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Discrete Wavelet Transform-Based Image Processing: A Review

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Abstract

The field of image processing has seen remarkable advancements over the past few decades, and Discrete Wavelet Transform (DWT) has emerged as a powerful tool within this domain. This review article provides a comprehensive overview of previously published works that focus on DWT's application in image processing. DWT offers multi-resolution analysis capabilities, making it particularly useful for various image processing tasks such as de-noising, compression, enhancement, and feature extraction. This review explores the fundamental principles of DWT and its mathematical foundations. We delve into its advantages over traditional Fourier Transform methods, particularly its ability to handle non-stationary signals and provide both frequency and spatial information simultaneously. The article surveys the application of DWT in different image processing techniques. It discusses how DWT contributes to the efficiency and effectiveness of image compression algorithms, such as the JPEG 2000 standard, and its role in reducing noise while preserving important image features.

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1. Introduction

Wavelet analysis is a time-frequency domain analysis of radiofrequency ultrasound signals using a novel mathematical model for assessing local changes in the geometrical profile of time-series signals to identify plaque components [1]. Wavelets have the effective advantages of multi-resolution properties and different basis functions. Before 35 years ago, Morlet produced a new apprehensible for wavelet analysis to reach a better swapping approach between time and frequency resolution automatically [2], and this proposal concept became the generation of ideas promoted by Haar in 1910 and Gabor in 1946 [3], then in 1976, Zweig based his search field depending on Morlet concepts [4]. In general, a wavelet is used to describe a function that has compact support, i.e. it is non-zero only over a finite interval. The set of time dependent data on a wavelet basis is represented to give an unmatched structure of information that is localized simultaneously in the time and frequency domains which cannot occur in the Fourier Transform representation, where specific frequencies cannot be associated with a particular time interval since the basic functions have constant resolution on the entire domain. The wavelet basis representation generates a set of wavelet coefficients organized at various resolution levels. Each parameter is associated with a specific level of accuracy and a point in the time domain. At the lower resolution level, the coefficients describe low-frequency features of data spanning wide time periods, while at the higher resolution level, the coefficients are associated with very local high-frequency features [5].

2. The Discrete Wavelet Transforms (DWT)

The discrete Wavelet transform (DWT) is considered one of the most powerful tools for image processing in both time and frequency domains which gives a better analysis of image localization features. The term wavelet was first presented in "the early 1980s" by Grossman and Morlett. The researchers used the

French word "omelette", as a representation of a small wave [6]. A wavelet can be defined as a small oscillatory function that is well localized in both time and frequency domains. Nowadays, wavelets are considered a mathematical tool to present many image applications because a wavelet is a versatile tool with wide fields of applications [7]. The first utilization of discrete waves was for seismology to give a time estimation to seismic spectroscopy, which results in steady information, but on the other side, it is considered not accurate approach to give Information with activities that are largely unexpected from previous information. The use of discrete wavelets in the seismology field came after the failure of the Fourier transform examination to give the desired results [8]. As well as seismology, the researchers used discrete wavelet transform in other wide applications such as signal and image processing, communications, the field of quantum mechanics and mathematical applications [9]. However, computer imaging and animation are not the exclusive application areas for discrete wavelet transformations. The biorthogonal wavelet bior4_4 (FBI) can encrypt about one million of fingerprints depending on the saved database. Other wavelet types were used in medical fields like breast cancer diagnosis examination, DNA analysis, pressure of blood, EEG and heart rate. Other field applications were for weather forecasts, verifying fingerprints, internet traffic descriptions, speech recognition, computer graphics, etc [10].

a. The discrete wavelet transformation analysis

The basis of DWT transformation relies on analyzing a signal into a range of small waves that have limited duration and inequalities frequency. Then by applying the IDWT (Inverse Discrete Wavelet Transform) technique to the comprised signal, the original signal will be fully reconstructed depending on the wavelet transform coefficients [11]. The DWT will analyse the original image into sub-images [12], one sub-band (LL) is for image zoom, and the other three sub-bands (LH, HL and HH) for highlighting the image edges. As shown in (figure 1) [13] where:

- **LL (Approximation sub-band):** has low-frequencies within the level and vertical directions. It represents the coarser approximation of the original image and retains the significant features and energy of the image.
- **LH (Horizontal detail sub-band):** has low-frequencies within the even heading and high-frequencies within the vertical direction. It highlights the vertical edges and details of the original image.
- **HL (Vertical detail sub-band):** has high-frequencies within the level direction and low frequencies within the vertical direction. It highlights the horizontal edges and details of the original image [12].
- **HH (Diagonal detail sub-band):** has high-frequencies in both the horizontal and vertical directions. It highlights the diagonal edges and fine details of the image. All the sub-bands have Size $= \frac{M}{2} \times \frac{N}{2}$. (For an image size (M×N) [13].

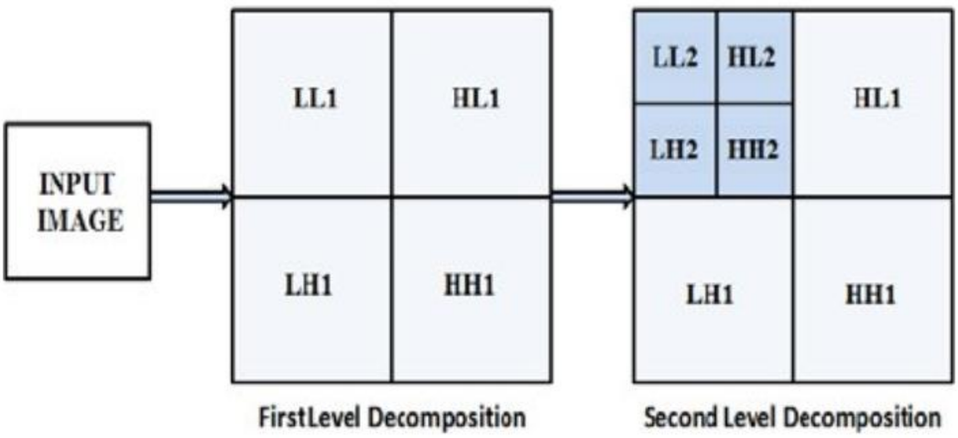

Figure 1. The Sub-bands after two levels of wavelet decomposition [13].

The sub-band LL is the most important and racy one because it contains most of the image with an estimation for it. The high-frequency sub-bands (LH, HL, and HH) can include watermarks because of their lesser sensitivity to human vision. By increasing the number of level sub-bands, the watermark force will be increased with no effects on image quality. For each decay level in the DWT process, the first level will be executed within the vertical direction and then in the level course. In deterioration, the first level will give four sub-bands or ranges: LL1, LH1, HL1, and HH1. As a sequential process, the LL sub-band from the olden level will be the input for each next decay level and it will be divided into another four sub-bands to process the coarse wavelength coefficients [14]. Sequentially, these operation steps will be repeated many times depending on the specific image application [15].

b. The discrete wavelet transform function

A wavelet function is a mathematical function used in wavelet transforms to decompose a signal into a set of basic functions consisting of contractions, expansions, and translations of a mother function [16]. The integral value of the mathematical wavelet function is always zero in the interval (∞, -∞) [1], and the wavelet basis or orthonormal basis or can be defined as the equations below depending on assuming *Ψ (x)* as the mother wavelets or basic wavelets:

$$\Psi_{(j,k)}(x) = 2^{j}/_{2}\ \Psi(2^{j}x - k) \quad \dots (1)$$

and the scaling function is given as

$$\Phi_{(j,k)}(x) = 2^{j}/_{2}\ \Phi(2^{j}x - k) \quad \dots (2)$$

where:

Ψ represents the wavelet function, j and k are the integers that represent the wavelet basis scaling and dilation. The factor "j" represents the width scale index, and the factor "k" represents the position. In terms of coefficients, the wavelet equation can be written as follows:

$$\Psi(x) = \sum_{k}^{n-1} g_k \left(\sqrt{2\Phi(2x - k)}\right. \quad \dots (3)$$

where g_0 and g_1: are the high pass wavelet coefficients that represent the time series changing at a specific certain resolution [17].

