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The Neutron Halo Structure of ^{11}Be and ^{14}Be Nuclei Studied by the Binary Cluster Model

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Abstract

We have calculated some of the basic structural properties such as the ground state proton, neutron and matter densities and elastic form factors of halo nuclei namely, ^{11}Be and ^{14}Be using the binary cluster model (BCM) within the Gaussian and harmonic oscillator wave functions. The halo nuclei have been considered as a compact core with loosely bound valence neutron(s) forming the halo. The comparison between the calculated and experimental matter density distributions supports the halo structure for ^{11}Be [^{14}Be] where the valence one neutron [two neutrons] occupy the $1p_{1/2}$ [$(2s_{1/2})^2$] orbital. We have also calculated the reaction cross sections (σ_R) for these nuclei using the Glauber model with an optical limit approximation at low and high energies. The calculated (σ_R) of these nuclei give a good description of experimental data at both low and high energies.
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Keywords: Glauber model, halo nuclei, binary cluster model.

Introduction

The study of neutron-rich drip line nuclei and neutron (proton) halo is one of most prominent branches of modern nuclear physics [1,2]. A variety of advanced techniques have been introduced to study the structural properties of neutron and proton halo nuclei [3-8]. The separation energy of the outer nucleon(s) in nuclei near the drip line becomes extremely small. In such a weakly bound nuclei the density distributions show an extremely long tail, called a halo. The density of a halo has strongly effects on the reaction cross sections although it is very low and leads to new features for these nuclei [9]. The charge and nuclear matter distributions as well as root mean square (rms) radii of nuclear matter contain an important insight on nuclear wave functions and nuclear potentials. The matter density distributions and radii of halo nuclei have been studied by measurements of the reaction and interaction cross sections [10]. The relation between the nuclear density distribution and the total reaction cross section is studied by the Glauber model [11].

The neutron-rich nuclei ^{6,8}He, ^{11,14}Be and ^{11}Li are the best investigated and established neutron-halo nuclei. Abdullah [12] has been used the three body model of (Core +2n) within the radial wave functions of Woods-Saxon (WS) potential to investigate the ground state densities, the corresponding rms radii and elastic form factors of neutron-rich ^{6}He,^{11}Li,^{12}Be and ^{14}Be halo nuclei. The obtained results were discussed and compared with the experimental one. The long tail manner was obviously revealed in the calculated neutron and matter densities of these nuclei. Abdullah [13] has been investigate the ground state properties such as, the binding energy per nucleon, the ground state densities and the corresponding rms radii of two-neutron ^{6}He,^{11}Li,^{12}Be and ^{14}Be halo nuclei by means of the Skyrme-Hartree-Fock (SHF) method with MSK7 Skyrme parameter.

Hamoudi *et al.* [14] have been studied the ground state densities of unstable proton-rich ^{9}C,^{12}N and ^{23}Al exotic nuclei in the framework of the two-frequency shell model (TFSM) and the binary cluster model (BCM). In BCM, the internal densities of the clusters are described by single particle Gaussian wave functions. They found that the calculated results are in very good agreement with experimental data. Hamoudi and Abdullah [15] have been studied the ground state densities of unstable neutron-rich ^{11}Li and ^{12}Be halo nuclei in the framework of the binary cluster model (BCM). The internal densities of the clusters were described by the single particle harmonic oscillator wave functions. The long tail performance was clearly noticed in the calculated neutron and matter density distributions of these nuclei. The structures of the two valence neutrons in ^{11}Li and ^{12}Be were found to be mixed configurations with dominant $(1p_{1/2})^2$.

In the present work, we will use the binary cluster model (BCM) within the Gaussian and harmonic oscillator wave functions to study some of the basic structural properties such as the ground state proton, neutron and matter densities and elastic form factors of halo nuclei namely, ^{11}Be and ^{14}Be. We will also study the reaction cross sections for these nuclei using the Glauber model with an optical limit approximation at low and high energies.

Theory

In the case of halo nuclei, it is reasonable to parameterize the core and halo densities separately. In BCM [16], the halo nuclei are considered as composite projectiles of mass A_p and described, in Fig.(1), as core and valence clusters, of masses A_c and A_v bounded with a state of relative motion. It is assumed that $A_c \geq A_v$. The matter density of the composite projectile is given by [17]:

$$\rho_m(r) = \rho_c(r) + \rho_v(r), \quad \text{(1)}$$

where $\rho_c(r)$ and $\rho_v(r)$ are the densities of core and valence clusters, respectively.

