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Bullet Solutions Using Two Integration Architectures

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Abstract

In this study, the exact Bullet solutions for the (3+1)-dimensional Schrödinger equation which demonstrates the Bullet behaviour in optical fibers can be accumulated through the extended simple equation method, and Csch method. The applied strategies may retrieve several kinds of optical Bullet solutions within one frame, which is quite simple and reliable. As a result, we are able to develop a variety of traveling wave structures namely the periodic, singular Bullet wave. The extended simple equation method, and Csch method approaches were implemented perfectly and can be extended to deal with many advanced models in contemporary areas of science and engineering.

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1. Introduction

Nonlinear partial differential equations (NLPDE) are often encountered in establishing the basic principles of life as well as in the mathematical examination of an extensive span of issues simply occurring in weather, hydrodynamics, blood plasma, atmosphere crests, computational life sciences, chemical engineering, and also in materials research [1-3]. Numerous amazing models serve as examples of how innovative Bullets can be. Nonlinear Schrödinger equations (NLSE) are used for modeling a majority of spontaneous events [4]. The most significant kind of NLPDE that can be used in both computational and classical physics is the NLSE [5]." The Sine-Gordon expansion method [6], the modified simple equation method [7], the generalized exponential rational function method [8], the Riccati equation method [9], the auxiliary equation method [10], the unified method [11], the improved F expansion technique [12], the $\exp(-\zeta(\xi))$ expansion technique [13], and the Khater method [14]" are just a few of the many techniques used to more thoroughly study various nonlinear dynamical structures.

In this study, the exact Bullet solutions for the (3+1)-dimensional Schrödinger equation which demonstrates the Bullet behavior in optical fibers can be accumulated by using two strategies in our investigation to acquire some new exact solutions. These are the extended simple equation method, and Csch method. We further state that the

Al-Nahrain Journal of Science
ANJS, Vol.27 (4), Special Issue – October, 2024, pp. 1 – 6

techniques used in this model are original and freshly drawn, to the best of my knowledge.

2. The governing Equation

In this article, a (3+1) dimensional Schrödinger equation which shows wave behaviour in optical fibers is [15]:

$$i\frac{\partial\psi}{\partial t}-\frac{\partial^2\psi}{\partial x^2}-\frac{\partial^2\psi}{\partial y^2}-\frac{\partial^2\psi}{\partial z^2}+p\psi|\psi|^2+b_1\frac{\partial^2\psi}{\partial x\partial y}+b_2\frac{\partial^2\psi}{\partial y\partial z}+b_3\frac{\partial^2\psi}{\partial z\partial x}+i\left(c_1\frac{\partial\psi}{\partial x}+c_2\frac{\partial\psi}{\partial y}+c_3\frac{\partial\psi}{\partial z}\right)=0 \quad \dots(1)$$

where $i=\sqrt{-1}$ and $\psi=\psi$ (x, y, z, t) is a complex function and x, y, z, and t are the independent space parameters and time parameter accordingly. p, b_i , and c_i are constants.

Further if put $c_1=c_2=c_3=0$, then the above equation (1) can be converted to the following (3+1)dimensional Schrödinger equation [15].

$$i\frac{\partial\psi}{\partial t}-\frac{\partial^2\psi}{\partial x^2}-\frac{\partial^2\psi}{\partial y^2}-\frac{\partial^2\psi}{\partial z^2}+p\psi|\psi|^2+b_1\frac{\partial^2\psi}{\partial x\partial y}+b_2\frac{\partial^2\psi}{\partial y\partial z}+b_3\frac{\partial^2\psi}{\partial z\partial x}=0 \quad \dots(2)$$

Both equations (1) and (2) demonstrate ultra-short impulses of light spread via extremely nonlinear mediums. Also both these equations are used in the transmission of data in fibers optical. Using the extended simple equation method [16], and Csch method [17]. Which were not applied to this model in the past, one can say that the obtained solution are newly made and robust. The bright and dark wave solution of equation (1) are obtained by [18], further the generalized exponential rational function method is used to attain wave solutions [19].

