\omega x_\omega\|_Y \longrightarrow 0$, as $n \longrightarrow \infty$. Where x_ω is Continuous functions depend on ω, x_ω^n is a sequence of continuous functions depend on ω.

Notice that:

$$\|F_\omega x_\omega^n - F_\omega x_\omega\|_Y = \sup_{0 \le t \le t_1} \|(F_\omega x_\omega^n)(t) - (F_\omega x_\omega)(t)\|_X$$

$$= \sup_{0 \le t \le t_1} \|E^{-1}R(t)Ex_0 + E^{-1}\int_0^t R(t-s)\Big[f(s, x_\omega^n(s)) + \int_0^s h(s-\tau)g(\tau, x_\omega^n(\tau))d\tau + B\omega(s)\Big]ds - E^{-1}R(t)Ex_0 - E^{-1}\int_0^t R(t-s)\Big[f(s, x_\omega(s)) + \int_0^s h(s-\tau)g(\tau, x_\omega(\tau))d\tau + B\omega(s)\Big]ds\|_X \quad (6)$$

After a series of simplifications and using the conditions A_2, A_6 and A_8 with equation (5), we get:

$$\|F_\omega x_\omega^n - F_\omega x_\omega\|_Y \le (L_f + L_{t_1} L_g)L_E M\|x_\omega^n - x_\omega\|_Y\, t_1$$

Since $\|x_\omega^n - x_\omega\|_Y \longrightarrow 0$, as $n \longrightarrow \infty$, which implies that:

$$\lim_{n \to \infty} \|F_\omega x_\omega^n - F_\omega x_\omega\|_Y = 0, \text{ i.e.,}$$

$$\|F_\omega x_\omega^n - F_\omega x_\omega\| \longrightarrow 0, \text{ as } n \longrightarrow \infty.$$

Now, assume that $\tilde{S} = F_\omega(\tilde{S})$, and for fixed $t \in [0, t_1]$, let $\tilde{S}(t) = \{(F_\omega x_\omega)(t) : x_\omega \in S_\omega\}$. To show that $\tilde{S}(t)$ is a precompact set, for every fixed $t \in [0, t_1]$.

For $t = 0$ we have $\tilde{S}(0) = \{(F_\omega x_\omega)(0) : x_\omega \in S_\omega\} = \{x_0\}$, which is a precompact set in X.

Now, for $t > 0$, $0 < \varepsilon < t$, define:

$$(F_\omega^\varepsilon x_\omega)(t) = E^{-1}R(t)Ex_0 + E^{-1}\int_0^{t}R(t-s)\Big[f(s,x_\omega(s)) + \int_0^s h(s-\tau)g(\tau,x_\omega(\tau))d\tau + B\omega(s)\Big]ds$$

For arbitrary $x_\omega \in S_\omega$, then:

$$(F_\omega^\varepsilon x_\omega)(t) = E^{-1}R(t)Ex_0 + E^{-1}T(\varepsilon)\int_0^{t-\varepsilon}R(t-s-\varepsilon)\Big[f(s,x_\omega(s)) + \int_0^s h(s-\tau)g(\tau,x_\omega(\tau))d\tau + B\omega(s)\Big]ds$$

from the compactness of the resolvent operator R(t) and equation(8), which implies that the set $\tilde{S}_\varepsilon(t) = \{(F_\omega^\varepsilon x_\omega)(t) : x_\omega \in S_\omega\}$ is precompact in X for every ε, $0 < \varepsilon < t < t_1$.

Moreover, for any $x_\omega \in S_\omega$, we have:

$$\|(F_\omega x_\omega)(t) - (F_\omega^\varepsilon x_\omega)(t)\|_X = \|E^{-1}R(t)Ex_0 + E^{-1}\int_0^t R(t-s)\Big[f(s,x_\omega(s)) + \int_0^s h(s-\tau)g(\tau,x_\omega(\tau))d\tau + B\omega(s)\Big]ds - E^{-1}R(t)Ex_0 - E^{-1}T(\varepsilon)\int_0^{t-\varepsilon}R(t-s-\varepsilon)\Big[f(s,x_\omega(s)) + \int_0^s h(s-\tau)g(\tau,x_\omega(\tau))d\tau + B\omega(s)\Big]ds\|$$

After a series of simplifications and using the conditions A_3, A_8 and A_9 with equation (5),

We get:

$$\|(F_\omega x_\omega)(t) - (F_\omega^\varepsilon x_\omega)(t)\|_X \le L_0 M(N_1 + h_{t_1} N_2)\varepsilon,$$

then:

$\|(F_\omega x_\omega)(t) - (F_\omega^\varepsilon x_\omega)(t)\|_X \longrightarrow 0$, as $\varepsilon \longrightarrow 0$, i.e.,

$\lim_{\varepsilon \to 0} (F_\omega^\varepsilon x_\omega)(t) = (F_\omega x_\omega)(t)$, which imply that $\tilde{S}(t)$ is totally bounded, that is, $\tilde{S}(t)$ is precompact in X, see [13],[16].