In this study, we have applied two parameterizations of matter density distributions labeled as G+G (Gaussian–Gaussian) and HO+ HO (Harmonic oscillator) to study the ground state densities of halo nuclei.

In the G+G parameterization, the densities of core and valence clusters are described by the Gaussian wave functions as [16]

$$\rho_m(r) = A_c\, g^{(3)}(\hat{\alpha}_c, r) + A_v\, g^{(3)}(\hat{\alpha}_v, r), \quad \text{(2)}$$

where $g^{(3)}$ is the normalized 3-dimensional Gaussian function

$$g^{(3)}\left(\hat{\alpha}_{c(v)}, r\right) = \frac{1}{\pi^{3/2}\hat{\alpha}_{c(v)}^{3}}\ e^{-r^2/\hat{\alpha}_{c(v)}^{2}}, \quad \text{(3)}$$

$$\int g^{(3)}\left(\hat{\alpha}_{c(v)}, r\right) d\vec{r} = 1 \quad \text{(4)}$$

While in the HO+HO parameterization, the densities of core and valence clusters are described by the HO wave functions as [15,18,19]:

$$\rho_c(r) = \frac{1}{4\pi}\sum_{n\ell} X_c^{n\ell} \left|R_{n\ell}(r, \hat{b}_c)\right|^2 \quad \text{(5)}$$

$$\rho_v(r) = \frac{1}{4\pi} X_v^{n\ell} \left|R_{n\ell}(r, \hat{b}_v)\right|^2 \quad \text{(6)}$$

where $X_{c(v)}^{n\ell}$ represents the number of protons or neutrons, in the sub-shell nlj.

The size parameters of the Gaussian $\left(\hat{\alpha}_c{}^2,\ \hat{\alpha}_v{}^2\right)$ and harmonic oscillator $\left(\hat{b}_c{}^2,\ \hat{b}_v{}^2\right)$ wave functions are given by [16]:

$$\hat{g}_c{}^2 = g_c{}^2 + \left(\frac{A_v g}{A_v + A_c}\right)^2 \quad \text{(7)}$$

$$\hat{g}_v{}^2 = g_v{}^2 + \left(\frac{A_c g}{A_v + A_c}\right)^2, \quad g \equiv \alpha, b \quad \text{(8)}$$

The matter density of Eq. (1) can be written as [20]:

$$\rho_m(r) = \rho^p(r) + \rho^n(r), \quad \text{(9)}$$

where $\rho^p(r)$ and $\rho^n(r)$ are the proton and neutron densities of halo nuclei, respectively and written as [20]:

$$\rho^p(r) = \rho_c^p(r) + \rho_v^p(r) \quad \text{(10)}$$

and

$$\rho^n(r) = \rho_c^n(r) + \rho_v^n(r) \quad \text{(11)}$$

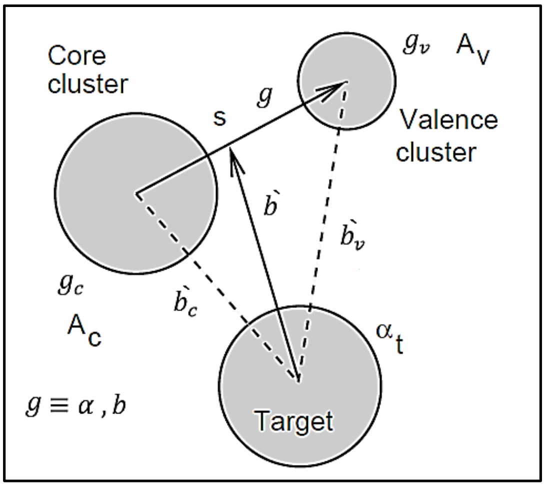

Fig.(1): The coordinates of the two-cluster projectile and target [16].

