Effect of MHD on Unsteady Flow with Fractional Maxwell Model

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Abstract

The aim of this paper is to study the effect of magnetic hydrodynamic (MHD) on unsteady flow of Maxwell fluid with fractional derivative due to a constant acceleration plate. The fractional calculus process is introduced to establish the constitutive relationship of fluid model, by using Laplace transform and Fourier sine transform, we obtained closed solutions for velocity field and shear stress. Lastly, the solutions are present by integral and series form in terms of the generalized G and R functions. The influence of the parameters on the velocity field spotlighted by means of the several graphs.

Keywords: Fractional derivatives, Laplace transform, Fourier transform, Generalized G and R functions.

Introduction

Now the most ofscientists and engineers have a great interest to real fluids that does not exhibit a linear relationship between stress and rate of strain, this attraction has mature considerably throughout the past few decades because of their varied applications in industry and engineering,these applications ranges from oil and well drilling to well completion operation,from technique involving waste fluids, synthetics fibers, extrusion of liquid plastic, artificial, exotic lubricants and natural gels and additionally as some flows of compound solutions,several fluids including with butter, cosmetics and toiletries, paints, soaps, oils, blood, shampoo fluids including withcosmetics and toiletries, butter, paints, soaps, oils, blood, shampoo and marmalades have rheological characteristics and are stated as the non-Newtonian fluids and marmalades have rheological characteristics and are stated as the non-Newtonian fluids, the rheological properties of those fluids cannot be explained by using a single constitutive relationship between stress and shear rate that is kind of different than the viscous fluids, the modeling of equations governing the non-Newtonian fluids provides rise to actuallynonlinear differential equations, generally physics properties of materials are such by their so known as constitutive equations, the only constitutive equation for a fluid isa Newtonian one, and the classical Navier - Stokes theory is based, the mechanical behavior of the many fluids is drawn by this theory [1-4], in general, the classification of the non-Newtonian fluid models is given below by three classes that are known as differential, integral and rate in general, the classification of the non-Newtonian fluid models is given below by three classes that are known as differential, integral and rate varieties, among them, the viscoelastic rate type model, that is used wide, is the Maxwell model, among them, the viscoelastic rate type model, that is used wide, is the Maxwell model.[5]

Fractional calculus has encountered much success within the description of the complicated dynamical systems [6]. The begining point of the fractional derivative model of viscoelastic fluid is often a classical differential equation a classical equation that is modified by substitution the time derivative of an integer order by the Riemanm - Liouville fractional calculus operator.

This generalization permits one to outline precisely non-integer order integrals or derivatives a very sensible agreement is achieved with experimental knowledge once the fractional Maxwell model is utilized with its initial order derivatives replaced by the fractional– order derivatives [7].

In recent years, the interest in unsteady flow of such a viscoelastic fluid with fractional derivative Maxwell model has magnified considerably and plenty of exact solutions are determined [8-12]. Khan[13],discussed some exact solutions for fractional generalized

Burgers' fluid in porous space. Xue and Nie [14] studied the exact solutions of Rayleigh – stokes problem for heated generalized Maxwell fluid in a porous half –space. Zheng et al [15] discussed the exact solution for MHD flow of generalized Oldroyd-B fluid. C.Fetecau [16] discussed the exact solutions for the unsteady flow of a viscoelastic fluid with fractional derivative Maxwell model produced by an infinite constantly accelerating plate. In the present work, we have to study the effect MHD of unsteady flow of a generalized Maxwell fluid with fractional derivative because of a constantly accelerating plate. The precise solutions for the velocity field and shear stress are obtain by using the Fourier sine transform and discrete Laplace transform.

1-Description of the problem:-

Consider an incompressible Maxwell fluid with fractional derivative which is, also known as generalized Maxwell fluid (GMF) lying over associate degree of infinite flat plate. At first ,the fluid is at rest and at time t=0^+ the infinite plate begins to slide in its plane with the velocity (At),where A is a constant due to the shear, and the fluid higher than the plate is step by step moved. The associated boundary and initial conditionare

$$U(y,t), \frac{\partial U(y,t)}{\partial y} \to 0 \; as \; y \to \infty \,, t > 0 \; ....(1)$$

$$U(y,0) = \frac{\partial U(y,0)}{\partial t} = 0, y > 0 \; ....(2)$$

$$U(0,t) = At \,, t \geq 0 \; ....(3)$$

In order to solve the problem under consideration, we shall use the Fourier sine and Laplace transformation.

