

Fig.(3) Restored images of Fig.(1-e) after:
 (a) 1 iteration (b) 10 iterations (c) 30 iterations

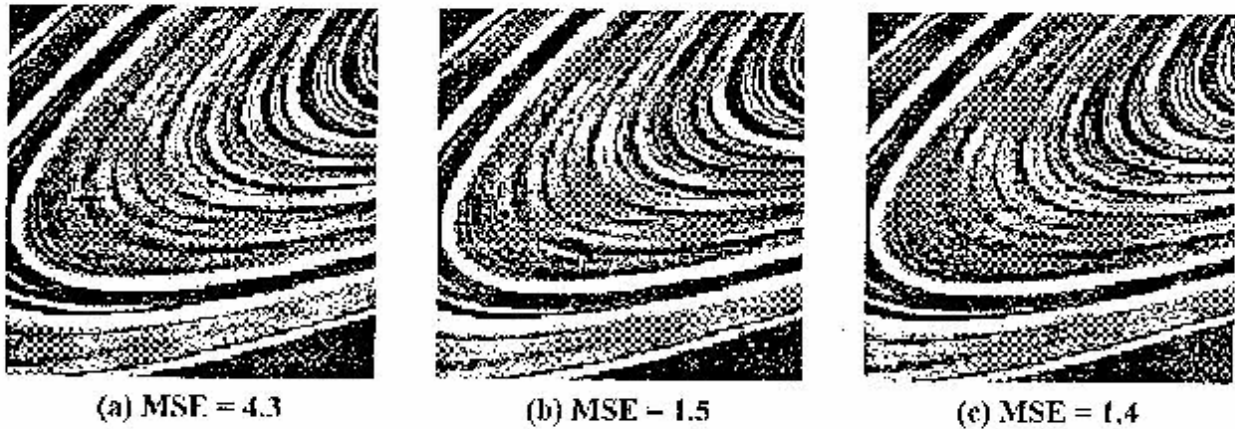


Fig.(4) Restored images of Fig.(1-d) after:
 (a) 1 iteration (b) 10 iterations (c) 30 iterations

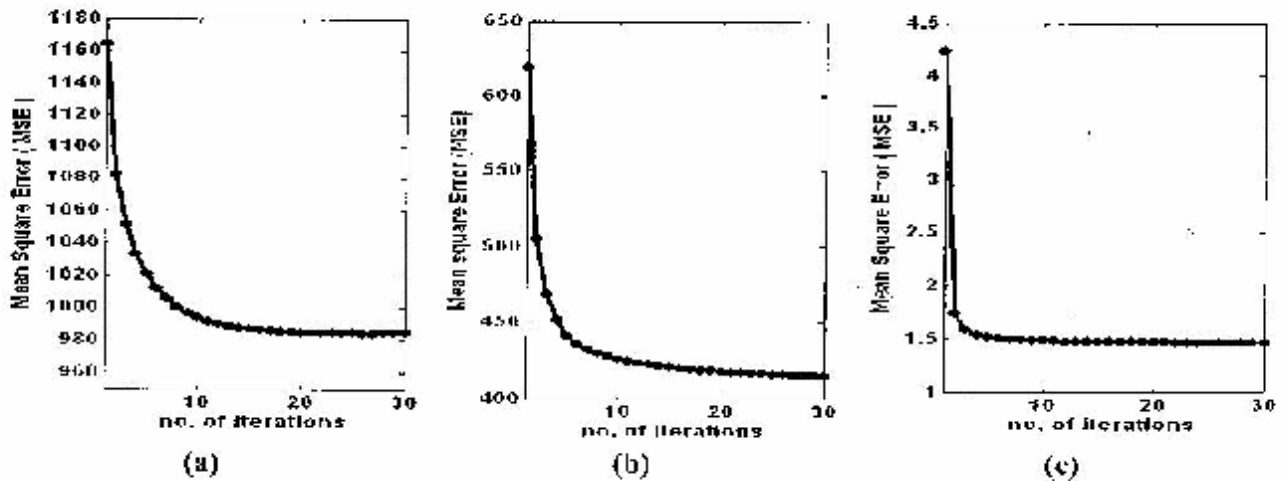


Fig.(5) Mean Square Errors (MSE) Versus no. of iterations For:
 (a) Restored images of the degraded image shown in Fig.(1-b)
 (b) Restored images of the degraded image shown in Fig.(1-c)
 (c) Restored images of the degraded image shown in Fig.(1-d)