3. Traveling wave solution

Consider the following nonlinear partial differential equation with four independent variables x, y, z, and t is written as

$$F\left(\psi,\psi_t,\psi_x,\psi_y,\psi_z,\psi_{tt},\psi_{xx},\psi_{yy},\psi_{zz},\psi_{xy},\psi_{yz},\dots\right)=0 \quad \dots(3)$$

where $\psi(x,\ y,\ z,\ t)$ denote a unknown function, F express a polynomial of $\psi=\psi(x,\ y,\ z,\ t)$ along with its partial derivatives.

let

$$\psi(x,y,z,t)=e^{i\theta}\,u(\xi) \quad \dots(4)$$

Where:

$$\xi=\alpha_1x+\beta_1y+\sigma_1z+\omega_1t \quad \dots(5a)$$

$$\theta=\alpha_2x+\beta_2y+\sigma_2z+\omega_2t \quad \dots(5b)$$

Using equations (4) and (5) to transfer the nonlinear partial differential equation (3) to nonlinear ordinary differential equation

$$Q(\psi,\psi',\psi'',\dots\dots\dots\dots.)=0 \quad \dots(6)$$

The ordinary differential equation (6) is then integrated as long as all terms contain derivatives, with zero constant of integration.

4. Methodology of the Extended Simple Equation Method (ESEM)

In this part, the extended form of the Simple Equation Method (ESEM) is introduced in order to derive the traveling wave solutions [15]. **Step 1** : Examining the structure of the answer to equation (6):

$$Q(\xi)=\sum_{j=-1}^{1}B_j\,f^j(\xi) \quad \dots(7)$$

here, B_j is real constant.

Step 2: Using the balance rule between the highest-order derivative and the non-linear terms in Equation (6), find the positive integer N that occurred in Equation (7).
Step 3: Assume that the following differential equation is satisfied by f:

$$f'(\xi)=b_0+b_1f(\xi)+b_2\,[f(\xi)]^2 \quad \dots(8)$$

where $b_0,\ b_1,\ b_2$ are arbitrary constants

Step 4: The following are the solutions to Equation (8) for various values of b_i:

In the event that $b_0=0$

$$f(\xi)=\frac{b_1\,e^{b_1(\xi+\xi_0)}}{1-b_2\,e^{b_1(\xi+\xi_0)}},\quad b_1>0 \quad \dots(9a)$$

$$f(\xi)=-\frac{b_1\,e^{b_1(\xi+\xi_0)}}{1+b_2\,e^{b_1(\xi+\xi_0)}},b_1<0 \quad \dots(9b)$$

When $b_1=0$:

Al-Nahrain Journal of Science
ANJS, Vol.27 (4), Special Issue – October, 2024, pp. 1 – 6

$$f(\xi) = \frac{\sqrt{b_0 b_2}\tan(\sqrt{b_0 b_2}\,(\xi+\xi_0))}{b_2}, \quad b_0 b_2 > 0 \quad \dots(10a)$$

$$f(\xi) = \frac{\sqrt{-b_0 b_2}\tanh(\sqrt{-b_0 b_2}\,(\xi+\xi_0))}{b_2}, \quad b_0 b_2 < 0 \quad \dots(10b)$$

The general solution of Equation (7) is

$$f(\xi) = \frac{\sqrt{4b_0 b_2 - b_1^2}\tan\left(\frac{1}{2}\sqrt{4b_0 b_2 - b_1^2}\,(\xi+\xi_0)\right) - b_1}{2b_2}, \quad 4b_0 b_2 > b_1^2 \quad \text{and} \quad b_2 > 0 \quad \dots(11a)$$

$$f(\xi) = \frac{\sqrt{4b_0 b_2 - b_1^2}\tan\left(\frac{1}{2}\sqrt{4b_0 b_2 - b_1^2}\,(\xi+\xi_0)\right) + b_1}{2b_2}, \quad 4b_0 b_2 > b_1^2 \quad \text{and} \quad b_2 < 0 \quad \dots(11b)$$

Step 5: A system of equations is produced by replacing equation (7) in equation (6) with equation (8) and setting the coefficients of powers of f^j to zero. After the set of equations is solved, the constant parameter values are found. Equation (6) can be solved by carrying these constant values along with the f(ξ) values in Equation (7).

5. Bullet Solution for the (3+1)dimensional Schrödinger Equation

To study the integrality aspects of the governing equations for (2), the following solution structure is taken into consideration. Using the transformation in equations(4)-(5). to get:

$$\begin{aligned} i\omega_1 u'(\xi) - \omega_2 u(\xi) &- [\alpha_1^2 u''(\xi) + 2i\alpha_1\alpha_2 u'(\xi) - \alpha_2^2 u(\xi)] \\ &- [\beta_1^2 u''(\xi) + 2i\beta_1\beta_2 u'(\xi) - \beta_2^2 u(\xi)] \\ &- [\sigma_1^2 u''(\xi) + 2i\sigma_1\sigma_2 u'(\xi) - \sigma_2^2 u(\xi)] + p\,u^3 \\ &+ b_1[\alpha_1\beta_1 u''(\xi) + i(\alpha_1\beta_2 + \alpha_2\beta_1) u'(\xi) - \alpha_2\beta_2 u(\xi)] \\ &+ b_2[\beta_1\sigma_1 u''(\xi) + i(\beta_1\sigma_2 + \beta_2\sigma_1) u'(\xi) - \beta_2\sigma_2 u(\xi)] \\ &+ b_3[\sigma_1\alpha_1 u''(\xi) + i(\sigma_1\alpha_2 + \sigma_2\alpha_1) u'(\xi) - \sigma_2\alpha_2 u(\xi)] \\ &= 0 \quad \dots(12) \end{aligned}$$