We study the elastic form factors for considered nuclei using the Plane Wave Born Approximation (PWBA) within the proton density distribution obtained by HO+HO parameterization. In the PWBA, the elastic form factors are written as [21]:

$$F(q) = \frac{4\pi}{Z}\int_0^\infty \rho_p(r) j_0(qr) r^2 dr, \quad \text{(12)}$$

where $j_0(qr)$ is the zero-order spherical Bessel function and q is the momentum transfer from the incident electron to the target nucleus. Inclusion the corrections of the finite nucleon size $F_{fs}(q) = exp(-0.43q^2/4)$ and the center of mass $F_{cm}(q) = exp(b^2q^2/4A)$ in the calculations needs multiplying the form factor of Eq. (12) by these corrections [22].

The reaction cross section of considered nuclei is studied by the Glauber model within an optical limit (OL) approximation which can be expressed as [23]:

$$\sigma_R = 2\pi \int [1 - T(b)]\, b\, db \left(1 - \frac{B_c}{E_{cm}}\right), \quad .... (13)$$

where E_{cm} is the kinetic energy in the center of mass system, B_c is Coulomb barrier and $T(b)$ is the transparency function at impact parameter b.

In the optical limit (OL) approximation the $T(b)$ is written as [16]

$$T(b) = \left|S_{el}^{OL}(b)\right|^2, \quad .... (14)$$

where $S_{el}^{OL}(b)$ is the elastic $S-$matrix for the target-projectile system given as [16]:

$$S_{el}^{OL}(b) = exp[iO_{PT}(b)] \quad .... (15)$$

$$O_{PT}(b) = \int_{-\infty}^{\infty} dR_3 \int dr_1 \int dr_2 \rho_P(r_1)\rho_T(r_2) f_{NN}\left(\left|R + r_1 - r_2\right|\right) \quad .... (16)$$

is the overlap of the ground state densities of projectile and target (ρ_P and ρ_T, respectively).

Results and discussion

We have calculated some of the basic structural properties such as the ground state proton, neutron and matter densities and elastic form factors of halo nuclei namely, ^{11}Be and ^{14}Be using the binary cluster model (BCM) within the Gaussian and harmonic oscillator wave functions. We have also calculated the reaction cross sections for these nuclei using the Glauber model with an optical limit approximation at low and high energies. We have assumed that ^{11}Be [^{14}Be] consists of a core ^{10}Be [^{12}Be] plus one halo neutron [two halo neutrons]. In HO+HO parameterization, the HO wave functions have been used to describe the density distributions of core and valence clusters. The core ^{10}Be and ^{12}Be nuclei have the configuration $\{(1s_{1/2})^4, (1p_{3/2})^6\}$ and $\{(1S_{1/2})^4, (1P_{3/2})^6, (1P_{1/2})^2\}$, respectively. We have considered that the outer one neutron of ^{11}Be occupies the orbit $1p_{1/2}$, whereas the outer two neutrons of ^{14}Be occupy the orbit $2s_{1/2}$. In G+G parameterization, the Gaussian wave functions have been used to describe the density distributions of core and valence clusters. Table (1) illustrates some properties of halo nuclei ^{11}Be and ^{14}Be. Table (2) shows the values of the Gaussian and HO size parameters utilized in our study for the considered nuclei obtained by Eqs. (7) and (8).

Table (1)
Some properties for the selected nuclei.

Halo nucleus	J^π [24]	Type of halo nucleus	Half life time $(\tau_{1/2})$ [24]	Separation Energy (MeV) [25]
^{11}Be	$1/2^+$	One neutron halo	13.76 s	S_n=0.501
^{14}Be	0^+	Two- neutron halo	4.35 ms	S_{2n}=1.27

Table (2)
The values of the Gaussian and HO size parameters utilized in our study.

Halo nucleus	Core nucleus	G+G		HO+HO	
		$\hat{\alpha}_c$ (fm)	$\hat{\alpha}_v$ (fm)	$\hat{b}_c$ (fm)	$\hat{b}_v$ (fm)
^{11}Be	^{10}Be	1.68	5.13	1.75	4.18
^{14}Be	^{12}Be	2.15	4.24	1.87	2.99

Fig.(2) illustrates the calculated matter densities of ^{11}Be [Figures-2(a) and 2(b)] and ^{14}Be [Figures-2(c) and 2(d)] together with their experimental matter densities (the grey area) taken from Refs. [26, 27] for [^{11}Be,^{14}Be]. The red, blue and dashed curves correspond to core, valence (halo) and matter densities, respectively. These densities are obtained by the Gaussian (left panel) and harmonic oscillator (right panel) parameterizations. As shown from these figures there is a very good accordance between the calculated (the dashed curves) and experimental matter densities for the above selected nuclei.

