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Contra ω_{pre} –Continuous Functions

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Abstract

In this paper, we present some concepts related to ω_{pre}-open set and study some of its basic properties, facts and some examples are given to illustrate our work. Several theoretical results are stated and proved throughout this paper.

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1. Introduction

In this work, (X, τ_X), (Y, τ_Y) are supposed to be topological spaces (for short X and Y), which have no separation exams except whenever state. For a subset A its interior and closure are denoted by $int(A)$ and $cl(A)$, respectively. Also, A is said b-open if $A \subseteq int(cl(A)) \cup cl(int(A))$ and A is ω-open if for every point in it, there is an open set U containing x with $U - A$ is countable [6], while A is ω_{pre}-open (shortly ω_p-open) whenever replacing the open set to be pre-open [7] and every pre-open, ω-open set is ω_p-open. The ω_p-closed and ω_p-interior defined as in $cl(A)$ and $int(A)$, respectively. Dontchev introduced the notion of contra continuity. He defined a function f: $X \longrightarrow Y$ contra continuous if the inverse image of V is closed in X whenever V is open set in Y is contra continuous [4]. A function f: $X \longrightarrow Y$ is said to be almost contra ω-continuous [2] (resp.; almost contra-precontinuous [5]) if $f(V)$ is ω-closed (resp.; pre-closed) for every regular open set V in Y.

2. Contra Continuous Via ω_p-Open Sets

Dontchev introduced the notion of contra continuity. He defined a function $f: X \to Y$ is contra continuous if the inverse image of each open set in Y is closed in X. By the same context, we can define the following:

Definition 2.1, [2]. A function $f: X \to Y$ is called ω-continuous if for each $x \in X$ and each open set V of Y containing $f(x)$, there exists ω-open set U containing x, such that $f(x) \subseteq U$.

Definition 2.2. A function $f: X \to Y$ is said to be contra ω_p continuous if the inverse image of open set in Y is ω_p-open in X.

Remark 2.1. (1) Every contra ω-continuous function contra ω_p-continuous.
(2) Every contra continuous function is contra ω_p-continuous but the convers not true.

Example 2.1. Let the identity function $f: (Z, \tau_{ind}) \to (Z, \tau_D)$, then f is contra ω_p-continuous but not contra continuous.

Proposition 2.1. A function $f: X \to Y$ is contra ω_p-continuous if and only if for every closed subset F of Y, then $f^{-1}(F)$ is ω_p open in X.

Proof. Given f is contra ω_p-continuous function and F is closed in Y then $Y - F$ is open in Y but f is contra ω_p continuous, then $f^{-1}(Y - F)$ is ω_p -closed, but $x - (f^{-1}(Y - F) = f^{-1}(F)$ is ω_p open set.

Conversely, by the same way of above. ■

Lemma 2.1. A subset U of space X is ω_p-open if and only if every point in U is ω_p-interior point to U.

Lemma 2.2, [6]. The following properties hold for subsets A, B of a space X:
(1) $x \in$ Ker (A) if and only if $A \cap F \neq \emptyset$ for only closed set F containing x.
(2) $A \subseteq$ Ker (A) and $A =$ Ker (A) if A is open in X.
(3) if $A \subseteq B$, then Ker $(A) \subseteq$ Ker (R).

Proof. Let U be ω_p-open set and $x \in U$, then U an ω_p-neighborhood for each it's point.

Conversely, for each $x \in U$, we get ω_p-open set V_x containing x and contained in U, that is $U = \cup_{i\in\wedge} U_i$, but the arbitrary union of ω_p-open sets is also ω_p open. so U is ω_p-open. ■

Proposition 2.2. A function $f: X \to Y$ is contra ω_p-continuous if for each $x \in Z$ and each closed set F containing $f(x)$, there exists ω_p-open set U containing x such that $f(U) \subseteq F$.

Proof. Assume that f is contra ω_p-continuous and F is closed set containing $f(x)$ for some $x \in X$, so that $x \in f^{-1}(F)$, then $f^{-1}(F)$ is ω_p-open set in Z (by Proposition 2.2). If $f^{-1}(F) = U$, then U is ω_p open set containing x, such that $f(U) = f\left(f^{-1}(F)\right) \subset F$.

Conversely, let F be any closed set of Y if $f^{-1}(F) = \emptyset$, then there is nothing to prove. Suppose that $f^{-1}(F) \neq \emptyset$ and $x \in f^{-1}(F)$, then$f(x) \in f\, f^{-1}(F) \subset F$ which implies that there exists ω_p-open set U containing x, such that $x \in U \subset f^{-1}(F)$. So $x \in \omega_p - int\left(f^{-1}(F)\right)$. Thus $f^{-1}(f)$ is ω_p-open set in x. ■

Proposition 2.3. If a function $f: X \to Y$ is contra ω_p-continuous, then $f(\omega_p\text{-}cl(A)) \subseteq$ Ker ($f(A)$), for every subset A of X.

