Al-Nahrain Journal of Science
ANJS, Vol.23 (4), December, 2020, pp. 35–39

Study of the Exotic Structure of Neutron-Rich ^{14}B and ^{17}C Nuclei Using the Binary Cluster Model

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Articles Information

Received: 14.06.2020
Accepted: 01.10.2020
Published: 01.12.2020

Keywords:
Glauber model
Exotic nuclei
Binary cluster model
Neutron-rich nuclei

Abstract

The neutron, proton, and matter densities of the ground state of the ^{14}B and ^{17}C exotic nuclei are analyzed using the binary cluster model (BCM). Two density parameterizations are used in BCM calculations namely; Gaussian (GS) and harmonic oscillator (HO) parameterizations. According to the calculated results it found that, the BCM gives a good description of the nuclear structure for above neutron-rich exotic nuclei. The elastic form factors of the unstable ^{14}B and ^{17}C exotic nuclei and those of their stable isotopes ^{10}B and ^{13}C are determined using the plane-wave Born approximation. The main difference between the elastic form factors of unstable nuclei and their stable isotopes is due to the difference in the center of mass correction. Moreover, the Glauber model is used to calculate the matter rms radii and reaction cross section of these exotic nuclei. The calculate results of the mentioned nuclei give a good accordance with the experimental data.

DOI: 10.22401/ANJS.23.4.06

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1. Introduction

A modern and fascinating field of nuclear physics was developed with the introduction of experimental radioactive ion beam systems. Nuclear radius has been found to be observable by the interaction of high energy collisions with heavy-ion. It is a special way to determine the radius of matter for unstable nuclei. The first discovered in this field by Tanihata *et al*, they were noted that the total reaction cross-section was remarkably enhanced by unstable nuclei that impact stable targets [1]. An odd rms radius was found far bigger than other nearby isotopes for the ^{11}Li-rich nucleus of neutrons, showed a long tail in the distribution of nuclear matter density. Then the concept of neutron halo was defined [2-4]. A neutron halo is essentially the threshold effect of a bound state along the continuum; primarily due to its closeness to the particle continuum at the Fermi level. Owing to the pairing correlation, the valence nucleons may easily be scattered through the continuum states. The wave function's tail extends farther out to the central nuclear confining potential, contributing to the formation of a diffuse nuclear cloud around the cor. The well-acceptable parameters are limited binding energy and weak angular momentum for the phenomenon of the nuclear halo. The halo nucleus structure is expected to consist of a closely bound nucleus plus one or two loosely nucleons (The two-nucleon halo is known as Borromean [5], as neither core nor two nucleon binary subsystems are located in a bound state). These nuclei are often called exotic because of their huge number of neutrons (N) or protons (Z) that give them peculiar features such as halo and skin. The N/Z ratio is limited in stable nuclei between 1 and 1.5 while the nucleon separation energy of a proton, or a neutron, is nearly usually between 6 and 8 MeV; while N / Z in the unstable nuclei can be ranged between 0.6 and 4 MeV and the separating energy can vary from 40 MeV to 0 MeV. Half-life times are usually below one second for halo nuclei [6].

Hamoudi and Abdullah [7] have studied the ground state densities of unstable neutron-rich ^{11}Li and ^{12}Be halo nuclei in the framework of the BCM. The internal densities of the clusters were described by the single particle harmonic oscillator wave functions. The long tail performance was clearly noticed in the calculated neutron and matter density distributions of these nuclei. Daham and Abdullah [8] calculated some of the basic structural properties such as the ground state proton, neutron and matter densities and elastic form factors of halo nuclei namely, ^{11}Be and ^{14}Be using the BCM within the Gaussian and HO wave functions. They also calculated the σ_R for these nuclei using the Glauber model with an optical limit approximation at low and high energies.

In this work, The BCM will be used to analyze the features of ground state such as proton, neutron and matter densities and elastic form factors for the neutron-rich ^{14}B and ^{17}C exotic nuclei. Two density parameterizations will be used in BCM calculations namely; GS and HO parameterizations. Moreover, the Glauber model will be used to calculated the matter rms radii and σ_R of these exotic nuclei.