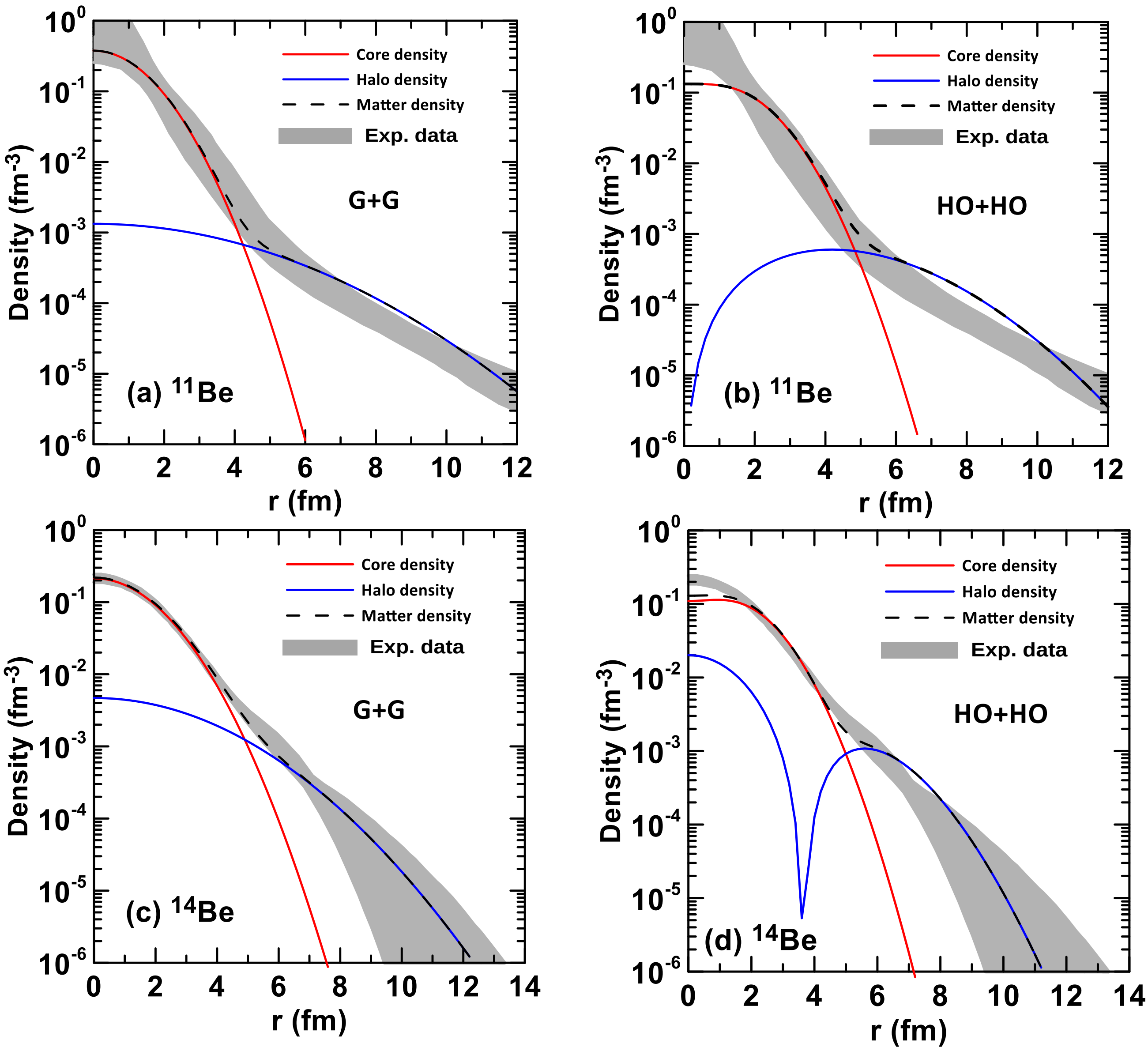

Fig.(2): The core, halo and matter density distributions for halo nuclei ^{11}Be and ^{14}Be.