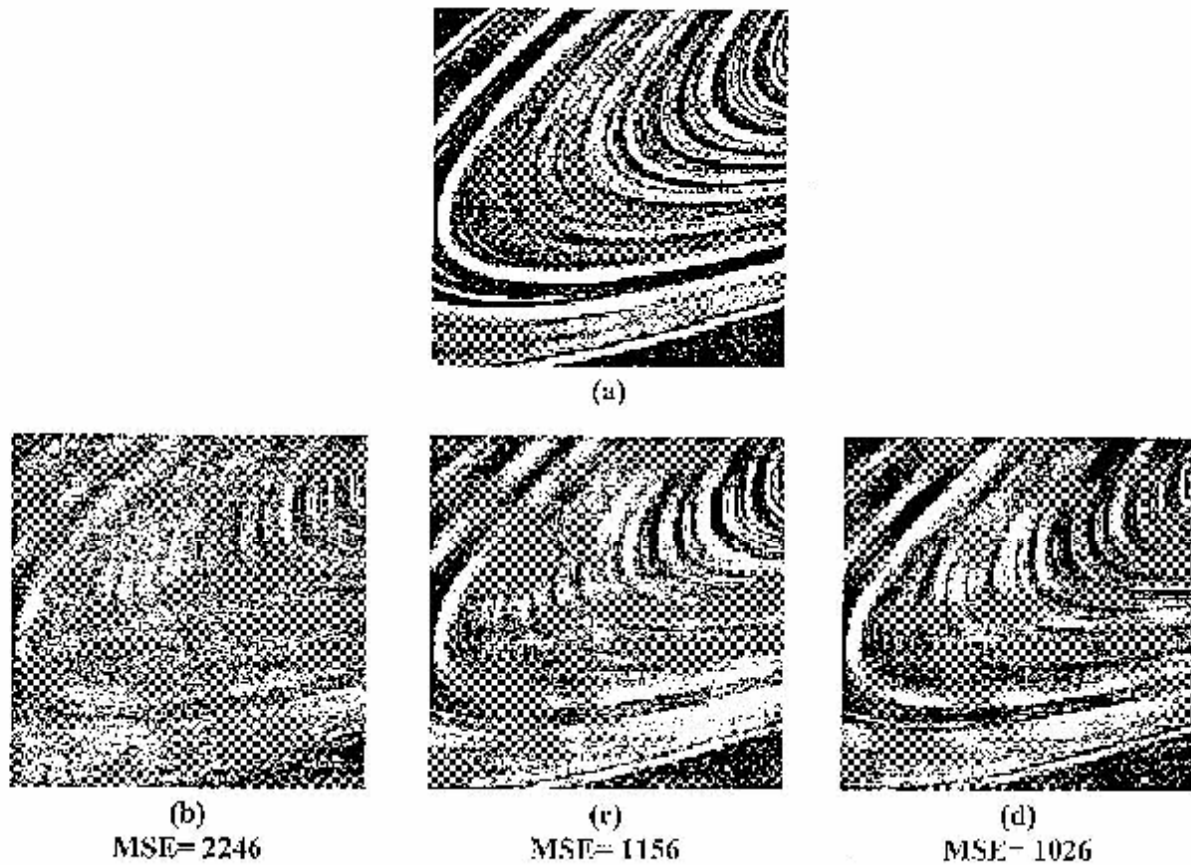


Fig.(1)

- (a) Original image, "Saturn rings" of size 128x128
- (b) Degraded image with Gaussian blur function of $\sigma = 1$ and with additive Gaussian noise with SNR=10 dB
- (c) Degraded image with Gaussian blur function of $\sigma = 1$ and with additive Gaussian noise with SNR=20 dB
- (d) Degraded image with Gaussian blur function of $\sigma = 1$ and with additive Gaussian noise with SNR= 50 dB

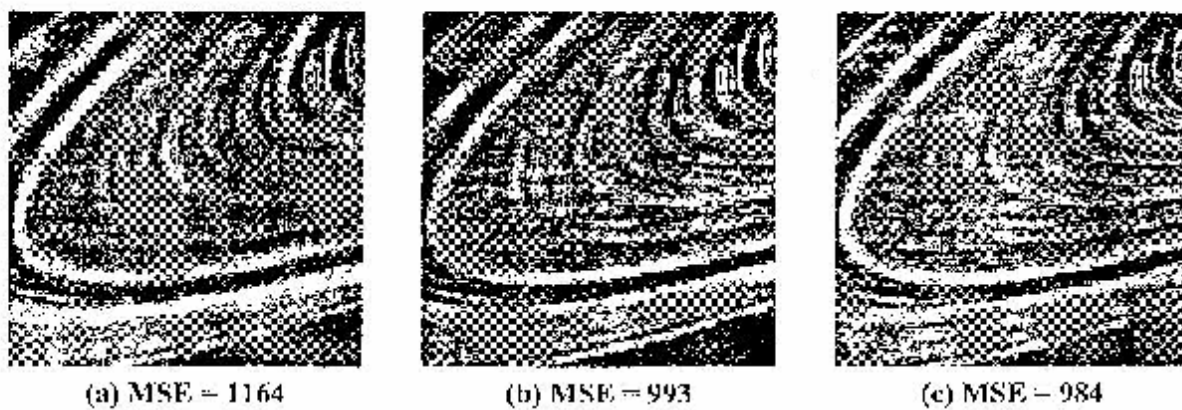


Fig.(2) Restored images of Fig.(1-b) after:
 (a) 1 iteration (b) 10 iterations (c) 30 iterations