Proof. Let A be any subset of X and assume that $Y \notin$ Ker $(f(A))$ then there is a closed set F containing $f(x)$ in Y such that $f(A) \cap F = \emptyset$, then $A \cap f^{-1}(F) = \emptyset$, but f is contra ω_p-continuous then by Proposition 2.2, $f^{-1}(F)$ is ω_p open set containing x so x is not ω_p-adherent point to A that is, x$\notin$ ω_p $cl(A)$ Y=f(x) $\notin$ f(ω_p $cl(A)$) . Therefore $f(\omega_p$ $cl(A))$$\subseteq$ Ker($f(A)$). ■

Proposition 2.4. A function $f: X \to Y$ is ω_p-continuous if and only if for each $x \in X$ and each open set V containing $f(x)$, there exists ω_p-open set U containing x, such that $f(U) \subseteq V$.

Proof. Assume that f is ω_p-continuous and V is open set containing $f(x)$, for some $x \in X$, so that $x \in f^{-1}(V)$ then $f^{-1}(V)$ is $\omega_p -$open set in X (since f is ω_p continuous) now put $f^{-1}(V) = U$ with $x \in U$ then U is ω_p open set containing x, such that $f(U) = f(f^{-1}(V)) \subset V$. Hence $f(U) \subset V$.

Conversely; let V beany open set of Y, if $f^{-1}(V) = \emptyset$, then there is nothing to prove. Suppose $f^{-1}(V) \neq \emptyset$ and $x \in f^{-1}(V)$, then $f(x) \in f(f^{-1}(V)) \subset V$, then there exists $\omega_p$open set U containing x such that $x \in U \subset f^{-1}(F)$, so $x \in \omega_p - int(f^{-1}(V))$ so (by Lemma 2.1) $f^{-1}(V)$ is ω_p-open set in X. ■

Proposition 2.5. A function $f: X \to Y$ is contra ω_p-continuous, then f is $\omega_p -$ continuous whenever Y is regular.

Proof. Let V be an open set of Y containing $f(x)$ for some $x \in X$, then there is an open set W in Y, such that $f(x) \in W \subseteq \overline{W} \subseteq V$ (since Y regular so by Proposition 2.2, we obtain $\omega_p -$open set U containing x with $f(U) \subset cl(W) \subset V$. Therefore f is $\omega_p -$continuous. ■

Proposition 2.6. Let $f: X \to Y$ be a contra $\omega_p$continuous function then Y is not a discrete space, whenever X is ω_p-connected space.

Proof. Assume the domain is discrete and there is a nonempty clopen set A of it, then $f^{-1}(A)$ is nonempty proper ω_p-open and ω_p-closed subset of X so X is not $\omega_p -$ connected space, which is a contrary with our hypothesis. ■

Proposition 2.7. If $f: X \to Y$ is surjective contra $\omega_p -$continuous function and X is $\omega_p -$connected space, then Y is connected.

Proof. Suppose there are two nonempty disjoint open sets V_1 and V_2 and there union equal to Y. So $f^{-1}(V_1)$ and $f^{-1}(V_2)$ are disjoint ω_p-open sets in X and their union equal to X. So X is ω_p-disconnected, which is contrary with our hypothesis, therefor Y is connected. ■

Definition 2.3. A function f from X into Y is said to be almost contra $\omega_p -$continuous if $f^{-1}(V) \in \omega_p$-closed set in X, for every regular open V in Y.

Definition 2.4. A subset A of space X is said to be regular open if $A = \overline{A}^{\circ}$.

Theorem 2.1. Let f be a function from X into Y, then the following are equivalents:

(1) f is almost contra $\omega_p -$continuous.

(2) $f^{-1}(F)$ is $\omega_p$open in X, whenever F is regular closed in Y.

(3) For every $x \in X$ and every regular closed set F containing $f(x)$ in Y, there exists an ω_p $-$open U in X containing x with $f(U) \subseteq F$.

(4) For every regular open set V non containing $f(x)$ in Y for some $x \in X$, there is an ω_p-closed set K in X with $x \notin K$, such that $f^{-1}(V) \subseteq K$.

Proof. (1) → (2) let F be a regular closed set in Y then F^c is regular open in Y, but by our assumption that f is almost contra $\omega_p -$continuous. So we get $X - f^{-1}(F) = f^{-1}(Y - F)$ is $\omega_p -$closed in X and hence $X - (X - f^{-1}(F)) = f^{-1}(F)$ is $\omega_p -$open in X.