c. The DWT families

There are two basic different properties of wavelet families, the orthogonal and bi-orthogonal wavelets. In general, orthogonality eliminates transform coefficients by minimizing redundancy. Symmetry provides linear phase and minimizes edge artifacts [18]. The other important properties of wavelet functions in image compression applications are symmetry, compactness, degree of smoothness and regularity [19]. In addition, orthogonality is considered an important aspect in the context of signal processing because it is an easy way to build controls and data structures in languages [20]. Due to its simple nature, it has many orthogonal designs with lower expectations [21]. Orthogonality always reflects right angles, that is, 90°. The orthogonal wavelet is related to the wavelet transform and they are orthogonal to each other [22]. Orthogonality has the advantage of avoiding interference so that the error-free output can be obtained. Because of this feature, it is mostly a preferable method in signal processing. The orthogonal wavelets are considered as the wavelet transform's adjoints. But when this condition fails, they result in biorthogonal wavelets. At this point, biorthogonal wavelets are formed, and the orthogonality of wavelets get vanishes. In other words, an orthogonal wavelet means using a single wavelet with a single-scaling function. While the biorthogonal wavelet means generating one wavelet and one scaling function for the image decomposition process and an additional one wavelet with one scaling function for image reconstruction process. Both scaling functions are responsible for generating different multi-resolution analyses. Generally, the biorthogonal wavelet is more advanced than the orthogonal wavelet [23]. However, it requires extra time-consuming and effort to accomplish the computation process. The basic difference between the orthogonal and the biorthogonal wavelets is the variation in the wavelet's length during the analysis and the synthesis process. In orthogonal wavelets, the same length filters are used and in the biorthogonal wavelet, different lengths filters are used [24]. The most common types of orthogonal wavelet families are (Haar, Daubechies (dbxx), Coiflets (coifx), and Symmlets (symx)), while the biorthogonal wavelets families are (Biorthogonal (biorx_x)), where x indicates the order of the wavelet. first (x) number represents the number of decomposition filters' vanishing moments, and the second (x) number represents the number of reconstruction filters' vanishing moments [25].

D. Wavelet-based applications in image processing

Recently, the DWT has played a main role in image processing field because of providing powerful tools for compression, denoising, edge detection, and watermarking [26]. The scientific researchers keep on enhancing its capabilities through new wavelet bases, adaptive methods, wavelet packets, and integration with deep learning [27]. The DWT applications in image processing can be summarized in:

I. Image Compression Techniques:

- JPEG 2000: the JPEG2000 standard uses dwt for image compression, offering better compression ratios and image quality compared to the traditional jpeg standard. Taubman and Marcellin (2002) detailed the benefits of dwt in achieving high compression efficiency [28].
- Set Partitioning in Hierarchical Trees (SPIHT): Said and Pearlman (1996) proposed the SPIHT algorithm, which exploits the hierarchical nature of DWT to efficiently encode images [29].
- Embedded Zerotree Wavelet (EZW):it is another wavelet-based image compression algorithm that exploits the spatial relationship between wavelet coefficients. It encodes significant coefficients and their descendants in a hierarchical tree structure, leading to efficient compression [30].

ii. Denoising Techniques and Methods:

- Thresholding Techniques: Donoho and Johnstone introduced the concept of wavelet thresholding for image denoising, where coefficients below a certain threshold are set to zero, effectively removing noise while preserving important features [31].
- Bayesian Methods: Portilla et al. developed a Bayesian denoising method using Gaussian scale mixtures in the wavelet domain, achieving state-of-the-art denoising performance [32].

iii. Edge Detection and Feature Extraction:

- Multi-resolution Analysis: Mallat and Zhong demonstrated the use of DWT for edge detection, leveraging its multi-resolution capability to identify edges at different scales [33].
- Texture Analysis: Ma and Manjunath utilized DWT for texture segmentation, capitalizing on its ability to capture texture information across multiple scales [34].

iv. Watermarking:

- Robust Watermarking: Sharma et al. explored the use of DWT for digital watermarking, showing that wavelet-based watermarking schemes are robust against various attacks and offer high imperceptibility [35].

