Al-Nahrain Journal of Science
ANJS, Vol.25 (2), June, 2022, pp. 29-33

Probabilistic Approach for Identifying Longest Fuzzy Critical Path

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Article's Information

Received: 17.01.2022
Accepted: 22.03.2022
Published: 30.06.2022

Keywords:
Critical path method
Probability
Fuzzy activities duration
Stochastic activities duration

DOI: 10.22401/ANJS.25.2.06

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Abstract

This paper, considers the occurring problem of fuzzy project network either in some or whole activities which are interrupted. A probability density function is constructed from the activities of the membership of the fuzzy set. Then an approach is proposed for identifying the longest critical path, by determining in each activity the criticality index depending on the costs yielded from project lateness caused by the interruption activities. A case study is presented and simulated to explain our approach.

1. Introduction

Projects planning and optimal timing, with a margin of uncertainty is very critical for organizations. Using an accurate mathematical model as an important tool for decisions making will be helpful for project managers. This highlights the importance of a reliable model leading to uncertainty and risk reduction. It is well known that in any project a team needs to develop a detailed plan for risk assessment and management. the Identification of the most critical activities regarding schedule risk is a problem for all project managers. However, in many cases the activity times may not be presented in an accurate manner, in which PERT based on the probability theory can be employed and has detailed critiques illustrated in [1]. The main advantages of fuzzy theory methodologies are not requiring prior predictable regularities or posterior frequency distributions. The majority are extensions of deterministic CPM in a straightforward way. However, many issues had been aroused regarding the failures and drawbacks particularly in estimating each path degree of criticality also the fuzziness of activity times were not completely conserved, thus some useful insights and valuable information may be lost [2,3].

Numerous papers have been written about the PERT method, with stochastic activity durations. See [4-6]. Stochastic projects are problematic when it comes to the identification of critical activates. Many approaches were suggested for the purpose of identifying the critical activities yet none of the methods were accurate enough which in return led to incorrect and inaccurate identification of these activates. Because of that project managers are facing project delays as they lack the appropriate methods to identify and categorize the sources that are causing the delay, and with identifying activates that models the best chances to address successfully the schedule risk. The literature review in [7], presenting some methods for determining critical activates in networks of stochastic projects. The review considers a number of methods that are used to asses criticality and sensitivity.

In this paper, we will convert fuzzy activities into stochastic activities, i.e., those that are interrupted for an uncertain amount of time. Also, both the interrupted and final processes time length are uncertain. In this study, we shall build a probabilistic approach to identify the longest critical path, which has critical activities taking into consideration their cost of lateness due to their interruptions, based on [8], a new criticality index expressions for evaluating the total expected interruption time, and the overall expectation of lateness, provided that the sum of latest finishing time is greater than or equal to the given date.

2. Fuzzy Basic Concepts

In this section, we shall present the basic definitions of fuzzy set theory.

Definition 2.1. Let U be a universal set, a fuzzy set $\tilde{A}$ of U is defined by a membership function $\mu_{\tilde{A}}(x): U \to [0,1]$, indicates the degree of x in A, which is defined as follows:

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, (-\infty, a_1] \\ f_1(x), [a_1, a_2] \\ 1, [a_2, a_3] \\ f_2(x), [a_3, a_4] \\ 0, [a_4, +\infty) \end{cases}$$

where a_1, a_2, a_3 and a_4 are real number, note that $f_1(x)$ and $f_2(x)$ are may be linear or convex nonlinear function.

Definition 2.2, [11]. A fuzzy number $\tilde{A}$ is a convex normalized fuzzy set of the real line $\mathbb{R}$, such that:

1. There exists exactly one $x_0 \in \mathbb{R}$ with $\mu_{\tilde{A}}(x_0) = 1$ (x_0 is called the mean value of $\tilde{A}$).
2. $\mu_{\tilde{A}}(x)$ is piecewise continuous.

Consider a mapping f defined by $f(x) = c\mu_{\tilde{A}}(x)$ is a probability density function associated with $\tilde{A}$, from space of fuzzy action defined over an output universe into a space of non-fuzzy (crisp) actions, and by using the property that $\int_{-\infty}^{\infty} f(x)dx = 1$, we obtain c according to the type of fuzzy number, such as triangular, trapezoidal etc., to have $f_{\tilde{A}}(x)$, and the problem become fuzzy stochastic problem, thus, by the use of the Mellin transform to obtain the expected value from a probability density function, and creating a procedure, using the μ-cut, for calculating the expected value of fuzzy stochastic variable $\tilde{\tilde{A}}$ denoted by $EV(\tilde{\tilde{A}})$. For more details see [8].