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2. Theory

In BCM [9], we assumed that the halo nuclei as composite projectiles of mass A_p and described, in Figure 1, as core and valence clusters, of masses A_c and A_v bounded with a state of relative motion. It is assumed that $A_c \geq A_v$. The matter density of the composite projectile is given by [7]:

$$\rho_m(r) = \rho_c(r) + \rho_v(r) \quad (1)$$

where $\rho_c(r)$ and $\rho_v(r)$ are the core and valence (halo) densities, respectively.

In this study, two density parameterizations are used namely; Gaussian (GS) and harmonic oscillator (HO) parameterizations.

In the GS parameterization, density distributions of the core and halo clusters are parameterized with Gaussian wave function [9]:

$$\rho_m(r) = A_c\, g^{(3)}(\hat{\alpha}_c, r) + A_v\, g^{(3)}(\hat{\alpha}_v, r) \quad (2)$$

where $g^{(3)}$ is the normalized 3-dimensional Gaussian function

$$g^{(3)}(\hat{\alpha}_{c(v)}, r) = \frac{1}{\pi^{\frac{3}{2}}\hat{\alpha}^3_{c(v)}}\, e^{-\frac{r^2}{\hat{\alpha}^2_{c(v)}}} \quad (3)$$

$$\int g^{(3)}(\alpha_{c(v)}, r)\, d\vec{r} = 1 \quad (4)$$

whereas in the HO parameterization, density distributions of the core and halo clusters are parameterized with HO wave function as [10]:

$$\rho_c(r) = \frac{1}{4\pi}\sum_{n\ell} X_c^{n\ell}\left|R_{n\ell}(r, \hat{b}_c)\right|^2 \quad (5)$$

$$\rho_v(r) = \frac{1}{4\pi} X_v^{n\ell}\left|R_{n\ell}(r, \hat{b}_v)\right|^2 \quad (6)$$

The Gaussian $({\hat{\alpha}_c}^2, {\hat{\alpha}_v}^2)$ and HO $({\hat{b}_c}^2, {\hat{b}_v}^2)$ size parameters are given by [7,9]:

$$\hat{g}_c{}^2 = g_c{}^2 + \left(\frac{A_v g}{A_v + A_c}\right)^2 \quad (7)$$

$$\hat{g}_v{}^2 = g_v{}^2 + \left(\frac{A_c g}{A_v + A_c}\right)^2, g \equiv \alpha, b \quad (8)$$

The matter density of equation (1) can be written as [11]:

$$\rho_m(r) = \rho^n(r) + \rho^p(r) \quad (9)$$

where $\rho^n(r)$ and $\rho^p(r)$ are the neutron and proton densities, respectively written as [11]:

$$\rho^n(r) = \rho_c^n(r) + \rho_v^n(r) \quad (10)$$

and

$$\rho^p(r) = \rho_c^p(r) + \rho_v^p(r) \quad (11)$$

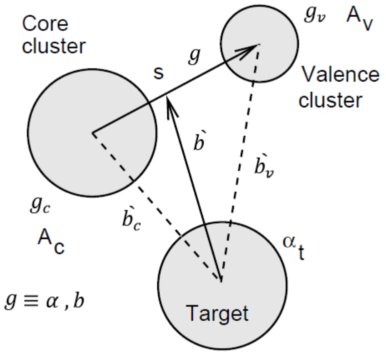

Figure 1. The coordinates of the target and two-cluster projectile [9].

The elastic form factors for considered nuclei are studied by the plane wave Born approximation (PWBA) within the proton density distribution obtained by HO parameterization. In the PWBA, the elastic form factors are written as [12]:

$$F(q) = \frac{4\pi}{Z}\int_0^\infty \rho_p(r) j_0(qr) r^2 dr \quad (12)$$

where $j_0(qr)$ is the zero-order spherical Bessel function and q is the momentum transfer from the incident electron to the target nucleus. Inclusion the corrections of the finite nucleon size $F_{fs}(q) = exp\left(-\frac{0.43q^2}{4}\right)$ and the center of mass $F_{cm}(q) = exp\left(\frac{b^2q^2}{4A}\right)$ in the calculations needs multiplying the form factor of equation (12) by these corrections.