2- Governing equations:-

The constitutive equations for associate incompressible Maxwell fluid with fractional derivative is given by

$$T = -pI + S \; .... (4)$$

Where T is Cauchy stress tensor, p is a pressure and I is an identity tensor, and

$$S + \lambda(D_t^\alpha S + V.\nabla S - LS - SL^T) \; = \mu A \; ....(5)$$

Where S is that the additional – stress tensor, V is that the velocity vector, L is that the velocity gradient, A = L+L^T is that the first Rivlin–Ericksen tensor, ∇ is that the gradient operator,λ is that the relaxation time, μ is that the dynamic viscosity and D_t^α is that the Riemann-Liouville fractional differential operator oforder α with respect to twhich in defined as

$$D_t^\alpha f(t) = \frac{1}{\Gamma(1-\alpha)} \frac{d}{dt} \int_0^t \frac{f(\tau)}{(t-\tau)^\alpha} d\tau \,, 0 < \alpha \leq 1 \; ....(6)$$

Where Γ(.) is the Gamma function.This model reduces to the standard Maxwell model when α=1 and therefore the Newtonian model once α=1 and λ=0.

For the problem under consideration, we have a tendency to get a velocity field of the form

$$V = V(y,t) = U(y,t)i, \; .... (7)$$

Where U(y,t) is that the velocity at intervals the x-coordinate direction which i is that the unit vector within the same direction. For this velocity field.Consider that the conducting fluid is permeated by an imposedmagnetic field B zero that acts in the positive y- coordinate .in the low-magnetic Reynolds number approximation, the magnetic body force is represented by σB_0^2,where σ is that the electrical conductivity of the fluid. Substituting Eq(7) into the Eq(4),Eq(5) and taking in to account the initial condition:

$$S(y,0) = 0 \,, y > 0 \; ....(8)$$

Such thatthe fluid being at rest up to the time t=0, we have$S_{yy} = S_{zz} = S_{xz} = S_{yz} =$ 0and $S_{xy} = S_{yx}$,we get

$$(1 + \lambda D_t^\alpha)\tau(y,t) = \mu \frac{\partial}{\partial y} U(y,t), \; ....(9)$$

Where $\tau(y,t) = S_{xy}(y,t)$ is that the shear stress that is all totally different of zero,in the absence of body forces and a pressure gradient at intervals the flow direction,the balance of momentum finishes up within the relevant equation

$$\rho \; \frac{\partial U(y,t)}{\partial t} = \frac{\partial}{\partial y} \tau(y,t) \; ....(10)$$

Wherever ρ is that the constant density of the fluid.

Eliminating τ(y,t) betweenEq(9)and Eq(10),we findthe governing equation

$$(1+\lambda\, D_t^{\alpha})\frac{\partial U(y,t)}{\partial t}=\nu\frac{\partial^2 U(y,t)}{\partial y^2}-M\,U(y,t) \quad ......(11)$$

Wherever ν = μ/ρ is the kinematic viscosity of the fluid and $M=\frac{\sigma B_0^2}{\rho}$

3-Calculation of the velocity field:-

The velocity field can be obtained bymultiplying both sides of Eq(11) by $\sqrt{2/\pi}\sin(y\xi)$integrating then with respect to y from 0 to ∞ and having in mind the initial and boundary conditions (1),(2)and (3), we find that

$$(1+\lambda D_t^{\alpha})\frac{\partial U_s(\xi,t)}{\partial t}+\nu\xi^2 U_s(\xi,t)=\nu A\xi t\sqrt{\frac{2}{\pi}}-MU_s(\xi,t) \quad ......(12)$$

Where the Fourier sine transform of U(y,t) has to satisfy the conditions

$$U_s(\xi,0),\frac{\partial U(\xi,0)}{\partial t}=0\; for,\xi>0 \quad ......(13)$$

Applying the Laplace transform to Eq(12) and using the Laplace transform formula for serial fractional derivatives [6], we acquire the image function $\overline{U}_s(\xi,q)\; of\; U_s(\xi,t)$ below the shape

$$\bar{U}_s(\xi,q)=\sqrt{\frac{2}{\pi}}\frac{\nu A\xi}{q^2(\lambda q^{\alpha+1}+\nu\xi^2+q+M)} \quad ......(14)$$