3. Problem Statement

Assume that we have a set of N-activities, each has an uncertain processing duration time t_i^p, and its uncertainty comes from its fuzzy interruption time $(\tilde{t}_i^r)$, $i = 1,2,\dots,N$. All fuzzy times $(\tilde{t}_i^r)$ are converted into fuzzy stochastic variables $(\tilde{\tilde{t}}_i^r)$, the expected values for all these variables $t_i^r \cong E(\tilde{\tilde{t}}_i^r)$ are obtained, and the problem can be considered as a stochastic problem.

In a stochastic project network, the time of activates (durations) are considered random variables. This makes the times of starting and ending, and slack activities random variables as well. A new way for determining the critical path activities (CPA) in stochastic project network (SPN) is based on a philosophy that differs from deterministic project network (DPN), where each critical activity must correspond to zero time slack activity, while it is not necessary in such condition in (SPN), where the index of critically, is defined as the probability of activity depending on a critical path may not introduced risk of project delay (i.e., schedule risk) into the project network (see [9]).

Now, if we define the following notation:

γ_i = The set of immediate predecessors of activity *i*.

Γ_i = The set of immediate successors of activity *i*.

$p_i(t)$ = The activity time probability density function PDF activity *i*.

$P_i(t)$ = The activity time cumulative distribution function CDF activity *i*.

$p_{E,i}(t)$. = The earliest start time PDF for activity *i*.

$p_{L,i}(t)$ = The latest start time PDF for activity *i*.

$P_{E,i}(t)$ = The earliest start time CDF for activity *i*.

$P_{L,i}(t)$ = The latest start time CDF for activity *i*.

$f_{E,i}(t)$ = The earliest finish time PDF for activity *i*.

$f_{L,i}(t)$ = The latest finish time PDF for activity *i*.

$F_{E,i}(t)$ = The earliest finish time CDF for activity *i*.

$F_{L,i}(t)$. = The latest finish time CDF for activity *i*.

By assuming the early start schedule distributions to be continuous distributions, we can define the following:

$$P_{E,i}(t) = \prod_{j\epsilon\gamma_i} F_{E,j}(t)$$

$$p_{E,i}(t) = \frac{dP_{E,i}(t)}{dt}$$

$$F_{E,i}(t) = \int_0^t P_i(t - t_1) p_{E,i}(t_1) dt_1$$

$$f_{E,i}(t) = \frac{dF_{E,i}(t)}{dt}$$

$$P_{L,i}(t) = \int_{t_1=0}^{t} \int_{t_2=t}^{\infty} p_i(t_2 - t_1) f_{L,i}(t_2) dt_2 dt_1$$

$$p_{L,i}(t) = \frac{dP_{L,i}(t)}{dt}$$

$$f_{L,i}(t) = \frac{dF_{L,i}(t)}{dt}$$

For every activity the time distribution of early starting and ending is determined by proceeding forward through the network sequentially, while, by setting $F_{L,n} = F_{E,n}$ and processing backwards through the network sequentially, Starting with activity n and finishing with activity 1, the late start schedule distributions can be calculated. In [9], the sources of schedule risk in a stochastic project network is identified, and the general expression for determining a late starting and ending time distributions in the activity are developed to determine the critical activities by the use of the activity critically index to the ones found using stochastic activity metrics. In [10] the early and late starting time distributions are used to calculate expected total slack time of the activity i as follows:

$$E[TS_i] = \int_0^{\infty} \int_0^{z_1+t} (z_2 - z_1) p_{E,i}(z_1)\, p_{L,i}(z_2)\, dz_2 dz_1$$

Then, the identification of activates ranking, whether are most (least) likely to introduce a delay into a project, having lowest (largest) E(TS), respectively. Also, claiming that considering $E(TS|L)$ and $E(L|TS)$ yield different ranking criticality index, and poor performance due to ignoring many factors that we are constructed in this paper.