Splitting into real and imaginary parts leads to a pair of relations. The real part reads as follows:

$$p\,u^3 + [b_1\alpha_1\beta_1 + b_3\sigma_1\alpha_1 + b_2\beta_1\sigma_1 - \alpha_1^2 - \beta_1^2 - \sigma_1^2]\,u''(\xi) + [\beta_2^2 + \alpha_2^2 - \omega_2 + \sigma_2^2 - b_1\alpha_2\beta_2 - b_2\beta_2\sigma_2 - b_3\sigma_2\alpha_2]\,u(\xi) = 0 \quad \dots(13)$$

The imaginary part equation implies

$$\omega_1 - 2(\alpha_1\alpha_2 + \beta_1\beta_2 + \sigma_1\sigma_2) + b_1(\alpha_1\beta_2 + \alpha_2\beta_1) + b_2(\beta_1\sigma_2 + \beta_2\sigma_1) + b_3(\sigma_1\alpha_2 + \sigma_2\alpha_1) = 0 \quad \dots(14)$$

Then:

$$\omega_1 = 2(\alpha_1\alpha_2 + \beta_1\beta_2 + \sigma_1\sigma_2) - b_1(\alpha_1\beta_2 + \alpha_2\beta_1) - b_2(\beta_1\sigma_2 + \beta_2\sigma_1) - b_3(\sigma_1\alpha_2 + \sigma_2\alpha_1) \quad \dots(15)$$

Equation (13) can be written as:

$$p\,u^3 + c_1 u''(\xi) + c_2 u(\xi) = 0 \quad \dots(16)$$

where:

$$c_1 = [b_1\alpha_1\beta_1 + b_3\sigma_1\alpha_1 + b_2\beta_1\sigma_1 - \alpha_1^2 - \beta_1^2 - \sigma_1^2] \dots(17)$$

$$c_2 = [\beta_2^2 + \alpha_2^2 - \omega_2 + \sigma_2^2 - b_1\alpha_2\beta_2 - b_2\beta_2\sigma_2 - b_3\sigma_2\alpha_2] \dots(18)$$

6. Extended Simple Equation Method (ESEM)

Apply the homogeneous balance principle to Equation (16) to determine the values of N. When u^3, N+2= 3N, then N = 1. As a result, u(ξ) has the form shown below:

$$u(\xi) = \frac{B_{-1}}{f(\xi)} + B_0 + B_1 f(\xi), \quad B_1 \neq 0 \quad \dots(19)$$

$$u'(\xi) = \left[-\frac{b_0 B_{-1}}{f^2} - \frac{b_1 B_{-1}}{f} - b_2 B_{-1} + b_0 B_1 + b_1 B_1 f + b_2 B_1 f^2\right] \quad \dots(20)$$

$$\begin{aligned} u''(\xi) = \Bigg[&\frac{2b_0^2 B_{-1}}{f^3} + \frac{3b_0 b_1 B_{-1}}{f^2} + (b_1^2 + 2b_0 b_2)\frac{B_{-1}}{f} + (b_1 b_2 B_{-1} + b_0 b_1 B_1) \\ &+ B_1(2b_0 b_2 + b_1^2) f + 3b_1 b_2 B_1 f^2 + 2b_2^2 B_1 f^3\Bigg] \quad \dots(21) \end{aligned}$$

By using Equations (19) and Equation (21) in equation (16) we get the following equation:

Al-Nahrain Journal of Science
ANJS, Vol.27 (4), Special Issue – October, 2024, pp. 1 – 6

$$p\left[\frac{B^3{}_{-1}}{f^3}+3B_0\frac{B^2{}_{-1}}{f^2}+3B_1\frac{B^2{}_{-1}}{f}+3\,{B_0}^2\frac{B_{-1}}{f}+6B_0B_1B_{-1}+{B_0}^3+\left(B_{-1}B_1+{B_0}^2\right)3B_1f+3B_0{B_1}^2f^2+{B_1}^3f^3\right]$$
$$+c_1\left[\frac{2\,{b_0}^2\,B_{-1}}{f^3}+\frac{3b_0b_1\,B_{-1}}{f^2}+\left({b_1}^2+2\,b_0b_2\right)\frac{B_{-1}}{f}+(b_1\,b_2B_{-1}+b_0b_1B_1)+B_1\left(2b_0b_2+{b_1}^2\right)f+3\,b_1b_2B_1\,f^2+2{b_2}^2B_1\,f^3\right]$$
$$+\;c_2\left[\frac{B_{-1}}{f(\xi)}+B_0+B_1f(\xi)\right]=0 \quad \ldots(22)$$