In order to get $U_s(\xi,t)=L^{-1}\overline{U}_s(\xi,q)$ and to avoid the long calculations of residues and contour integrals, we tend to apply the distinct inverse Laplace transform technique .However, for a additional appropriate presentation of the ultimate results, we first rewrite Eq(14) within the equivalent forms

$$\bar{U}_s(\xi,q)=A\sqrt{\frac{2}{\pi}}\frac{1}{\xi q^2}-A\,\nu\xi\sqrt{\frac{2}{\pi}}\frac{1+\lambda q^{\alpha}+Mq^{-1}}{q\nu\xi^2(\lambda q^{\alpha+1}+\nu\xi^2+q+M)}=A\sqrt{\frac{2}{\pi}}\frac{1}{\xi q^2}-\nu\xi\sqrt{\frac{2}{\pi}}\left(\frac{1}{(q+\nu\xi^2)q\nu\xi^2}\right)-A\xi\nu\sqrt{\frac{2}{\pi}}\frac{\lambda q^{\alpha-1}+M}{(q+\nu\xi^2)(q+\lambda q^{\alpha+1}+\nu\xi^2+M)} \quad ......(15)$$

The second factor of the last term of Eq.(15)can be written within the sort of a double series,(see Appendix (A1)).

$$\frac{\lambda q^{\alpha-1}+M}{q+\lambda q^{\alpha+1}+\nu\xi^2+M}=\sum_{\gamma=0}^{\infty}\frac{(-1)^{\gamma}}{\lambda^{\gamma+1}}\sum_{i,j\ge 0}^{i+j=\gamma}\frac{\gamma!}{i!\,j!}\frac{q^i M^j(\lambda q^{\alpha-1}+M)}{\left(q^{\alpha+1}+\frac{\nu\xi^2}{\lambda}\right)^{\gamma+1}}$$

We obtain,

$$(\xi,q)=A\sqrt{\frac{2}{\pi}}\frac{1}{\xi q^2}-A\nu\xi\sqrt{\frac{2}{\pi}}\left(\frac{1}{(q+\nu\xi^2)q\nu\xi^2}\right)-A\xi\nu\sqrt{\frac{2}{\pi}}\frac{1}{q+\nu\xi^2}\sum_{\gamma=0}^{\infty}\frac{(-1)^{\gamma}}{\lambda^{\gamma+1}}\sum_{i,j\ge 0}^{i+j=\gamma}\frac{\gamma!}{i!\,j!}\frac{q^i M^j(\lambda q^{\alpha-1}+M)}{\left(q^{\alpha+1}+\frac{\nu\xi^2}{\lambda}\right)^{\gamma+1}} \quad ......(16)$$

Inverting the result by means that of the Fourier sine formula [17],we get that

$$\bar{U}(y,q)=\frac{A}{q^2}-\frac{2A\nu}{\pi}\int_0^{\infty}\left(\frac{1}{(q+\nu\xi^2)q\nu\xi^2}\right)\sin(y\xi)\,d\xi-\frac{2A\nu}{\pi}\int_0^{\infty}\frac{\xi\sin(\xi y)}{q+\nu\xi^2}\sum_{\gamma=0}^{\infty}\frac{(-1)^{\gamma}}{\lambda^{\gamma+1}}\sum_{i,j\ge 0}^{i+j=\gamma}\frac{\gamma!}{i!\,j!}\left(G_1(q)(\lambda M^j)+G_2(q)M^{j+1}\right)d\xi \quad ......(17)$$

Where [18],

$$G(q)=\frac{q^b}{(q^a-d)^c}=q^{b-ac}\left(1-\frac{d}{q^a}\right)^{-c},Re(ac-b)>0, \quad ......(18)$$

With $a_1=\alpha+1, b_1=\alpha-i+1, a_2==\alpha+1, b_2=i$, and $c_1=c_2=k+1$ are not constrained to be integers and $d_1=d_2=-\nu\xi^2/\lambda$, using the binomial theorem, we can also write

$$G(q)=\sum_{s=0}^{\infty}\frac{\Gamma(1-c)q^{b-ac-as}}{\Gamma(s+1)\Gamma(1-s-c)}(-d)^s\; if\; |d|>|q^a| \quad ......(19)$$

This last expression may be term by term inverse transformed, yielding [18]

$$G_{a,b,c}\left(\frac{-\nu\xi^2}{\lambda},t\right)=L^{-1}\{G(q)\}=\sum_{s=0}^{\infty}\frac{\Gamma(1-c)q^{(c+s)a-b-1}}{\Gamma(s+1)\Gamma((c+s)a-b)}\left(\frac{\nu\xi^2}{\lambda}\right)^s \quad ......(20)$$