In this paper, we are simplifying the method presented in [10], by creating an expression for determining the total expected interruption time $E[Tt^r]$ and the total expected lateness, given that the total latest finishing time is greater than or equal to the known due date D, as follows:

$$E(Tt^r) = \sum_i E[t_i^r]$$

where:

$$E[t_i^r] = \int_0^{\infty} \int_0^{z_1+t_i^r} (z_2 - z_1) p_{E,i}(z_1) p_{L,i}(z_2) dz_2 dz_1$$

$$E(L|F_{L,n} \geq D) = \int_D^{\infty} (t - D) p_{L,n}(t|Tt^r) dt$$

Then, we compute the total interruption cost of the project resulting from activity j as follows:

$$r_j = \frac{c_j^r}{E[Tt^r]} * E(L|F_{L,n} \geq D)$$

where $L = F_{L,n} - D$ represents project lateness, and c_j^r is the interrupted cost of activity j.

Now, our approach, consist of the following steps:

Step 1: Convert all fuzzy times variables into fuzzy stochastic variables by calculate the mean of each fuzzy activity.

Step 2: Calculate $F_{L,n}$ based on classical PERT.

Step 3: Compute the following:

$E[t_j^r], E(L|F_{L,n} \geq D)$ and r_j

Step 4: Identify the critical activities with the highest values of r_j, that consist a longest critical path.

4. Case Study

We are considering a real-life project network Figure 1, as a case study. As shown in Table 1; the 30 activities are listed with their operation and fuzzy interruption times.

Table 1. Construction project.

Activity item	Activity description	Pre activity	Fuzzy interruption time (per day) $(\tilde{t}_i^r)$	Expected interruption time t_i^r	Uncertain processing time t_i^p	Critical index
P_1	Concrete foundation	–	(25,28,30,35)	27	60	0.97
P_2	Insulation works	P_1	(3,4,4,5)	4	18	1.0
P_3	Parking + Roads + Landscape	P_2	(25,29,30,35)	28.6	65	0.93
P_4	Back filling works	P_3	(3,7,12,15)	11.1	30	0.91
P_5	Sub-base	P_4	(5,6,6,10)	5.62	20	0.96
P_6	Steel structure erection	P_5	(26,30,35,40)	30.6	70	0.93
P_7	Underground drainage	P_5	(7,10,10,13)	10	30	0.99
P_8	Water tank - civil works	–	(15,21,21,25)	22.5	55	0.71
P_9	Steel structure testing	P_6	(2,3,4,5)	3.5	10	0.90
P_{10}	Roofing works	P_6	(9,10,12,15)	10	30	0.517
P_{11}	Water tank - finishing	P_8	(6,7,8,10)	7.37	26	0.90
P_{12}	HVAC works - 1st fix	P_9	(12,14,14,16)	14	40	0.9
P_{13}	Firefighting 1st fix	P_9	(7,9,11,12)	10.3	32	0.45
P_{14}	Electrical works - 1st fix	P_{12}, P_{13}	(5,6,7,10)	6	22	0.44
P_{15}	Flooring	P_{14}	(7,9,11,12)	10.3	32	0.55
P_{16}	HVAC work-2nd fix	P_9	(12,14,14,16)	14	40	0.96
P_{17}	Firefighting - 2nd fix	P_9	(7,9,11,12)	6	22	0.56
P_{18}	Cladding works	P_9	(15,24,25,30)	20.7	55	0.49
P_{19}	Electrical works - 2nd fix	P_{16}, P_{17}	(5,6,7,10)	6.87	25	0.56
P_{20}	Water tank - MEP	P_{11}	(9,11,12,14)	11.4	35	0.52
P_{21}	Finishing works	P_{15}	(15,18,18,20)	18	50	0.67
P_{22}	HVAC works - 3rd	P_9	(12,14,14,16)	14	40	0.56
P_{23}	Firefighting - 3rd fix	P_9	(7,9,11,12)	9.12	30	0.58
P_{24}	Electrical works -3rd fix	P_{22}, P_{23}	(5,6,7,10)	6.5	25	0.98
P_{25}	Plumbing works - 1st fix	P_{14}	(5,6,6,8)	6.12	25	0.43
P_{26}	Plumbing works - 2nd fix	P_{19}	(5,6,6,8)	6.12	25	0.52
P_{27}	Plumbing works - 3rd fix	P_{24}	(5,6,6,8)	6.12	25	0.51
P_{28}	Water tank testing	P_{20}	(1,2,2,3)	2	10	0.56
P_{29}	Testing and commission	P_{28}	(1,2,2,3)	2	10	0.57
P_{30}	Snag list & Initial handle	P_{29}	(5,7,7,9)	7	25	0.55