Set of Equations is obtained for different orders of

$$f^j\;,j=-3,-2,-1,0,1,2,3$$
$$p\,B^2{}_{-1}+2c_1\,{b_0}^2=0$$
$$p\,B_0B_{-1}+c_1b_0b_1=0$$
$$p\left[3B_1B_{-1}+3\,{B_0}^2\right]+c_1\left[\left({b_1}^2+2\,b_0b_2\right)\right]+\;c_2=0$$
$$p\left[6B_0B_1B_{-1}+{B_0}^3\right]+c_1\left[b_1\,b_2B_{-1}+b_0b_1B_1\right]+c_2B_0=0$$
$$\left\{p\left[\left(B_{-1}B_1+{B_0}^2\right)3\right]+c_1\left[\left(2b_0b_2+{b_1}^2\right)\right]+\;c_2\right\}B_1=0$$
$$\{p\,B_0B_1+c_1\,b_1b_2\}B_1=0$$
$$\left\{p\,{B_1}^2+c_1\,2{b_2}^2\right\}B_1=0 \quad \ldots(23)$$

Constant values of B_{-1}, B_0, B_1, b_0, b_1, b_2, are obtained for the following cases:

Case I:
$b_0=0$
Then :

$$B_0=\mp i\sqrt{\frac{c_2}{p}}\,B_{-1}=0,\;B_1=\mp ib_2\sqrt{\frac{2\,c_1}{p}},\;\;b_1=\sqrt{\frac{2\,c_2}{c_1}}$$

$$\psi_1(\xi)=\;\mp\frac{i}{\sqrt{p}}\left\{\sqrt{c_2}+b_2\sqrt{2\,c_1}\;\;\frac{b_1\,e^{b_1(\xi+\xi_0)}}{1-b_2\,e^{b_1(\xi+\xi_0)}}\right\}e^{i\,\theta},\quad b_1>0 \quad \ldots(24a)$$

$$\psi_2(\xi)=\;\mp\frac{i}{\sqrt{p}}\left\{\sqrt{c_2}-b_2\sqrt{2\,c_1}\;\;\frac{b_1\,e^{b_1(\xi+\xi_0)}}{1-b_2\,e^{b_1(\xi+\xi_0)}}\right\}e^{i\,\theta},\quad b_1<0 \quad \ldots(24b)$$

where:

$c_1\;and\;p\neq 0$, $\xi=\alpha_1x+\beta_1y+\sigma_1z+\omega_1t$, $\theta=\alpha_2x+\beta_2y+\sigma_2z+\omega_2t$ (25)

Case II:
$b_1=0$

$$B_{-1}=\;\mp i\,b_0\sqrt{\frac{2c_1}{p}}\;,\;B_0=0,$$
$$B_1=\;\mp i\,3\sqrt{\frac{2\,c_1}{p}}\,,\;\;b_0=\;\frac{c_2}{12\,c_1}\;\;b_2=\;3$$

$$\psi_3(\xi)=\;\mp i\sqrt{\frac{6b_0\,c_1}{p}}\left\{\cot\left(\sqrt{3b_0}\,(\xi+\xi_0)\right)+\tan\left(\sqrt{3b_0}\,(\xi+\xi_0)\right)\right\}e^{i\,\theta},\quad b_0>0 \quad \ldots(26a)$$

$$\psi_4(\xi)=\;\mp i\sqrt{\frac{6b_0\,c_1}{p}}\left\{\coth\left(\sqrt{3b_0}\,(\xi+\xi_0)\right)+\tan\left(\sqrt{3b_0}\,(\xi+\xi_0)\right)\right\}e^{i\,\theta},\;\;b_0<0 \quad \ldots(26b)$$

where:
$p\neq 0$,
$\xi=\alpha_1x+\beta_1y+\sigma_1z+\omega_1t,$
$\theta=\alpha_2x+\beta_2y+\sigma_2z+\omega_2t \quad \ldots(27)$