3. Literature Review

Wavelet theory emerged in the 1980s, with key contributions from researchers like Ingrid Daubechies and Stéphane Mallat. DWT, a discrete implementation of the continuous wavelet transforms, gained prominence due to its practicality in digital signal processing. Below is the literature review of discrete wavelet transform-based image processing applications [36].

A. Applications of image compression

Image compression is a crucial aspect of digital image processing, enabling the reduction of storage space and transmission bandwidth. DWT has become a popular technique for image compression due to its ability to represent images efficiently by decomposing them into multi-resolution sub-bands [37]. Image compression is considered as the main image processing application type that is based on discrete wavelet transform because image compression that is processed using DWT will offer better image quality and compression rates than using JPEG algorithm [38]. Many of scientific researchers focused on this field and some of them used the multispectral images (from satellite imagery) as test compressed images using different wavelet bases and methods in their researche [39]. Khashman and Dimililer [40] used neural systems in compressing an image depending on varying Haar wavelet in order to originate an ideal framework for the compressed image. The varying of discrete wavelet will successfully effect on image-coding applications because of decay. Elamaran and Praveen [41] proposed a constitutive rule for the compressed image, they depended on reducing the actual number of bits per pixel to represent the image in full clarity. Noticing that DCT ((discrete cosine transform) and DFT (discrete Fourier transform) failed in processing an image with the low-frequency domain. Taujuddin et al. [42] proposed a method by computing DCT (discrete cosine transform) with special main capacities, in compressing the 2D images. The primary image is divided into eight-by-eight pieces, and it can be reversely reconstructed into eight-by-eight squares but with an error ratio.

Seetharaman [43] used the plots of compressed image with discrete wavelet difference-based pruning proposal. It results in sufficient and high-compression proportions with no noticeable decrease in image quality. Hacihaliloglu and Kartal [44] made a comparison between the compression techniques namely discrete cosine transform and discrete wavelet transform to investigate RADARSAT and SPOT images of different regions of different characteristics in some different regions (sea areas, forest areas, built environment residential and industrial areas) which defined different patterns of urban land use. The studies showed that compression ratios changed according to the pixel classification. The obtained results showed the evaluated compression ratios and capacities of the various wavelets that achieved calculating the compression ratio (CR), cruel square error, bits per pixel (BPP) and PSNR (Peak signal-to-noise ratio) for different wavelets [10]. Sahnoun and Benabadji [45] in their search paper used discrete wavelet shift noise suppression to compress the fake images. They proposed an image compression strategy based on the evidential hypothesis and k-nearest neighbor (k-NN) to calculate the compression parameters. Moreover, they used a distinct compression method using the Fourier transform and Huffman coding method. Also, Memane and Ruikar [46] used shifting of a wavelet in their work. The implementation analyses are based on using various wavelets using partisanship compressed image. Susilo and Bretschneider [47] Depict the image compression strategy for X-sat images by using hardware-based arranging to perform loss levels of image compression.

b. Applications of image de-noising

Image processing, which covers a palatial range of processes like digitization, duplication, transmission, visualization etc. Unfortunately, this processing spoils the overall image quality by covering up many types of noise. So, to reconstruct the basic structure of the image, unwanted embedded noise must be decoded and removed. In image processing, noise suppression is implemented by using of filter-based noise reduction approaches. Wavelet transforms support many small operators and a small number of large ones. The basic stages for denoising methods based on using wavelet transform are:

i. Calculating the WT of the selected noisy signal.
ii. Changing the nuisance wavelet parameters depending on basic denoise rules.
iii. Calculating the inverse wavelet transform using modified operators [48].