If Re ((-b)+ac)) > 0, Re(q) > 0 and$|qa| > \frac{\nu\xi^2}{\lambda}$ the form of Eq(20) presents analysis difficulties,since once c is associate integer, Γ(1-c) and Γ(1-s-c) will become infinite. However, it may be rewritten within the following calculable from [18],

$$G_{a,b,c}\left(\frac{-\nu\xi^2}{\lambda},t\right) = \sum_{s=0}^{\infty}\frac{(c)_s q^{(c+s)a-b-1}}{\Gamma(s+1)\Gamma\big((c+s)a-b\big)}\left(\frac{-\nu\xi^2}{\lambda}\right)^s \quad \ldots\ldots(21)$$

Where $(c)_s$ is the Pochhammer Polynomial. Finally, applying the inverse Laplace transform to Eq(17) and taking under consideration all the previous results and useproperty (A3),it is not difficult to point out that

$$U(y,t) = At - \frac{2A}{\pi\nu}\int_0^{\infty}\big(1 - Exp(-\nu\xi^2 t)\big)\frac{\sin(y\xi)}{\xi^3}d\xi - \frac{2A\nu}{\pi}\int_0^{\infty}\xi\sin(y\xi)\int_0^t Exp(-\nu(t-\delta)\xi^2) \times\sum_{\gamma=0}^{\infty}\frac{(-1)^\gamma}{\lambda^{\gamma+1}}\sum_{i+j\ge 0}^{i+j=\gamma}\frac{\gamma!}{i!\,j!}\sum_{w=0}^{\infty}\left(\frac{\nu\xi^2}{\lambda}\right)^w \frac{\Gamma\big(1-(\gamma+1)\big)}{\Gamma(w+1)\Gamma\big(1-w-(\gamma+1)\big)} \left(\frac{\delta^{\alpha\gamma+\alpha w+\alpha+\gamma+w-i}}{\Gamma(\alpha\gamma+\alpha w+\alpha+\gamma+w-i+1)}M^{j+1} + \frac{\delta^{\alpha\gamma+\alpha w+\gamma+w-i+1}}{\Gamma(\alpha\gamma+w\alpha+\gamma+w-i+2)}\lambda M^j\right)d\delta d\xi \quad \ldots\ldots(22)$$

Or equivalently

$$U(y,t) = U_1(y,t) - \frac{2A\nu}{\pi}\int_0^{\infty}\xi\sin(y\xi)\int_0^t \mathrm{Exp}(-\nu(t-\delta)\xi^2)\sum_{\gamma=0}^{\infty}\frac{(-1)^\gamma}{\lambda^{\gamma+1}}\sum_{i+j\ge 0}^{i+j=\gamma}\frac{\gamma!}{i!\,j!}\sum_{w=0}^{\infty}\left(\frac{\nu\xi^2}{\lambda}\right)^w\frac{\Gamma\big(1-(\gamma+1)\big)}{\Gamma(w+1)\Gamma\big(1-w-(\gamma+1)\big)} \left(\frac{\delta^{\alpha\gamma+\alpha+w\alpha+\gamma+w-i}}{\Gamma(\alpha\gamma+w\alpha+\alpha+\gamma+1+w-i)}M^{j+1} + \frac{\delta^{\alpha\gamma+w\alpha+\gamma+1+w-i}}{\Gamma(\alpha\gamma+w\alpha+\gamma+w-i+2)}\lambda M^j\right) d\delta d\xi \quad \ldots\ldots(23)$$

Where [19],

$$U_1(y,t) = At - \frac{2A}{\pi\nu}\int_0^{\infty}\frac{\big(1-Exp(-\nu\xi^2 t)\big)\sin(y\xi)}{\xi^3}d\xi = 4Ati^2Erfc\left(\frac{y}{2\sqrt{t\nu}}\right), \quad \ldots\ldots(24)$$

Appear the velocity field comparable to a Newtonian fluid and *Erfc*(.) isthe integrals of the complementary Error function of Gauss[19].