Conclusion

In this paper, we construct an adaptive iterative Wiener filter to restore the degraded (blurred and noisy) images. The aim of this method is to estimate the power spectral density of the original image from degraded image using an iterative method.

From the above figures, we can conclude that the adopted iterative Wiener filter is an efficient method to restore the degraded images. We also conclude that the mean square error of the restored images decreases with the increasing the number of iterations until the result convergent. Moreover, the convergence, mostly, happens after 10 iterations, and also decreased with increasing SNR. Finally, we can conclude that the ratio of the MSE of the degraded image to the corresponding restored image at iteration 30 will increased with increasing SNR, i.e. this method has better performance for less degradation parameters, i.e. with high SNR.

References:

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Iterative Wiener Filter

Wiener filter requires a priori knowledge of the power spectral density of original image, which is, practically, often unavailable. Thus the estimation of power spectral density of the original image is a crucial problem in the restoration field. The second type of restoration techniques that removes some of the assumptions used in the derivation of the Wiener filtering [3, 4], which is known as an iterative solution. The iterative restoration filter has an interesting benefit over the one shot solution (non iterative filter) [5, 6]. This is because an iterative solution builds up the filter until a convergence endpoint [7]. Therefore, the main aim of this paper is to estimate the power spectral density of the original image using an adapted iterative method. The process of this algorithm based on three major steps, these are: First: estimation of the power spectral density of the original image was constructed from previous iteration [5, 8], using the autocorrelation function of the whole image instead of splitting the image. Second: Wiener filter that have been used to restore the degraded image were adapted in spacial domain [9]. Third: The iterations continue until the mean square error "MSE" converges..

The process of the adapted iterative Wiener filter is listed below:

Initial data:

$$R_g(i) = R_x = E[g \cdot g^*] \text{ and } H_s = H \text{ and } R_n = E[n \cdot n^*]$$

Construct Wiener filter:

$$R_f(i) = \sum f_r^* f_s$$

$$\Sigma(i) = [HR_f(i)H^* + R_n]^{-1}$$

$$W(i+1) = R_g(i)H^*\Sigma(i) \dots\dots\dots(10)$$

Restoring the image:

$$\hat{f}(i+1) = W(i+1)g \dots\dots\dots(11)$$

Evaluate MSE between $\hat{f}(i)$ and $\hat{f}(i+1)$

If $|MSE(i+1) - MSE(i)| \leq \mu$ then end, else continue. Where μ is a ve value, determined by experience and trial and error.

$$f(i+1) = \hat{f}(i+1) \dots\dots\dots(12)$$

Construct new image power spectral:

$$R_f(i+1) = E[f(i+1) \cdot f^*(i+1)]$$

$$= E[W(i+1)g(i+1) \cdot g^*(i+1)W^*(i+1)]$$

$$= W(i+1)R_gW^*(i+1)$$

$$= R_g(i)H^*\Sigma(i)R_g\Sigma(i)HR_f^*(i) \dots\dots\dots(13)$$

GOTO step 2.

Results and Discussion

A color image of 128*128 pixels size, Saturn rings-image, as shown in figure (1-a), was used to check the quality of the adaptive iterative Wiener filter.

The degraded (blurred and noisy) images are simulated as follows:

- 1-The blurred images were simulated by convolving the original image with Gaussian function of standard deviation (σ), one values of $\sigma = 1$ has been taken.
- 2-Random noise of Gaussian distribution with zero means was added to the blurred image (obtained in step 1). Different SNR = 10 dB, 20 dB, and 50 dB, have been taken.

Figure (1-b, 1-c, and 1-d) shows the original image after degraded with Gaussian blurring function of standard deviation ($\sigma = 1$) and additive Gaussian noise with signal to noise ratio (SNR) are 10 dB, 30 dB, and 50 dB, respectively. The figure shows, also, the Mean Square Error (MSE) of the degraded image with respect to the original image.

3- To restore the above simulated degraded images, we construct the procedure listed in section (2).

Figure (2) shows the restored images and the corresponding MSE for the degraded image that blurred with Gaussian blur of $\sigma = 1$ and SNR=10 dB, after 1 iteration, 10 iterations, and 30 iterations, respectively.