To prove that $\tilde{S} = F_\omega(S_\omega)$ is an equicontinuous family of functions, when $0 < r_1 < r_2$, we have:

$$\|(F_\omega x_\omega)(r_1) - (F_\omega x_\omega)(r_2)\|_X = \|E^{-1}R(r_1)Ex_0 - E^{-1}\int_0^{r_1}R(r_1-s)\Big[f(s,x_\omega(s)) + \int_0^s h(s-\tau)g(\tau,x_\omega(\tau))d\tau - B\omega(s)\Big]ds$$
$$E^{-1}R(r_2)Ex_0$$
$$E^{-1}\int_0^{r_2}R(r_2-s)\Big[f(s,x_\omega(s)) + \int_0^s h(s-\tau)g(\tau,x_\omega(\tau))d\tau + B\omega(s)\Big]ds\|_X \quad (7)$$

Hence:

$$\|(F_\omega x_\omega)(r_1) - (F_\omega x_\omega)(r_2)\|_X = \|E^{-1}(R(r_1) - R(r_2))Ex_0 + E^{-1}\int_0^{r_1}R(r_1-s)\Big[f(s,x_\omega(s)) + \int_0^s h(s-\tau)g(\tau,x_\omega(\tau))d\tau + B\omega(s)\Big]ds - E^{-1}\int_0^{r_2}R(r_2-s)\Big[f(s,x_\omega(s)) - \int_0^s h(s-\tau)g(\tau,x_\omega(\tau))d\tau + B\omega(s)\Big]ds\|_X \quad (8)$$

After a series of simplifications and using the following condition A_3, A_8 and A_9 with equation (5), we get:

$$\|(F_\omega x_\omega)(r_1) - (F_\omega x_\omega)(r_2)\|_X \le L_1\|(R(r_1) - R(r_2))Ex_0\|_X + M(N_1 + h_{t_1}N_2 + K_0K_1)(r_1 - r_2) \quad (9)$$

Since R(t) is compact resolvent operator which implies that R(t) is continuous in the uniform operator topology for $t > 0$, therefore the right hand side of equation (9) tends to zero as $r_1 - r_2$ tends to zero. Thus $\tilde{S}$ is equicontinuous family of functions. It follows from the "Arzela-Ascolies theorem" that $\tilde{S} = F_\omega(S_\omega)$ be relatively compact in Y and by applying "Schauder fixed point theorem", which implies $F_\omega : S_\omega \longrightarrow S_\omega$ has a fixed point, i.e., $F_\omega x_\omega = x_\omega$, for arbitrary control function is given in $L^2([0, T], \Omega)$, hence equation (1) has a local mild solution $x_\omega \in C([0, t_1], X)$. To show the uniqueness, let $\overline{x}_\omega$, $\overline{\overline{x}}_\omega$ be two local mild solutions of the semilinear initial value control problem given by equation(1) on the interval $[0, t_1]$, where $\overline{x}_\omega$, $\overline{\overline{x}}_\omega$ Continuous functions depend on ω.

We must prove that $\|\overline{x}_\omega(t) - \overline{\overline{x}}_\omega(t)\|_X = 0$, assume $\|\overline{x}_\omega(t) - \overline{\overline{x}}_\omega(t)\|_X \neq 0$, and notice that:

$$\|\bar{x}_{\omega}(t) - \bar{\bar{x}}_{\omega}(t)\|_X = \| E^{-1}R(t)Ex_0 + E^{-1}\int_0^t R(t-s)\Big[f(s,\bar{x}_{\omega}(s)) + \int_0^s h(s-\tau)g(\tau,\bar{x}_{\omega}(\tau))d\tau + B\omega(s)\Big]ds - E^{-1}R(t)Ex_0 - E^{-1}\int_0^t R(t-s)\Big[f(s,\bar{\bar{x}}_{\omega}(s)) + \int_0^s h(s-\tau)g(\tau,\bar{\bar{x}}_{\omega}(\tau))d\tau + B\omega(s)\Big]ds \|_X$$