4-Calculation of the shear stress:-

Applying the Laplace transform to Eq(9) and using the initial condition Eq (8),we get

$$\bar{\tau}(y,q) = \mu\left(\frac{1}{1+\lambda q^\alpha}\frac{\partial\bar{U}(y,q)}{\partial y}\right) \quad \ldots\ldots(25)$$

Where, the differentiation of Eq.(17) with respect to y,we get

$$\frac{\overline{\partial U}(y,q)}{\partial y} = -\frac{2A\nu}{\pi}\int_0^{\infty}\left(\frac{1}{(q+\nu\xi^2)q\nu\xi^2}\right)\frac{\xi cos(y\xi)}{\xi^3}d\xi - \frac{2A\nu}{\pi}\int_0^{\infty}\xi^2 cos(\xi y)\sum_{\gamma=0}^{\infty}\frac{(-1)^\gamma}{\lambda^{\gamma+1}}\sum_{i,j\ge 0}^{i+j=\gamma}\frac{\gamma!}{i!\,j!}\sum_{w=0}^{\infty}\left(\frac{\nu\xi^2}{\lambda}\right)^w \frac{\Gamma(-\gamma)}{\Gamma(-(\gamma+w))\Gamma(1+w)}\frac{1}{q+\nu\xi^2}\left(\frac{M^{j+1}}{q^{\alpha\gamma+\alpha+\alpha w+\gamma+w-i+1}} + \frac{\lambda M^j}{q^{\alpha\gamma+\alpha w+\gamma+w-i+2}}\right)d\xi \quad \ldots\ldots(26)$$

Substituting Eq (25) into Eq (26),it results that

$$\bar{\tau}(y,q) = \frac{\mu}{1+\lambda q^\alpha}\left(-\frac{2A\nu}{\pi}\int_0^{\infty}\left(\frac{1}{(q+\nu\xi^2)q\nu\xi^2}\right)\frac{\xi cos(y\xi)}{\xi^3}d\xi - \frac{2A\nu}{\pi}\int_0^{\infty}\xi^2 cos(\xi y)\sum_{\gamma=0}^{\infty}\frac{(-1)^\gamma}{\lambda^{\gamma+1}}\sum_{i,j\ge 0}^{i+j=\gamma}\frac{\gamma!}{i!\,j!}\sum_{w=0}^{\infty}\left(\frac{\nu\xi^2}{\lambda}\right)^w \frac{\Gamma(-\gamma)}{\Gamma(-(\gamma+w))\Gamma(1+w)}\frac{1}{q+\nu\xi^2}\left(\frac{M^{j+1}}{q^{\alpha\gamma+\alpha+\alpha w+\gamma+w-i+1}} + \frac{\lambda M^j}{q^{\alpha\gamma+\alpha w+\gamma+w-i+2}}\right)d\xi\right) \quad \ldots\ldots(27)$$

Inverting this result and use property (A3),we get theshear stress under the form

$$\tau(y,t) = \tau_1(y,t) + 2\,\frac{A\,\mu}{\pi}\int_0^\infty\!\!\int_0^t (Exp\left(-\nu\,\xi^2(t-\delta\,)\right) R_{\alpha,\alpha-1}\left(-\frac{1}{\lambda},0,\delta\right)\cos(y\xi)\,d\delta\,d\xi$$
$$-2\,\frac{A\nu}{\pi}\frac{\mu}{\lambda}\int_0^\infty \xi^2\cos(\xi y)\sum_{\gamma=0}^{\infty}\frac{(-1)^\gamma}{\lambda^{\gamma+1}}\sum_{i,j\ge 0}^{i+j=\gamma}\frac{\gamma!}{i!\,j!}\sum_{w=0}^{\infty}\left(\frac{\nu\xi^2}{\lambda}\right)^w\frac{\Gamma(-\gamma)}{\Gamma\left(-(\gamma+w)\right)\Gamma(1+w)}$$
$$\int_0^t\!\!\int_0^\sigma Exp\left(-\nu\,\xi^2(t-\delta)\right) R_{\alpha,0}(-\frac{1}{\lambda},0,t-\sigma)\left(\frac{M^{j+1}\delta^{\alpha\gamma+\alpha w+\gamma+w-i+1}}{\Gamma(\alpha\gamma+\alpha w+w-i+\gamma)}+\lambda\frac{M^j\delta^{\alpha\gamma+\alpha w+\gamma+w-i}}{\Gamma(\alpha\gamma+\alpha w+w-i+\gamma+2)}\right)$$
....................................(28)

Wherever the generalized function $R_{c,\,d}$ (a,b,t) [18],is defined by,

$$R_{c,d}(a,b,t) \;=\sum_{w=0}^{\infty}\frac{(a)^w(t-b)^{c(w+1)-d-1}}{\Gamma(L+1)\Gamma\left((w+1)c-d\right)}$$

And [19]

$\tau_1(y,t) = -2\frac{\rho A}{\pi}\int_0^\infty(1-Exp(-\nu t\xi^2)\frac{\cos(y\xi)}{\xi^2}d\xi = -2\rho A\sqrt{\nu t}iErfc\left(\frac{y}{2\sqrt{\nu t}}\right),$ ....(29)

Is the shear stress equivalent to a Newtonian fluid performing a same motion.