We can prove (2) → (1) by the same way to a bove.

(2) → (3) assume F be regular closed set in Y containing $f(x)$ in Y for some $x \in X$ and so by (2) $f^{-1}(F)$ ω_p-open in

X containing x . Put $f^{-1}(F)=U$, then $f\,(U)=ff^{-1}(F)\subseteq F$.

(3) → (2) let F be any regular closed set in codomain and x belong to $f^{-1}(F)$ so $f(x)\in ff^{-1}(F)\subset F$, then by (3) there is ω_p-open set U_x containing x, such that $f\,(U_x)\subset F$, then $U_x\subseteq f^{-1}(f(U_x))\subset f^{-1}(F)$, so x is ω_p -interior point to $f^{-1}(F)$ but x is an arbitrary, so $f^{-1}(F)$ is ω_p −open set (by Lemma 2.1).

(3)→(4) assume V is a regular open set with $f(x)\notin V$ in a space Y then $Y-V$ is a regular closed with $f(x)\in Y-V$, so by (3) there is ω_p −open set U in X with $x\in U$ and $f(U)\subset Y-V$ $U\subseteq f^{-1}f(U)\subset f^{-1}(Y-V)=f^{-1}(Y)-f^{-1}(V)=X-f^{-1}(V)$ that $U\subseteq X-f^{-1}(V)$ $X-(X-f^{-1}(V)\subseteq X-U$ $f^{-1}(V)\subseteq X-U$. Put $H=X-U$, so H is ω_p-closed set in X with $x\notin H$. ■

Definition 2.5. A space X is said to be:

(1) ω_p-compact if for all ω_p open cover of X has a finite subcover.

(2) Countably ω_p-compact if every countable cover of X through ω_p-open sets has finite subcover.

(3) ω_p -Lindelof if for all ω_p -open cover of X has a countable subcover.

(4) S-Lindelof [6] if for all cover of X by regular closed sets has a countable subcover.

(5) countably S-closed [15] if for all countable cover of X by regular closed sets has a finite subcover.

(6) S-closed [16] if for all regular closed cover of X has a finite subcover.

Theorem 2.2. Let f be a function from X into Y be an almost contra ω_p -continuous surjection. The following statements are holds:

(1) If X is ω_p-compact, then Y is S-closed.

(2) Y is S-Lindelof whenever X is ω_p-Lindelof.

(3) Y is countably S-closed whenever X is countably ω_p-compact.

Proof. If (1) hold then the other also holds.

Let $\{V_\alpha : \alpha\in I\}$ be any regular closed cover of Y. Since we have f is almost contra- ω-continuous, then $\{f^{-1}(V_\alpha)\alpha\in I\}$ is an ω_p-open contra of X and hence there is a finite subset $I_\circ$ of I, such that $X=\cup\{f^{-1}(V_\alpha):\alpha\in I_\circ\}$, so we have $Y=\cup\{V_\alpha:\alpha\in I_\circ\}$ and Y is S-closed. ■

Definition 2.6. The space X we call it:

(1) ω_p-closed compact if every ω_p-closed cover of X has a finite subcover.

(2) Countably ω_p-closed compact if every countable contra of X by ω_p closed sets has a finite subcover.

(3) ω_p Closed-Lindelof if for all cover of X by ω_p closed sets has a countable subcover.

(4) Nearly compact if for all regular open cover of X has a finite subcover.

(5) Nearly countably compact if for all countable cover of X by regular open sets has a finite subcover.

(6) Nearly Lindelof if for all cover of X by regular open sets has a countably subcover.

Theorem 2.3. Let f be a function from X into Y and f is almost contra-ω_p- continuous surjection. The following statements are hold:

(1) Y is nearly compact if X is ω_p-closed compact.

(2) Y nearly lindelof if X is ω_p-closed lindellof.

(3) Y is nearly countably compact if X is countably ω_p-closed compact.

Proof. If (1) holds then the other holds also, let $\{V_\alpha:\alpha\in I\}$ be regular open cover of Y, but f is almost contra-ω_p-continuous, then $\{f^{-1}(V_\alpha):\alpha\in I\}$ is an ω_p-closed cover of X. Since X is ω_p closed compact, then there exists a finite subset $I_\circ$ of I with $X=\cup\{f^{-1}(V_\alpha)\alpha\in I_\circ\}$ thus, we get $Y=\cup\{V_\alpha:\alpha\in I_\circ\}$ and Y is nearly compact. ■

Definition 2.7. A function f from space X in to a space Y is said to be almost contra ω_p-continuous if the inverse image of regular open in Y is ω_p-closed in X.

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