Fig.(3) shows the proton, neutron and matter densities displayed as red, blue and dashed curves, respectively. The long tail property is obviously revealed in blue curves due to the existence of neutron(s) in the halo orbits. As the protons does not exist in the halo orbits the steep slope behavior is clearly shown in red curves.

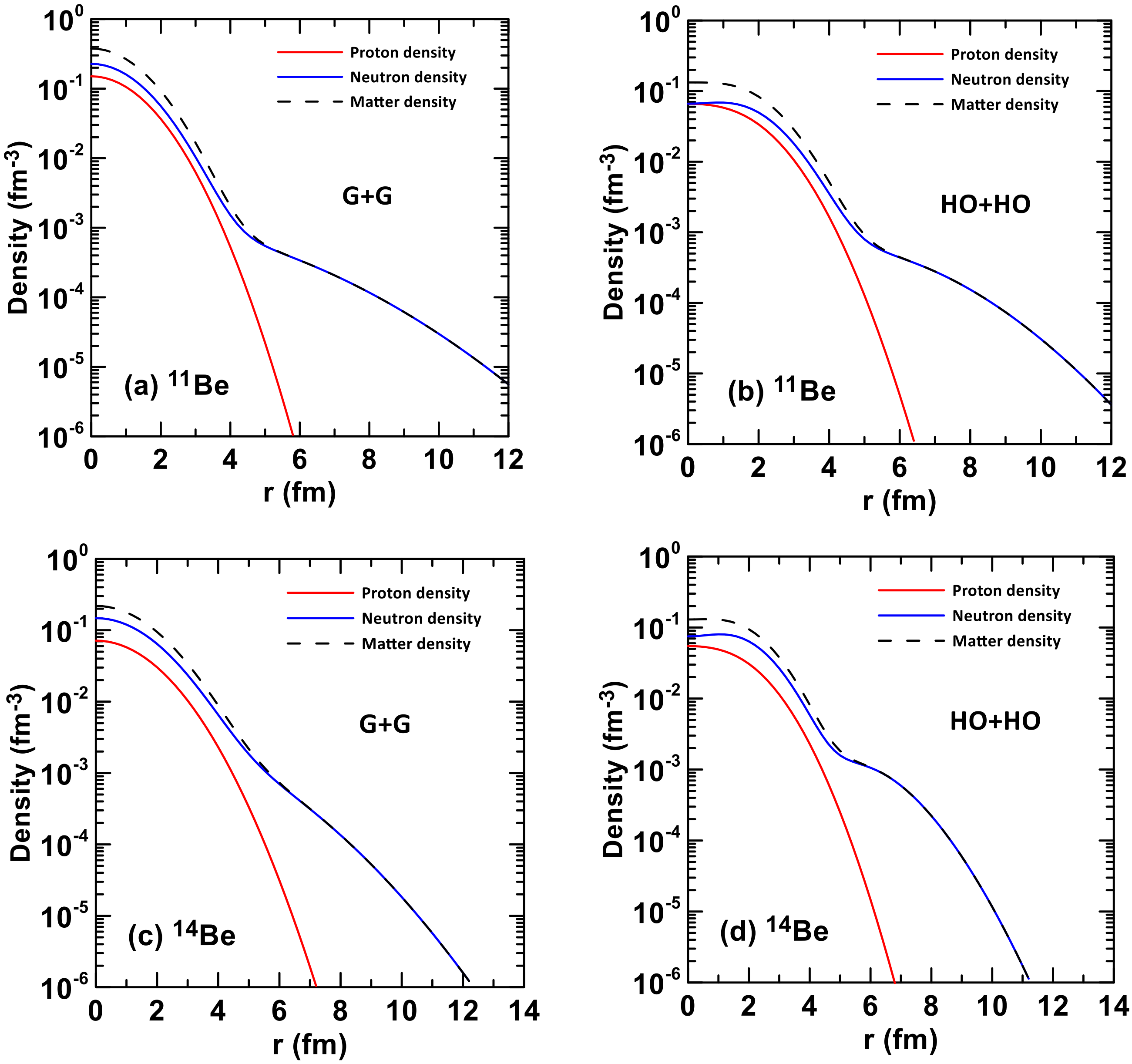

Fig.(3): The proton, neutron and matter density distributions for halo nuclei ^{11}Be and ^{14}Be.