The reaction cross section (σ_R) of considered nuclei is studied by the Glauber model within an optical limit approximation (OLA) which can be expressed as [13]:

$$\sigma_R = 2\pi\int[1 - T(b)]\, b\, db\, \left(1 - \frac{B_c}{E_{cm}}\right) \quad (13)$$

where E_{cm} is the kinetic energy in the center of mass system, B_c is Coulomb barrier and $T(b)$ is the transparency function at impact parameter b.

In the OLA the $T(b)$ is written as [9]:

$$T(b) = \left|S_{el}^{OL}(b)\right|^2 \quad (14)$$

Where $S_{el}^{OL}(b)$ is the elastic S-matrix for the target-projectile system given as [9]:

$$S_{el}^{OL}(b) = exp[iO_{PT}(b)] \quad (15)$$

$$O_{PT}(b) = \int_{-\infty}^{\infty} dR_3 \int d\vec{r}_1 \int d\vec{r}_2 \rho_P(r_1)\rho_T(r_2) f_{NN}(|\vec{R} + \vec{r}_1 - \vec{r}_2|) \quad (16)$$

is the overlap of the ground state densities of projectile and target ($\rho_P$and ρ_T, respectively).

3. Results and Discussion

The BCM is used to analyze the features of ground state such as proton, neutron and matter densities and elastic form factors for the neutron-rich ^{14}B ($S_n = 0.97$ MeV, $\tau = 12.5\, ms$) and ^{17}C ($S_n = 0.734$ MeV, $\tau = 193\, ms$) [14,15] exotic nuclei. Two density parameterizations are used in BCM calculations namely; Gaussian (GS) and harmonic oscillator (HO) parameterizations. Moreover, the Glauber model is used to calculated the matter rms radii and σ_R of these exotic nuclei.

We assumed that the nuclei ^{14}B ($J^\pi, T = 2^-, 2$) and ^{17}C $\left(J^\pi, T = \frac{3}{2}^+, \frac{5}{2}\right)$ have a structure of an inert cores ^{13}B $\left(J^\pi, T = \frac{3}{2}^-, \frac{3}{2}\right)$ and ^{16}C ($J^\pi, T = 0^+, 2$) plus one loosely bound neutron, consecutively. In the HO parameterization, the density distributions of the core and halo clusters are parameterized with HO wave function. $\{(1s_{1/2})^4, (1p_{3/2})^7, (1p_{1/2})^2\}$ and $\{(1s_{1/2})^4, (1p_{3/2})^8, (1p_{1/2})^2, (1d_{5/2})^2\}$ are the configurations of the core ^{13}B and ^{16}C nuclei, respectively. We considered that the outer neutron of ^{14}B and ^{17}C has mixed configuration of $1d_{3/2}$ and $2s_{1/2}$ using the following relation [16]:

$$\rho_v = \left\{\alpha\left|\phi_{2s_{\frac{1}{2}}}\right|^2 + (1-\alpha)\left|\phi_{1d_{\frac{3}{2}}}\right|^2\right\},\ \alpha \leq 1$$

where $\phi_{2s_{\frac{1}{2}}}$ and $\phi_{1d_{\frac{3}{2}}}$ refer to the halo neutron wave functions of $2s_{\frac{1}{2}}$ and $1d_{\frac{3}{2}}$ with occupation probabilities $\alpha = 0.6$ (in $2s_{\frac{1}{2}}$) and $\alpha = 0.4$ (in $1d_{\frac{3}{2}}$) for the halo neutron, respectively and α refer to the occupation probability of the $(2s_{1/2})$ orbital. In the GS parameterization, the density distributions of the core and halo clusters are parameterized with Gaussian wave function. The HO and Gaussian parameters utilized in these calculations for exotic nuclei are calculated by equation (7) and summarized in Table 1, whereas those of stable nuclei ^{10}B and ^{13}C are chosen to reproduce the measured matter rms radii for these nuclei and presented in Table 2.