The subsequent write-up talks about noise reduction using wavelet changes has many explanation steps, like using many different thresholding concepts for a palatial range of the testing images [49]. Remembering that t he image is often damaged by disturbance when it is captured and moved. Image noise reduction is used to remove embedded noise while protecting as many important signal highlights as possible [48]. Shifting a wavelet will reduce noise within an image. Because of its remarkable localization feature, wavelet changes are quickly becoming an important tool for signal and image preparation for a variety of applications, including noise and weight reduction. Wavelength noise reduction attempts to eliminate noise within the signal while protecting the properties of the signal, however, from repeating material. The initially proposing of wavelet thresholding was by Taujuddin et al. [42], who used signal estimation method (that misuses wavelet shifting capabilities) to reduce the noise of a signal. It gets rid of the noise by killing insignificant exchanges within some limits. Analysts have devised various ways of choosing turbulence minimization parameters. Raghavendra et al. [48] presented a workable variation of wavelet space noise filtering strategy that preserves edges and decimates noise. Noise is removed from changeable wave information in certain scale by making an information compassion, by correlating information at that scale with information at larger scales. Features are distinguished and retained because they are unambiguously related across the extension during the changeable wavelet field. The noise will be identified and then removed because of its weak correlated cross-band during the changeable wavelet field.

In general, Highlights will remain distortion-free because of their right places in space during the changeable wavelet field. Then, these image edges will remain steep after the process of sifting. Choi and Jeng [25] described Bisov spheres (a set of curved images in which Bissoff parameters are limited to radii) during many wavelets regions and predicted their crossable point using Projection on Curved Mass Calculation (POCS). This method resulted in an amazing advance over the known wavelength drag calculation method, depending on the single wavelength domain to give a strong edge within it. Yoon and Vaidyanathan [50] introduced a typical threshold scheme which outperformed traditional robust and accurate histograms. Boudiaf et al. [39] used multivariate deflation to develop traditional and olden wavelet strategies to several variation coefficients. Initially, a simple second-order orthogonal prefilter was used to give many higher frequencies waves (while maintaining a pre-orthogonal channel to another). So, the edge selections will be considered by using an unbiased Stein estimator for each resolution point. Gilman et al. [51]presented a proposed method on a novel technique for resampling a non-uniformly sampled image onto a uniform grid that can be used for fusion of translated input images that implemented as a finite impulse response filter of low order (10th order results in good performance). The technique is based on optimizing the resampling filter coefficients using a simple image model in a least squares fashion.

After discrete wavelet shift (DWT) based denoising obstacle analysis, using coupled complex wavelet shift (DTCWT) [39] to extract SPN from the image, which can obtain better SPN quality within the region around hard edges [52]. Otherwise, we point out that the extension of conventional iodinated boundaries is the source of the poor quality of SPN besides image boundaries [53]. Then, symmetric boundary expansion is introduced in SPN extraction, which can fundamentally improve the quality of SPN with image boundaries. Extensive tests show that the proposed strategy leads to better SCI implementation compared to the case of verbatim methods, like the strategies based on DWT [54].

c. Applications of image enhancement

Image enhancement is considered as the most scaling problem in high-quality image devices like HDTV and digital cameras. There are many issues based on image sharpness and clarity that requires the control, these issues like lighting, weather conditions, incorrect camera exposure or wrong aperture settings, etc., these may result in an unclear image [55] and obtaining the most accurate details of an image with useful data, especially data for the darker images [56]. To solve these problems, many techniques were developed to produce basic image information. These techniques focus on improving color images, like discrepancy, histogram enhancing formats, smooth filtration, single-band and multi-band retinex, and multi-band width. As known, the colored- images give more visual perception data than grayscale- images [57].