Fig.(4) exhibits the matter densities of ^{9,11}Be (top panel) and ^{9,14}Be (bottom panel) obtained by the Gaussian [Figures 4(a) and 4(c)] and harmonic oscillator [Figures 4(b) and 4(d)] parameterizations. The matter densities of the unstable (^{11}Be, ^{14}Be) nuclei and those of the stable ^{9}Be isotope are plotted with dashed and red distributions, respectively. It is seen from these figures that the red and dashed distributions are different. The dashed distributions extended more than the red distributions due to the weak binding of the outer neutron (s) in unstable nuclei ^{11}Be and ^{14}Be.

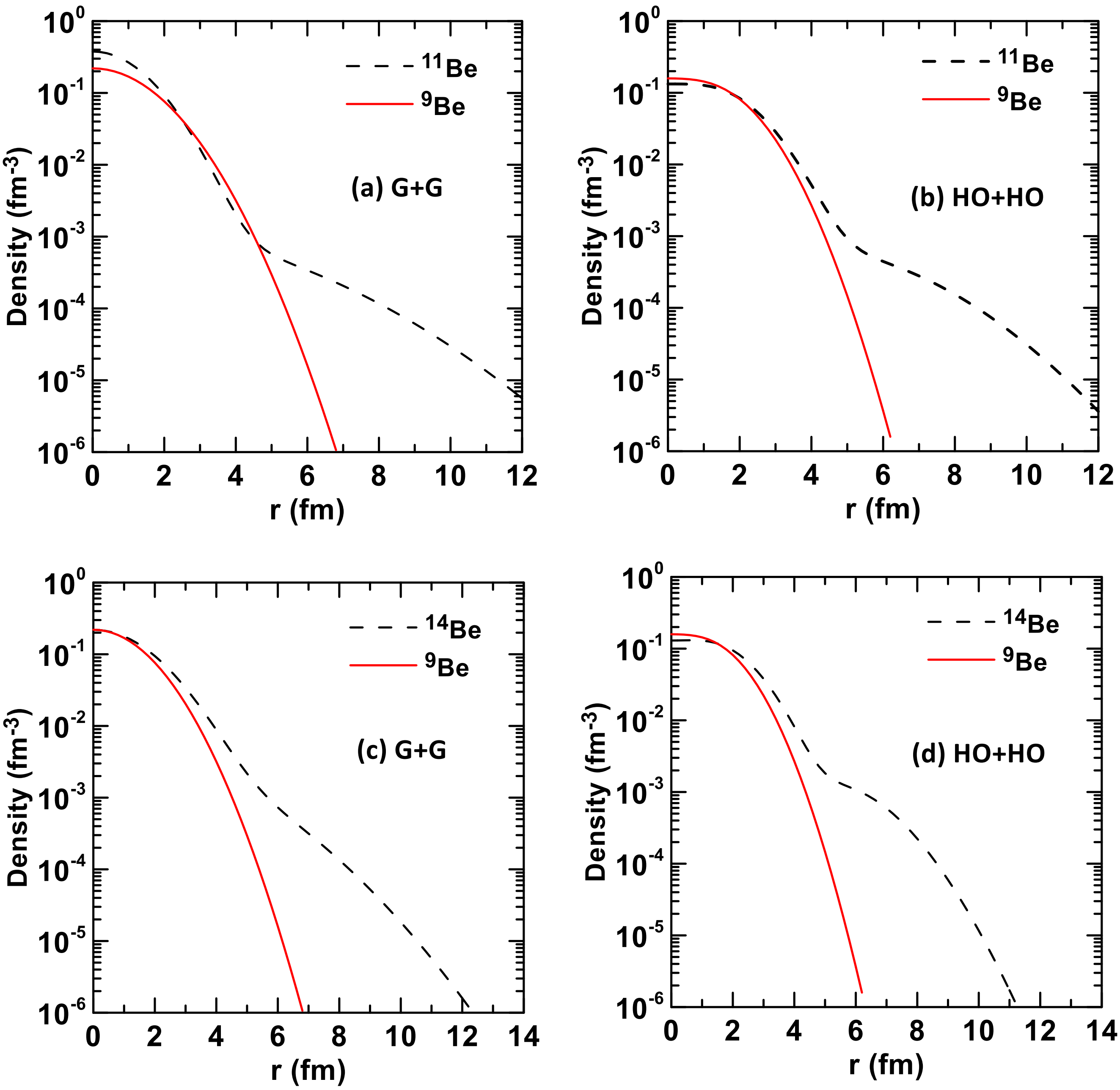

Fig.(4): Matter density distributions for ^{9,11}Be and ^{9,14}Be.