After a series of simplifications and using the conditions A_3 and A_8 with equation (5), we get:

$$\|\bar{x}_{\omega}(t) - \bar{\bar{x}}_{\omega}(t)\|_X \le (L_0 + h_{t_1} L_1)l_0 M \|\bar{x}_{\omega} - \bar{\bar{x}}_{\omega}\|_X t_1$$

By using the condition (A_{10},ii), we get:

$$\|\bar{x}_{\omega}(t) - \bar{\bar{x}}_{\omega}(t)\|_X < (L_0 + h_{t_1} L_1)l_0 M \|\bar{x}_{\omega} - \bar{\bar{x}}_{\omega}\|_X \frac{1}{(L_0 + h_{t_1} L_1)l_0 M}$$

Then $\|\bar{x}_{\omega}(t) - \bar{\bar{x}}_{\omega}(t)\|_X < \|\bar{x}_{\omega} - \bar{\bar{x}}_{\omega}\|_X$

By taking the supremum over $[0, t_1]$ of the both sides of the above inequality, we get:

$\|\bar{x}_{\omega} - \bar{\bar{x}}_{\omega}\|_X < \|\bar{x}_{\omega} - \bar{\bar{x}}_{\omega}\|_X$, which implies to the contradiction, so we get: $\bar{x}_{\omega}(t) = \bar{\bar{x}}_{\omega}(t)$, $\forall\, 0 \le t \le t_1$. Hence we have a unique mild solution $x_{\omega} \in C([0,t_1],X)$, for arbitrary control function $\omega(\cdot) \in L^2(0, t_1); O)$.

References

[1] C.Corduneanu "Integral equations and applications",Cambridge University Press,Cambridge,1991.

[2] G.Gripenberg "Volterra Integral and Functional Equations". Cambridge University Press, Cambridge, 1990.

[3] R.Grimmer "Resolvent operators for integral equations in a Banach space",Transactions of the American Mathematical Society. Vol.273, pp. 333-349, 1982.

[4] J.H.Liu "Resolvent operators and weak solutions of integrodifferential equations", Differential and Integral Equations, Vol.7, pp.523-534, 1994.

[5] W.E.Fitzgibbon "Semilinear functional differential equations in Banach space",Nonlinear Analysis;Theory,Methods and Applications, Vol.4, pp.745-760,1980.

[6] L.Byszewski "Theorem about the existence and uniqueness of solutions of a semilinear evolution nonlocal Cauey problem",Journal of Mathematical Analysis and applications, Vol.162, pp. 494-505,1992.

[7] Y.Lin and J.H.Liu " Semilinear integrodifferential equations with nonlocal Cauchy problem,Nonlinear Analysis;Theory,Methods and Applications, Vol.26, pp.1023-1033,1996.

[8] K.Balachandran "Nonlinear integrodifferential equations of Sobolev type with nonlocal conditions in Banach spaces",communications of the Korean mathematical society, Vol.14, pp.223-231,1999.

[9] K.Balachandran "Nonlocal Cauchy problem for delay integrodifferential equations of Sobolev type in Banach spaces", Applied Mathematical Letters, Vol.15, pp.845-854, 2002.

[10] M.L.Heard "An abstract semilinear hyperbolic Volterra integrodifferential equations",Journal of Mathematical Analysis and applications, Vol. 80, pp.175-202,1981.

[11] M.L. Heard "A semilinear parabolic Volterra integrodifferential equations",Journal of differential equation, Vol. 71, pp. 201-233,1988.

[12] E.Hernandez "Existence results for a class of semilinear evolution equations",Electric Journal of differential equations, Vpl. 2001, pp. 1-14, 2001.

[13] D. Bahuguna "Integrodifferential equations of parabolic type",Mathematical in Engg. And indistre in Narosa publish house, India,1997.

[14] Manaf A. Salah "Solvability and controllability of semilinear initial value control problem via semigroup approach", MSC Thesis in mathematics,Al-Nahrain University,Baghdad,Iraq,2005.

[15] Krishnan Balachandran "Regularity of solutions of Sobolev type semilinear integrodifferential equations in Banach spaces",Electronic Journal of differential equations, Vol.2003,No.114,pp. 1-8, 2003.

[16] K.Balachandran "Existence of solutions and controllability of nonlinear integrodifferential systems in Banach spaces",Hindawi publishing corporation.mathematical problems in engineering, Vol. 2 ,pp. 65-79,2003.

المستخلص

الهدف من هذا البحث هو إثبات وجود ووحدانية الحل العام (محلي) لمسألة سيطرة شبه خطية ذات قيمة ابتدائية في فضاء باناخ مناسب باستخدام مؤثر محتل و نظرية النقطة الثابتة لشودر.