In general, the two main categories of image enhancement methods are: spatial and frequency domain methods. The spatial strategies are based on image pixels [58]. The values of these pixels must be controlled in order to achieve the desired upscaling. in iteration space strategies, an image is initially interchanged in the iteration space [59] i.e. any Fourier changes of the image will be calculated initially to perform all the Fourier transform optimization of the image, and then an inverse Fourier transform is performed to obtain the resulting image. Sometimes certain image features are difficult to distinguish with the naked eye, we often must change the recently viewed images. There are many efficient strategies in differentiating optimization field, graph equalization is one of these useful strategies. its benefits based on images with impoverished spike distribution [60]. Another way to make the image understandable is to upgrade image edges, one great way to upgrade image discrimination [10]. Gupta [54] produced other works in the development of wavelet-based images, which integrate the work of stoschek and hegerle on noise reduction of electronic ct entertainment [32].

Abdulazeez et al. [61] appeared that wavelets were used to separately update and merge ct mammography extraction. the scientific researchers, madhankumar et al. [62] provided an updated discrete method for useful images using wavelet-based multi-scale in representation the image edge depending on the image edge detection and the wavelet classification features. The test images were based using different imaging modalities. for the medical applications, urooj and singh, [63], image edge detection was a characteristic application for monitoring the heart wall thickness in the middle of the cardiac cycle. for security fields applications, stromvig and ren [64] fingerprint images are compressed using the dwt (discrete wavelet transform) biorthogonal wavelet bior4.4 and orthogonal wavelet haar at different levels of transformation. The experiment is recognized through python and is divided into two parts: transformation testing and spiht (set partitioning in hierarchical trees) algorithm compression. rong et al. [65], used the histogram equalization technique on colored image to improve the contrast of the color heritage image and try to give the dramatic improvement in image and its structural details. There is not much improvement that has been made to different images.

D. Applications of image watermarking

Watermarking emerged in the middle of the 1990s within the broad scope in the information education field, presenting special ways to secure a digital image from any theft. It involves implanting a watermark within an advanced image at a certain time point or during its distribution. The features of this strategy depend on its ability to achieve three basic preconditions: security, power and intangibility [35]. The general viewing of a watermark consists of two parts: firstly, looking at the watermark and then detecting it, as shown in figure 2.

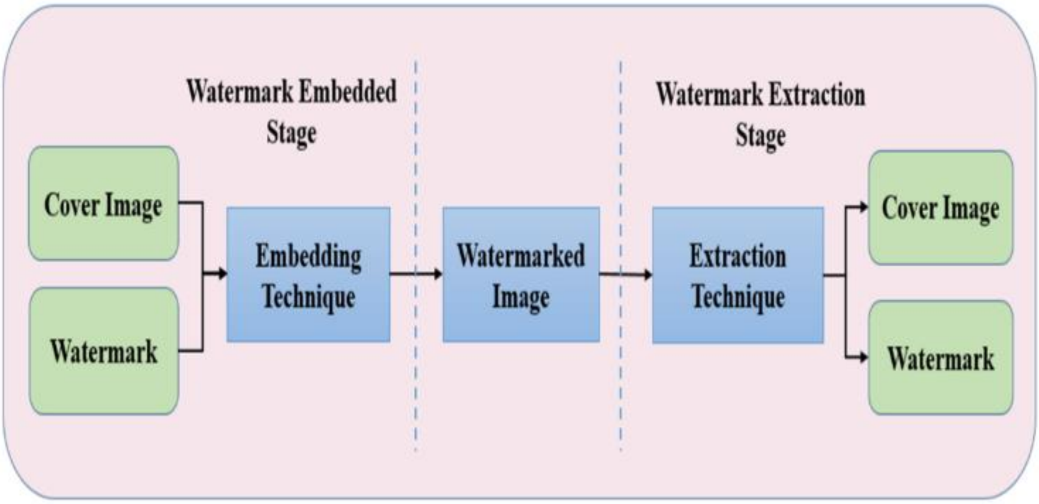

Figure 2. The general process of embedding and extraction operation [65].