5- Special case

Applyingthe limit (α→1) into Eq.(23) and Eq.(28), we tend to acquire the similar solutions

$$U(y,t) = U_1(y,t) - \frac{2A}{\pi\nu}\int_0^\infty\left(1-Exp(-\nu\xi^2 t)\right)\frac{sin(y\xi)}{\xi^3}d\xi$$
$$-\frac{2A\nu}{\pi}\int_0^\infty \xi\,sin(y\xi)\int_0^t Exp(-\nu(t-\delta)\xi^2)\times\sum_{\gamma=0}^{\infty}\frac{(-1)^\gamma}{\lambda^{\gamma+1}}\sum_{i+j\ge 0}^{i+j=\gamma}\frac{\gamma!}{i!\,j!}\sum_{w=0}^{\infty}\left(\frac{\nu\xi^2}{\lambda}\right)^w\frac{\Gamma\left(1-(\gamma+1)\right)}{\Gamma(w+1)\Gamma\left(1-w-(\gamma+1)\right)}$$
$$\left(\frac{\delta^{2\gamma+2w+1-i}}{\Gamma(2\gamma+2w+2-i)}M^{j+1}+\frac{\delta^{2\gamma+2w-i+1}}{\Gamma(2\gamma+2w-i+2)}\lambda M^j\right)d\delta d\xi$$
....................................(30)

And by using property (A6),we get

$$\tau(y,t) = \tau_1(y,t) - 2\,\frac{A\mu\lambda}{\pi}\int_0^\infty\frac{1}{(\nu\xi^2\lambda-1)}\times\left(Exp(-\nu\xi^2 t) - Exp\left(-\frac{t}{\lambda}\right)\right)\cos(y\xi)\,d\delta\,d\xi$$
$$-2\,\frac{A\nu\mu}{\pi\lambda}Exp\left(\frac{-t}{\lambda}\right)\int_0^\infty\xi^2\cos(y\xi)\int_0^t\!\!\int_0^\sigma Exp\left(-\nu\xi^2(\sigma-\delta)+\frac{\sigma}{\lambda}\right)\sum_{\gamma=0}^{\infty}\frac{(-1)^\gamma}{\lambda^{\gamma+1}}$$
$$\sum_{i,j\ge 0}^{i+j=\gamma}\frac{\gamma!}{i!\,j!}\sum_{w=0}^{\infty}\left(\frac{\nu\xi^2}{\lambda}\right)^w\frac{\Gamma(-\gamma)}{\Gamma\left(-(\gamma+w)\right)\Gamma(1+w)}$$
$$\left(\frac{M^{j+1}\delta^{2\gamma+2w-i+1}}{\Gamma(2\gamma+2w-i)}+\lambda\frac{M^j\delta^{2\gamma+2w-i+1}}{\Gamma(2\gamma+2w-i+2)}\right)$$
.......................(31)

6- Results and Discussion

In this paper, we presented an analysis for the unsteady MHD flow an incompressible generalized Maxwall fluid due to an infinite constantly accelerating plate.

The solutions for the velocity field and shear stress in terms of generalized G and R functions are obtained by using the Fourier sine and Laplace transforms .The characteristic of velocity field were analyzed in terms of the analytical solutions get in Eq(22)We take ν=0.001, A=1,in all Figures.

The influences of relaxation time on the velocity field is show in Fig.(1), effectof increasingλ is an decreasesofthe velocity field U(y,t) whenM=2,t=2, α=0.9

In Fig.(2) show the velocity change with the fractional parameter α when M= 2, t=2, λ=3,the velocity field decrease when α is increasing.

In Fig.(3) demonstrates the influence of the magnetic field M on the velocity when λ =3, α =0.9, t =2, you can see that the velocity field is decreasing with increase of the magnetic parameter.

Fig.(4) demonstrates the influence of flow of velocity field for various values of time. It is seen that the greater the time the more slowly the velocity field decays, when λ= 3, M = 2, α =0.9.

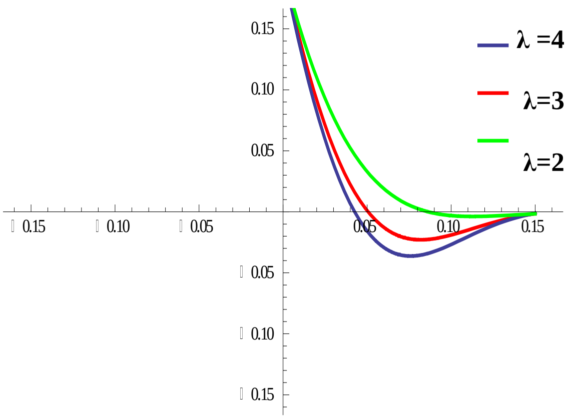

Fig.(1): Velocity Field U(y,t) for λ =4, 3, 2, when M=2, t=2, α=0.9.