Table 1. Gaussian and HO size parameters for the core and halo clusters.

Halo Nucleus	Core nucleus	GS		HO	
		$\hat{\alpha}_c$ (fm)	$\hat{\alpha}_v$ (fm)	$\hat{b}_c$ (fm)	$\hat{b}_v$ (fm)
^{14}B	^{13}B	1.857	4.852	1.647	3.634
^{17}C	^{16}C	1.874	4.238	1.746	3.367

Table 2. Gaussian and HO size parameters for the ^{10}B , ^{13}C stable nuclei.

Stable nucleus	α (fm)	b (fm)	$\langle r_m^2\rangle_{cal}^{\frac{1}{2}}$(fm)		$\langle r_m^2\rangle_{exp}^{\frac{1}{2}}$ (fm) [17]
			Gaussian	HO	
^{10}B	1.912	1.580	2.34	2.29	2.34±0.06
^{13}C	1.976	1.618	2.42	2.39	2.42±0.24

Figure 2 illustrates the contributions of the core nucleons (green curve) and the valence neutron (blue curve) to the matter density (dashed-dot red curve) for halo nuclei ^{14}B and ^{17}C obtained by the GS (left part) and HO (right part) parameterizations along with their experimental matter density (gray area) is taken from Refs. [18,19]. The top and bottom panels correspond to halo nuclei ^{14}B and ^{17}C, respectively. The halo nucleus distinctive property (*i.e.* the long tail behavior) is revealed in all dashed-dot red curves of Figure 2 which agree well with experimental data. Moreover, the dashed-dot red curve in Figures 2(a) and 2(d) is better describing the experimental data than that in Figures 2(b) and 2(c).

Figure 3 displays the densities of matter (dashed-dot red curve), neutron (blue curve), and proton (green curve). The long tail is the property that clearly revealed in the density of the neutron (blue curves) because it is found in the halo orbits. The steep slope behavior is clearly seen in the green curves since the protons are absent in the halo orbit and all the protons of these nuclei are located in their cores.

Figure 4 presents the distributions of matter density for the isotopes pairs ^{10,14}B (top panel) and ^{13,17}C (bottom panel) calculated by the two density parameterizations. The matter densities of unstable and stable nuclei are plotted with green and dashed-dot red curves, consecutively. From these figures, it can be seen that the matter density distributions for each isotopes pairs are different. The weak binding of the outer neutron in exotic ^{14}B and ^{17}C nuclei leads to the extended matter density distributions in them.

Figure 5 shows the calculated elastic form factors of ^{10,14}B and ^{13,17}C determined by PWBA with the proton densities obtained by HO parameterization along with the experimental data of the stable ^{10}B [20] and ^{13}C [21] isotopes. The form factors of the exotic nuclei, stable isotopes and experimental data of the stable nuclei are portrayed by the red curve, green curve and dotted symbols, consecutively. It obvious that the form factors for each isotopes pairs are quite different although they have the same proton number. The minima position of the red curve has left shift as compared with that of the green curve. This change is attributed to the difference in the center of mass correction, which depends on the mass number and the size parameter.