In the watermarking Embedded Stage, the watermark might be encoded with the cover image using a specific key for more security levels. This will result in a watermarked image to be moved to the recipient. Then, in the extraction (detection) stage, the watermark will be removed from the attacked cover signal. Hina Lala [11] proposed a 2D DWT handled by the cover image which decomposes this image into four sub-bands, including the estimation sub-band for low-frequencies, corner to corner sub-bands for high-frequencies and sub-band for vertical flat low-frequencies. Furthermore, 2D DWT will be performed on the watermark image that needs to be embedded in the cover image by using wavelet radiation. Abdulkareem et al. [66] proposed a stepwise watermark depending on using discrete wavelets based on pixels blocking. It is based on compatible properties of HVS watermark. So, adaptively this watermark will be embedded with further possible details and its effect can be calculated depending on HVS screen information. Hamid and Jamel [67] proposed a system of hiding a text in a digital grayscale image by using the algorithm system with adopted two transforms Integer Wavelet transform and Discrete Cosine transformed. They used (PSNR) to measure the effect of embedding text in the watermarked image. Quazi [58] made an accommodative wavelength watermarking plot scheme. Insertion is achieved during the higher sub-bands of the changeable wavelet, although this will obviously change the image definition. To avoid tactile weakness of the image, the watermark is carefully included while using HVS. Jabade and Gengaje [68] established an efficient off-the-shelf computation of dual micro-watermarks using advanced pixel blocking screen and state-of-the-art bit substitution based on pseudo-uniform clustering. The strategy includes a hard and precise watermark within the tough part and the sensitive part of coefficients, to avoid overlapping of the two watermarks. The recommendation of Jie et al. [69] was to use watermark plotting to obtain dazzling images based on wavelet tree metric evaluation. To get watermark bits a comparison is made based on values of the largest variance and the average value of the expanded variance with the most final evaluated variance coefficients. Kaibou et al. [70] contrast the implementation of several chaos-based digital image watermarking techniques in terms of imperceptibility, robustness, security and complexity. Most spatial and transform domain methods have been used namely Least Significant Bit (LSB), Discrete Cosine Transform (DCT), Discrete Wavelet Transform (DWT). Locally, a wavelet will be shifted, and a watermark will be implanted depending on a chaotic computable scheme [71]-[72]. Tuncer and Kaya [73] proposed a method to obtain a strong watermark. For this, the watermark will be included in the selected image low-frequency sub-bands. The first step is encoding the selected image based on Arnold transform; then logistic maps will be used for scrambling and as a known process, the decomposing of the selected image will be achieved using DWT [74]. Based on Arnold transform, the watermarked image will be mixed and then embedded within discrete wavelet domain low-frequency coefficients to give the final watermarked image with a perfect optical quality [61]. Also, the saturation of colors during hue-preserving color image enhancement can be improved without the gamut problem [75]. Many searches regarding soft threshold method are used to distinguish the useful information and noise of image [76]. Beside to 1D and 2D, the 3D images processing constitutes a challenging topic in many scientific fields such as medicine, computational physics, and informatics [77]. **Table 1** lists the overview study on some published review and articles related works.