Figures 5 (a) and 5 (b) demonstrates the elastic form factors of ^{9,11}Be and ^{9,14}Be calculated by PWBA with the proton densities obtained by HO parameterization. The elastic form factors of the unstable (^{11}Be, ^{14}Be) nuclei and those of the stable ^{9}Be isotope are plotted with dashed and red curves, respectively. The experimental data of ^{9}Be (filled circles) taken from Ref. [28] are also shown in these figures. From these figures it is evident that the dashed and red curves have one diffraction minimum. The minimum location of the dashed curve shifts to backward due to the difference in the center of mass correction which depends on the mass number and the size parameter.

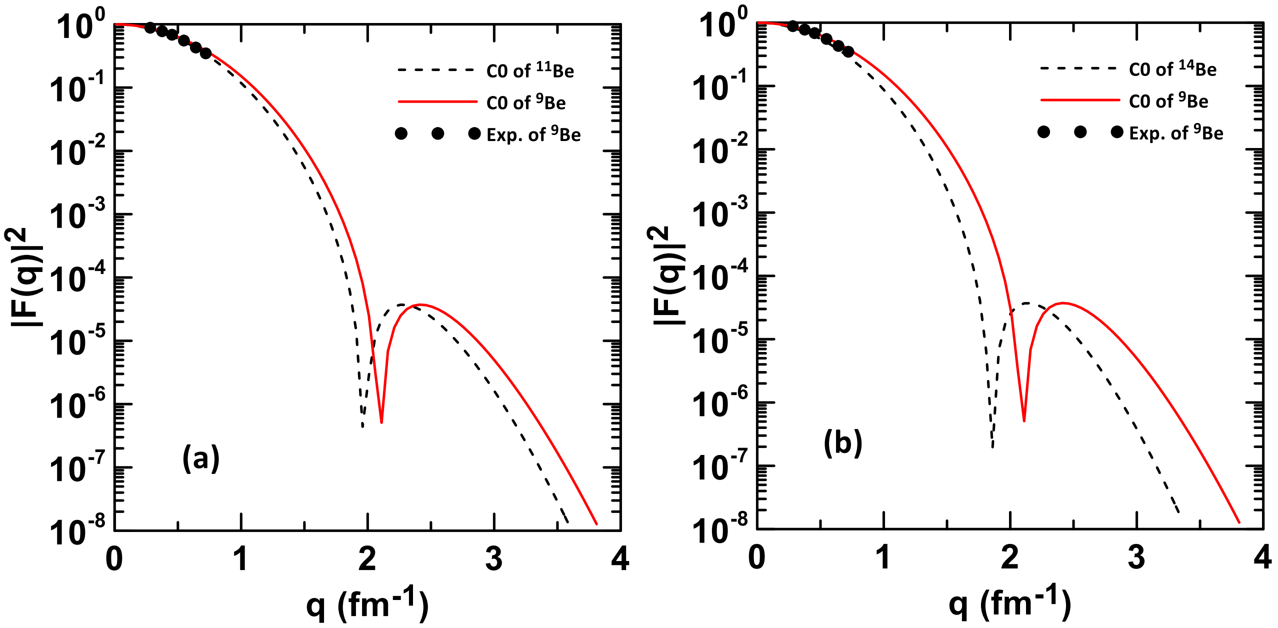

Fig.(5): Elastic form factors for ^{9,11}Be and ^{9,14}Be.

The OL approximation of the Glauber model within Gaussian wave functions has been employ to study the reaction cross section (σ_R) of neutron- rich ^{11}Be and ^{14}Be nuclei on ^{12}C target at both low and high energies. The calculated and experimental results of σ_R are tabulated in Table (3) and plotted in Figures 6 (a) and 6 (b) as a function of energy. Therein, the open and filled circle symbols corresponded to the calculated and experimental results, respectively. From these figures, we can see that there is a very good accordance between the calculated and experimental results within quoted error.