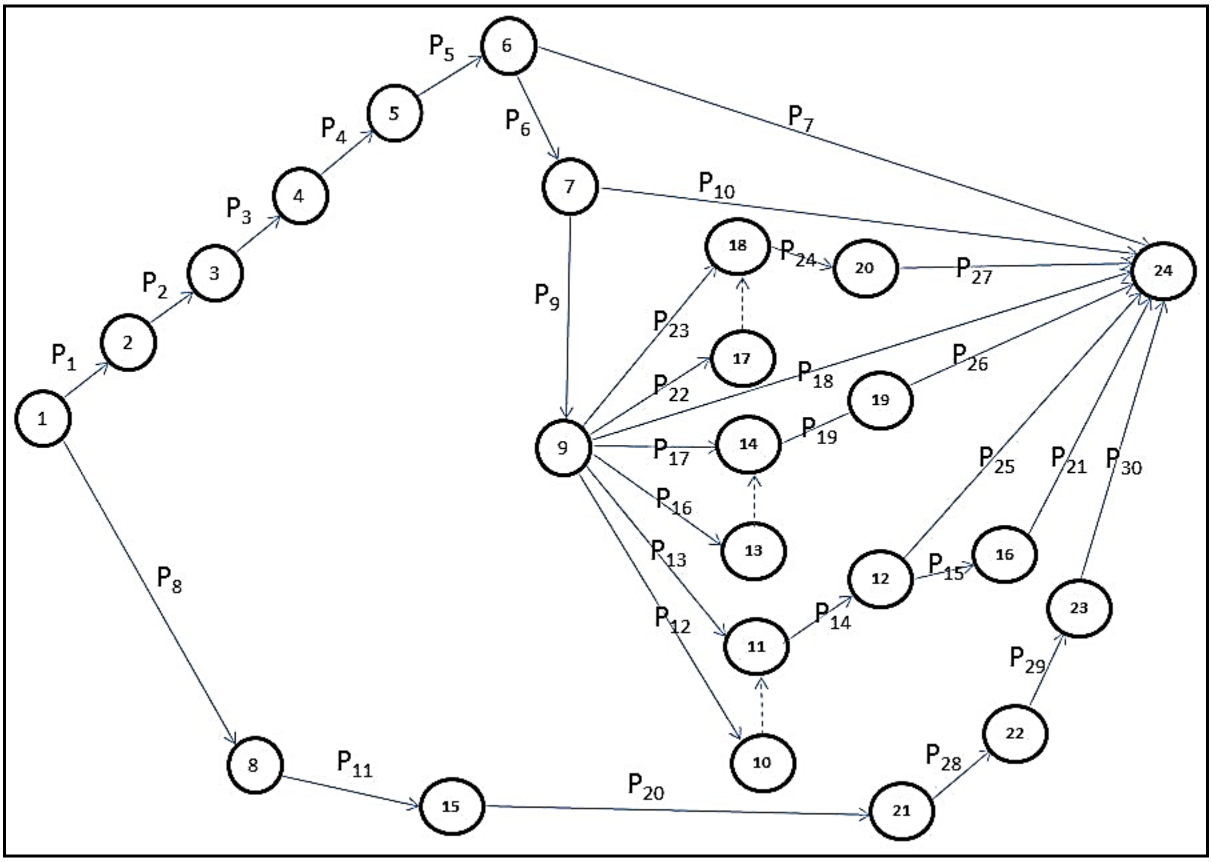

Figure 1. Project network.

In this work, we implemented our proposed approach to calculate the criticality index for each activity, showing that the longest critical path with activities 1, 2, 3, 4, 5, 6, 7, 9, 11, 12, 16, 24 approximately have critical indices from 0.9 to 1.0, and the others critical paths having critical activities with low critical indices.

5. Conclusion

In [11] and [12], different approaches are constructed and tested on nearly the same case study, shown the same longest critical path. Also, in this study, we addressing an important problem faced by many project managers, specifically, we introduced the concepts of fuzzy project, and sensitivity in stochastic activity network, in which we are believe that the approach presented in this paper is the first one to test which tasks introduced in the schedule by developing new efficient pointers.

Many applications of such problem can be found in the deterministic project planning and scheduling literature. Therefore, more studies have been required to test the efficiency of the proposed method compared with other stochastic methods, considering different activities distributions.

Conflicts of Interest

The authors declare that there is no conflict of interest.

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