Case III
$b_0\neq 0\,,b_1\neq 0$

then

$$B_{-1}=\mp i\,b_0\sqrt{\frac{2c_1}{p}},$$
$$B_0=\pm i\,b_1\sqrt{\frac{c_1}{2\,p}}\;,$$
$$B_1=\mp i\,b_2\sqrt{\frac{2\,c_1}{p}} \quad \ldots(28)$$
$$c_2=\left[\frac{1}{2}{b_1}^2+4\,b_2\,b_0\right]c_1 \quad \ldots(29a)$$

Or

$$c_2=\left[\frac{1}{2}{b_1}^2-\;8\;\;b_2\,b_0\;\right]c_1 \quad \ldots(29b)$$

Where $4b_0b_2>b_1^2$ and $b_2>0$,

$$\psi_6(\xi)=\mp i\sqrt{\frac{c_1}{2p}}\left[\frac{4b_0\,b_2}{\sqrt{4b_0b_2-b_1^2}\tan\left(\frac{1}{2}\sqrt{4b_0b_2-b_1^2}\,(\xi+\xi_0)\right)+b_1}+\sqrt{4b_0b_2-b_1^2}\tan\left(\frac{1}{2}\sqrt{4b_0b_2-b_1^2}\,(\xi+\xi_0)\right)\right]e^{i\,\theta} \quad \ldots(30)$$

Where: $4b_0b_2>b_1^2$ and $b_2<0$,

$$\xi=\alpha_1x+\beta_1y+\sigma_1z+\omega_1t,$$
$$\theta=\alpha_2x+\beta_2y+\sigma_2z+\omega_2t \quad \ldots(31)$$

Al-Nahrain Journal of Science
ANJS, Vol.27 (4), Special Issue – October, 2024, pp. 1 – 6

7. Csch function method

The solution to many nonlinear equations can be expressed in the form [17]:

$$V(\xi) = A\, csch^{\tau}(\mu\, \xi) \quad \ldots (32)$$

and their derivative.

$$V'(\xi) = -A\, \tau\, \mu\, csch^{\tau}(\mu\xi)\,.\, cot\, h(\mu\xi) \quad \ldots (33)$$

$$V''(\xi) = A\, \tau\, \mu^2 \left((\tau + 1)\, \mathrm{csch}^{\tau+2}(\mu\xi) + \tau\, \mathrm{csch}^{\tau}(\mu\xi) \right) \quad \ldots (34)$$

where μ and λ are the wave number and wave speed, respectively, and A, μ, and τ are parameters that need to be found. When we enter equations (32) and (34) into the simplified equation (16), we obtain

$$p\, A^2 csch^{3\tau}(\mu\, \xi) + c_1 \tau\, \mu^2 \left((\tau + 1)\, \mathrm{csch}^{\tau+2}(\mu\xi) + \tau\, \mathrm{csch}^{\tau}(\mu\xi) \right) + c_2 csch^{\tau}(\mu\, \xi) = 0 \quad \ldots (35)$$

Balance the terms of the csch functions in equation (35), i.e. $3\tau = \tau + 2$, then $\tau = 1$

Next, we gather all terms in equation (35) that have the same power in $csch^{k}(\mu\xi)$ and set their coefficients to zero in order to obtain an algebraic system of equations between the unknowns A and μ that leads to the following system:

$$p\, A^2 + 2\, c_1\, \mu^2 = 0, \quad c_1\, \mu^2 + c_2 = 0 \quad \ldots (36)$$

Solving the system of equations in (36) we get:

$$A = \mp \sqrt{\frac{2\, c_2}{p}}, \quad \mu = \mp i \sqrt{\frac{c_2}{c_1}} \quad \ldots (37)$$

Therefore:

$$\psi_7(x,t) = i\, e^{i(\alpha_2 x + \beta_2 y + \sigma_2 z + \omega_2 t)} \sqrt{\frac{2\, c_2}{p}}\, Csc\left(\sqrt{\frac{c_2}{c_1}}\, [\alpha_1 x + \beta_1 y + \sigma_1 z + \omega_1 t] \right) \quad \ldots (38)$$

where:

$p > 0, \; c_1 > 0$

8. Conclusions

In this article, we study the (3+1)-dimensional Schrödinger model by employing some well-known analytical techniques such as the extended simple equation method, and the Csch method. A variety of new Bullet solutions have been discovered that are periodic shaped, singular shaped Bullet wave solutions. As a result, it can be seen that the techniques used useful for dealing with a variety of other higher dimensional nonlinear evolution models that exist in the fields of hydrodynamics, plasma, mathematics, and other sciences.

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Al-Nahrain Journal of Science
ANJS, Vol.27 (4), Special Issue – October, 2024, pp. 1 – 6

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