Table (3)
The calculated and experimental results of σ_R for ^{11}Be and ^{14}Be halo nuclei.

Halo nuclei	Calculated σ_R (mb)	Experimental σ_R (mb) [9,29]	Energy (MeV) [9,29]
^{11}Be	1570	1560 ± 30	33
	934	942 ± 8	790
^{14}Be	1698	1690 ± 76	42
	1514	1508 ± 93	75
	1070	1063 ± 69	400
	1145	1139 ± 90	790
	1089	1082 ± 34	850

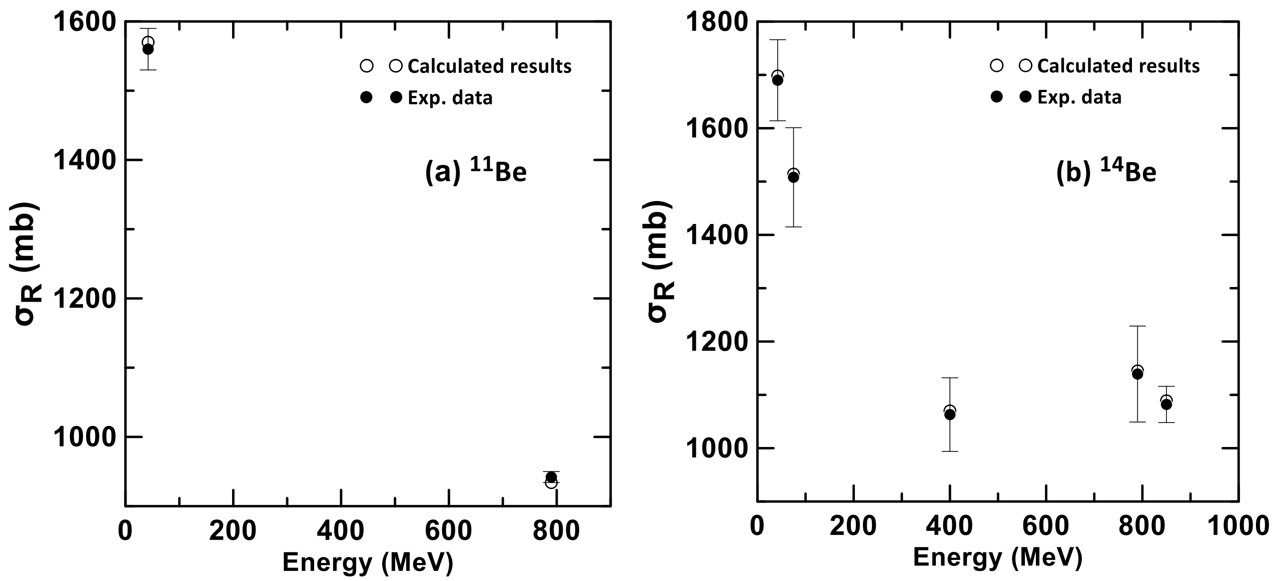

Fig.(6): The calculated and experimental results of σ_R for ^{11}Be and ^{14}Be halo nuclei.

Conclusions

This study draws the following conclusions:

1. It is found that the BCM within the Gaussian and harmonic oscillator wave functions is capable of providing theoretical predictions on the structure of exotic (halo) nuclei and be in a satisfactory description with those of experimental data.
2. The halo structure of one neutron (^{11}Be) and two neutrons (^{14}Be) exotic nuclei is emphasized through exhibiting the long tail performance in their calculated neutron and matter density distributions, where this performance is considered as a distinctive feature of halo nuclei.
3. The steep slope behavior is obviously seen in calculated proton density distributions of considered halo nuclei, as a result of the absence of protons in halo orbits, where all protons are found in their cores only.
4. The structure of the one valence (halo) neutron in ^{11}Be is in a pure $1p_{1/2}$ whereas the structure of the two valence neutrons in ^{14}Be are in a pure $(2s_{1/2})^2$.
5. It is found that the major difference between the calculated form factor of unstable (halo) nuclei and those of their stable isotope is attributed to the difference in the center of mass correction.
6. The calculated results of σ_R using the Glauber model with an optical limit approximation are in an good agreement with experimental data at both low and high energies.

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