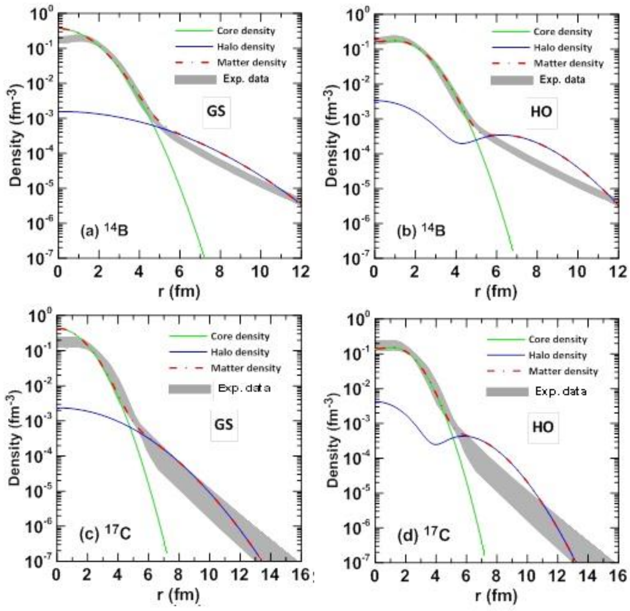

Figure 2. The matter, core and halo density distributions for halo nuclei ^{14}B and ^{17}C.

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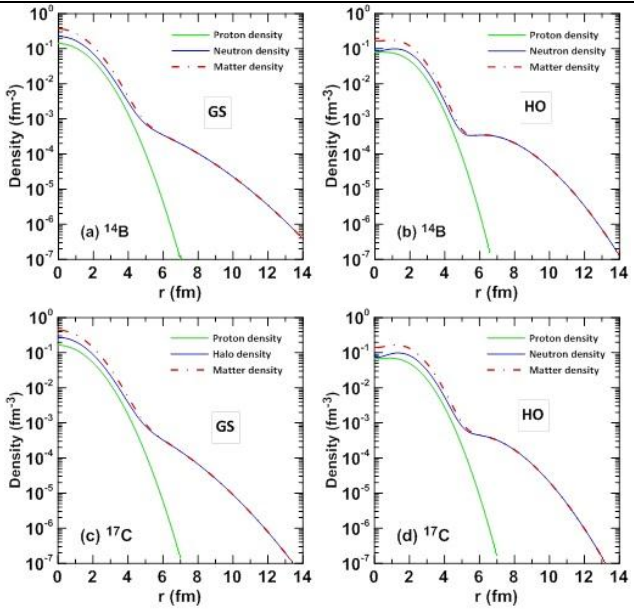

Figure 3. The matter, proton and neutron density distributions for^{14}B and ^{17}C halo nuclei.

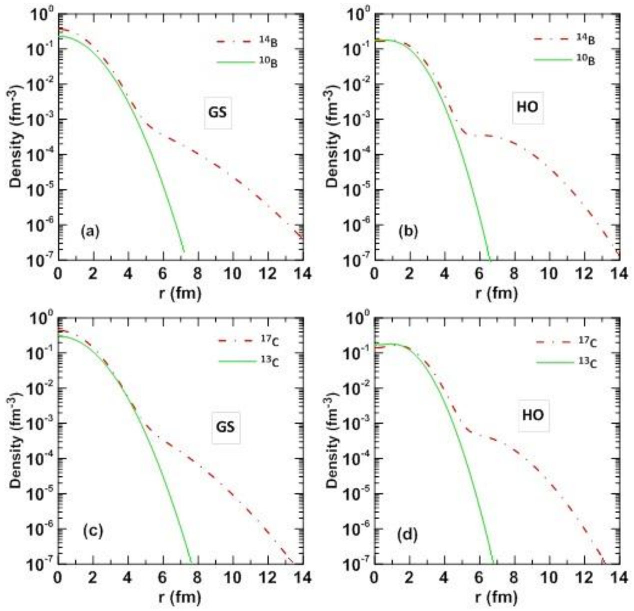

Figure 4. The distributions of the matter density for halo nuclei ^{10,14}B and ^{13,17}C.

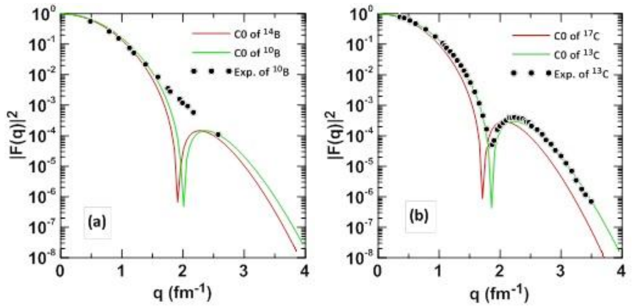

Figure 5. The elastic form factors for halo nuclei ^{10,14}B and ^{13,17}C.