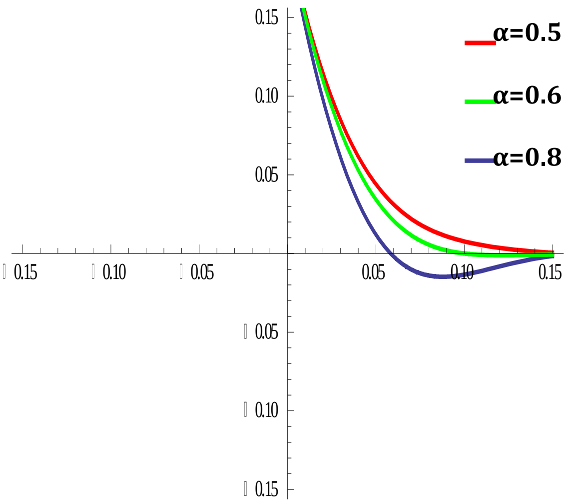

Fig.(2): Velocity Field U(y,t) forα = 0.5, 0.6,0.8 when M=2, t=2, λ =3.

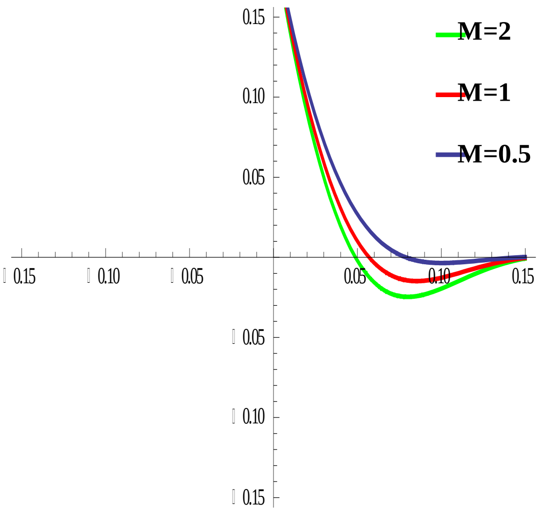

Fig.(3): Velocity Field U(y,t) for M= 2, 1, 0.5, when α= 0.9, t = 2, λ =3.

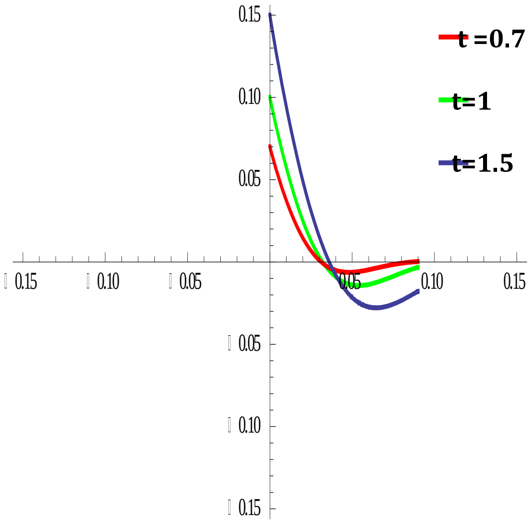

Fig.(4): Velocity Field U(y,t)for t=1.5, 1, 0.7when α=0.9, M=2, λ =3.

Appendix:-

$$(A1)\ D_t^p(t^a) = \frac{\Gamma(a+1)}{\Gamma(a+1-p)} t^{-p}$$

$$(A2)\ \frac{1}{z+a} = \sum_{r=0}^{\infty} (-1)^r \frac{z^r}{a^{k+1}} \sum_{m=0}^{r} C_r^m b^m = \sum_{m,l\geq 0}^{m+l=r} \frac{r!\, b^m}{m!\, l!}$$

$$(A3)\ \left(\frac{t^q}{\Gamma(q+1)}\right) = \frac{1}{p^{q+1}}\ ;\ Re(q) > -1, Re(p) > 0$$

$$(A4)\ L^{-1}\left(\frac{q^f}{(q^d-d)^e}\right) = G_{d,f,e}(p,t),$$
$$Re(de-f) > 0, Re(q) > 0$$

$$(A5)\ L^{-1}\left(\frac{p^B}{(p^a-q)}\right) = R_{a,B}(c,0,t), Re(a-B) > 0, Re(p) > 0$$

$$(A6)\ R_{1,0}\left(\frac{1}{\lambda},0,t\right) = \sum_{n=0}^{\infty}\left(\frac{1}{\lambda}\right)^n \frac{t^n}{\Gamma(n+1)} = Exp\left(-\frac{t}{\lambda}\right)$$