Figure (3) shows the restored images and the corresponding MSE for the degraded image that blurred with Gaussian blur of $\sigma = 1$ and SNR=30 dB, after 1 iteration, 10 iterations, and 30 iterations, respectively.

Figure (4) shows the restored images and the corresponding MSE for the degraded image that blurred with Gaussian blur of $\sigma = 1$ and with SNR=50 dB, after 1 iteration, 10 iterations, and 30 iterations, respectively.

Figure (5) shows the MSE for the restored images versus no. of iterations for the degraded image blurred with Gaussian blur of $\sigma = 1$ and with SNR=10 dB, 30 dB, and 50 dB, respectively.

Note that, by experience and for mathematical simplicity, we fixed the no. of iteration to 30, since the convergence is done less than 20th iteration. But, for more safety and certainty we take no. of iteration equal 30.

Restoration of Astronomical Images Using an Adaptive Iterative Wiener Filter

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Abstract

In this paper, an iterative Wiener filter, were adapted. The aim of this method is to estimate the power spectral density of the original image from degraded image using an iterative method. The adapted filter was designed for restoring astronomical images that are blurred with space-invariant point spread function and corrupted with additive noise. Different degradation parameters, i.e. different signal to noise ratio were considered. The results using an adaptive filter were compared, quantitatively, using mean square error (MSE). Results shows that this method has better performance for restoring the degraded images, especially for high signal to noise ratio.

Introduction

Mathematical degradation model is given by [1,2]:

$$g(x, y) = f(x, y) \otimes h(x, y) + n(x, y) \dots\dots\dots(1)$$

Where, $g(x, y)$ is the degraded image, and $f(x, y)$ is the original "ideal" image. $h(x, y)$ is the blurring function, called point spread function "PSF", which was assumed to be space invariant PSF [1]. The blur may be space-invariant PSF "SIPSF" or space variant PSF "SVPSF". For mathematical simplicity, many types of blurring can be approximated as SIPSF or isoplanctic process. Thus, the image can be represented by a convolution of space invariant PSF with the ideal image $f(x, y)$ [1], $n(x, y)$ is the noise function which was assumed to be additive, Gaussian distribution with zero mean, and independent with the object. \otimes denotes convolution process. In case of sampled image, equation (1) takes the form [1]:

$$g = Hf + n \dots\dots\dots(2)$$

Where, H is a blurring function, n is a white Gaussian noise, f is the original image, and g is the degraded image. We prefer this notation since it is compact and general. Restoration, process is that attempts to reconstruct an image that has been degraded using a priori knowledge about the degradation phenomenon.

Wiener Filter

Wiener filter, minimizing the mean squared error between the original image " $f(x, y)$ " and the restored image " $\hat{f}(x, y)$ ", is often used to restore degraded images. Wiener filter " $W(u, v)$ " can be described in the frequency domain as [2]:

$$W(u, v) = \frac{H_c^*(u, v)}{|H_c(u, v)|^2 + \left[\frac{S_n(u, v)}{S_f(u, v)} \right]} \dots\dots\dots(3)$$

Where $H_c(u, v)$ is the Fourier transform (FT) of the expected PSF, $*$ denotes complex conjugate, $S_n(u, v)$ and $S_f(u, v)$ are the power spectral density of noise and signal, respectively. Since, the power spectral ratio of the noise and signal, almost, unknown, therefore, this ratio is estimated by Signal to Noise Ratio "SNR" parameter, [1], i.e.

$$\frac{S_n(u, v)}{S_f(u, v)} = \frac{1}{(SNR)^2} \dots\dots\dots(4)$$

Hence, eq. (3) can be rewritten as follows:

$$W(u, v) = \frac{H_c^*(u, v)}{|H_c(u, v)|^2 + \frac{1}{(SNR)^2}} \dots\dots\dots(5)$$

Wiener filter, i.e. eq.(3), in matrix form " W " is given by [2]:

$$W = R_f H_c^* [H_c R_f H_c^* + R_n]^{-1} \dots\dots\dots(6)$$

Where $[.]^{-1}$ denotes matrix inverse, and bold representation means matrix multiplication. R_f and R_n represent the correlation matrices of f and n , respectively, defined respectively, by [2,3]:

$$R_f = E[f f^*] \text{ and } R_n = E[n n^*] \dots\dots\dots(7)$$

Where, $E[.]$ denotes the expectation operation.

Finally, the restored image " $\hat{f}(x, y)$ ", using Wiener filter, is given by:

$$\hat{f}(x, y) = FT^{-1} [W(u, v) G(u, v)] \dots\dots\dots(8)$$

Equation (8) in matrix form is given by:

$$\hat{f}(x, y) = W^* g \dots\dots\dots(9)$$