The calculated σ_R at high and low energies for ^{14}B+^{12}C and ^{17}C+^{12}C systems obtained by the Glauber model with OLA along with experimental data [17-19] are plotted in Figure 6 and listed in Table 3. The open red and filled blue circle symbols are the calculated and experimental results, consecutively. From the results, one can see clearly that a good description of the experimental σ_R is obtained by the calculated results for both halo nuclei at high and low energies.

Table 3. Calculated and experimental σ_R for ^{14}B+^{12}C and ^{17}C+^{12}C systems.

Halo nuclei	Energy (MeV) [17-19]	Calculated σ_R (mb)	Experimental σ_R (mb) [17-19]
^{14}B	790	938	929±26
	120	1080	1070±72
	70	1237	1228±57
	45	1534	1526±46
^{17}C	965	1060	1056±10
	79	1358	1350±21

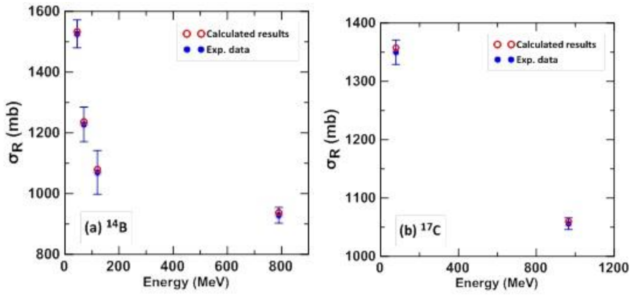

Figure 6. The experimental and calculated results of reaction cross section for halo nuclei ^{14}B and ^{17}C on ^{12}C target.

Figure 7 depicts the calculated σ_R (black line), obtained by the Glauber model within OLA, versus the matter rms radii for ^{14}B+^{12}C and ^{17}C+^{12}C systems at energy 790 and 965 MeV, respectively. The horizontal blue line shows the experimental σ_R (given in Table 3) with error bar plotted by the shaded area. The vertical red line represents the obtained matter radius, which equal $\langle r_m^2 \rangle^{\frac{1}{2}}$=2.52 [2.68] fm for ^{14}B [^{17}C] which agree well with the analogous experimental data 2.51±0.09 [2.72±0.03] fm [17].

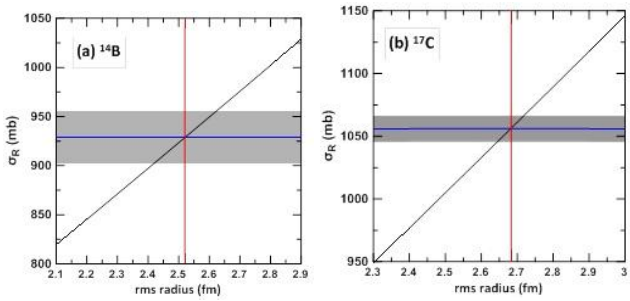

Figure 7. The reaction cross section versus the matter rms radii for the halo nuclei^{14}B and ^{17}C.

4. Summary and Conclusions

The neutron, proton, and matter densities of the ground state of the exotic ^{14}B and ^{17}C nuclei are analyzed using the BCM. Two density parameterizations are used in BCM calculations namely; GS and HO parameterizations. According to the calculated results it found that, the BCM gives a good description of the nuclear structure for above neutron-rich exotic nuclei. The elastic form factors of the ^{14}B and ^{17}C exotic nuclei and those of their stable isotopes ^{10}B and ^{13}C are determined using the plane-wave Born approximation. The main difference between the elastic form factors of halo nuclei and their stable isotopes is due to the difference in the center of mass correction. Moreover, the Glauber model is used to calculated the matter rms radii and reaction cross-section of these exotic nuclei. The calculate results of the mentioned nuclei give a good accordance with the experimental data.

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