Reference

[1] Dunn .J.E, Rajagopal. K.R, "Fluids of differential type-critical review and thermodynamic analysis", Int. J. Eng .Sci,(33) 689-729, 1995

[2] Rajagopal. K.R, "Mechanics of non – Newtonian fluids",Pitman Research Notes in mathematics,pp. 129-162, Longman Scientific & Technical, 1993.

[3]Dunn.J.E,Rajagopal.K.R, "Thermodynamics stability and boundedness of fluids of complexity 2 and fluids of second grade", Arch. Ration,Mech, Anal, (56)191-252, 1974.

[4] Hussain. M, Hayat .T, Asghar. S,and Fetecau .C, "Oscillatory flows of second grade fluid in a porous space", Nonlinear Analysis: Real World Applications, 11(2403–2414),2010.

[5] Maxwell .J.C, "On the dynamical theory of gases", Philos.Trans.Roy.Soc. Lond, A (157) 26-78.,1866

[6] PodlubnyI, "Fractional Differential equations", Academic press, New York,1999.

[7] Markis .N, Constatinou .M.C, "Fractional derivative model for viscous dampers", J. Struct. Eng. ASCE, (117) 2708-2724,1991.

[8] Tan.W., Pan.W, Xu M, "A note on unsteady flows of a viscolastic fluid with fractional Maxwell model between two parallel plates, Internat". J. Non-linear Mech,(38)645-650,2003.

[9] Hayat .T, Nadeem. S, Asghar. S, "Periodic unidirectional flows of a viscolastic fluid with fractional Maxwell model", Appl.Math. Comput, (151)153-161,2004.

[10] Qi. H, Jin .H, "Unsteady rotating flows of a viscoelastic fluid with fractional Maxwell model between coaxial cylinders", Acta Mech.Sinica, (22)301-305,2006.

[11] Yin .Y, Zhu .K.Q, "Oscillating flow of a viscoalastic fluid in a pipe with fractional Maxwell model",Appl .Math. Comput,(173) 231-242,2006.

[12] Shaowei .W, Mingyu. X, "Exact solution on unsteady Couette flow of generalized Maxwell fluid with fractional derivative", ActaMech,.(187)103-112,2006.

[13] Khan. M, Hayat. T, "Some exact solutions for fractional generalized Burgers' fluid in a porous space", Nonlinear Anal.RWA 9 (2288–2295),2008.

[14] Xue .C, Nie. J, "Exact solutions of Rayleigh-Stokes problem for heated generalized Maxwell fluid in a porous half-space", Math. Prob. Eng. (2008), 2008

[15] Zheng. L, Liu.Y, Hang .X. Z, "Exact solution for MHD flow of generalized Oldroyd-B fluid due to an infinite accelerating plate",Math,Comput,Model,54(780-788), 2011.

[16] Feteca Corina, Fetecau.C, Athar .M, "Unsteady flow of a generalized Maxwell fluid with fractional derivative due to a constantly accelerating plate",Comput.Math .Appl, (57) 596-603, 2009.

[17] Senddn.I.N, "Fourier Transform", McGraw Hill Company, New York, Toronto, London, 1951.

[18] Lorenzo .C .F, Hartley .T.T, "Generalized Functions for the fractional calculus", NASA/TP, 1999-209424 /RVE1, 1999.

[19] Abramowitz .M, Stegun .I. A, "Handbook of Mathematical Function", Dover, New York, 1965.

الخلاصة

الهدف من هذا البحث هو دراسة تأثير المجال المغناطيسي على التدفق اللامستقر لمائع ماكسويل ذو المشتقات الكسرية نتيجة لوحة متسارعة ثابتة، حساب التفاضل والتكامل الكسري قد استخدم لكتابة معادلات الحركة,باستخدام تحويلات فورييه و لإبلاس يتم عرض الحلول الدقيقة بصيغة التكامل ومتسلسلات بدلالة الدوالG وRأخيرا، تأثير المعلمات على حقل السرعة قد درست من خلال الإيضاحات التخطيطية.