Table 1. Overview study on some published review and articles related works

SN	Author Name and Year	The Technique	The work's Objective	The output Results
1	Grossmann and Morlet, 1984, [6]	L2 -theory Mathematical equations	To examine an appropriate family of square-integrable wavelets with a constant shape, which can be derived through shifts and dilations from a singular prototype.	The depiction of square-integrable coefficients associated with an irreducible representation of the nonunimodular group formed by the transformation ax +b
2	Canuto et.al., 2000, [4]	Wavelet Element Method (WEM)	To develop multiresolution frameworks and biorthogonal wavelet functions defined on arbitrary domains.	The foundational principles for the construction of the Wavelet Element Method (WEM) are articulated for one-dimensional, two-dimensional, and three-dimensional scenarios, incorporating additional characteristics such as symmetry, vanishing moments, and the minimal support of wavelet functions.
3	Dahmen, 2001, [3]	adaptive wavelet scheme for elliptic self-adjoint problems	The advancement of wavelet methodologies for the numerical resolution of operator equations focuses particularly on two key aspects: adaptive solution strategies and complex domain geometries.	The advancement of principles for Fourier-free construction and the establishment of criteria for assessing the validity of norm equivalences.
4	Usevitch, 2001, [18]	Wavelet coding techniques	The emphasis lies on the core principles of wavelet coding to enhance comprehension of the JPEG 2000 standard.	Demonstrating the implications of boundary effects serves to justify the application of biorthogonal wavelets, while concurrently presenting the symmetric wavelet transform.
5	Portilla et al., 2003, [32]	product of two independent random variables: a Gaussian vector and a hidden positive scalar multiplier.	To eliminate unwanted artifacts from digital images, a statistical model of the coefficients derived from an overcomplete multiscale oriented basis is employed.	The efficacy of additive white Gaussian noise exceeds that of previously documented methodologies, demonstrating superior results both visually and in terms of mean squared error.
6	Brifcani and Al-Bamerny, 2010, [7]	A novel coding (compression) methodology is suggested that integrates discrete wavelet transform (DWT), multistage vector quantization (MSVQ), and Huffman coding.	To decrease the bit rate associated with transmission or data storage while preserving an acceptable level of fidelity or image quality.	Increasing the compression ratio
7	Kumari and Vijay, 2011, [19]	Image compression algorithms	The examination of the efficacy of orthogonal and biorthogonal wavelet filters in the context of image compression is the focus of this analysis.	The efficacy of biorthogonal wavelets surpasses that of their orthogonal counterparts.

8	Kumar and Saini, 2012, [9]	Thresholding scheme and Gaussian Distribution (GGD) modeling	Proposing an innovative thresholding technique for image denoising within the wavelet domain.	The Coiflet wavelet exhibits a superior Peak Signal-to-Noise Ratio (PSNR) compared to alternative wavelet functions when employed for denoising the Lena image.
9	Elamaran and Praveen, 2012, [41]	Discrete Cosine Transform (DCT) and Discrete Wavelet Transform DWT	to conduct a comparative analysis of the Discrete Cosine Transform (DCT) and the Discrete Wavelet Transform (DWT), particularly in the context of compression methodologies.	Both transformations yield similar outcomes in terms of energy density performance.
10	Li and Chen, 2014, [2]	wavelet weighted residual method (WWRM), wavelet finite element method (WFEM)	To conduct an exhaustive evaluation and categorization of wavelet-based numerical analysis while identifying their advantages, limitations, and prospective developments.	Assisting scholars in pinpointing initial research avenues within the realm of wavelet-based numerical analysis.
11	Sharma et al., 2015, [35]	A watermarking technique is proposed that leverages two widely recognized transform domain methods, namely Discrete Wavelet Transform (DWT) and Discrete Cosine Transform (DCT).	To safeguard data within the medical domain to promote the utilization of telemedicine applications.	Strengthening the security of textual watermarking.
12	Rong and Dongnan., 2015, [65]	Histogram Equalization on Colored Image	To enhance the differentiation of the color attributes within the heritage image.	Enhancement of image contrast and the augmentation of structural details in heritage imagery.
13	Vyas and Paik, (2016) [10]	Wavelet transforms facilitate the analysis of a substantial quantity of minor components alongside a limited number of significant components.	Image control encompasses numerous functions, including digitization, replication, display, and transmission. However, it is important to recognize that these control processes typically deteriorate image quality by introducing various forms of noise.	The process of noise removal is achieved through the application of filter-based denoising methods.
14	Singh, 2016, [17]	wavelet transform Technique	To attain wavelet transformation in the contexts of image denoising, image segmentation, and image compression.	Determining frequency data while preserving temporal context.
15	Kumar et al., 2016,[74]	Wavelet based watermarking	To conduct an in-depth examination of wavelet-based watermarking methodologies, this study aims to compare and evaluate various wavelet functions employed in wavelet-based watermarking applications.	Conducting a comprehensive examination of the literature pertaining to wavelet-based watermarking methodologies.
16	LaLa, 2017, [11]	discrete wavelets transform technique	The integration and retrieval of a watermark within the original image is achieved through the technique of alpha blending.	The Discrete Wavelet Transform (DWT) technique demonstrates resilience to a range of commonly employed operations in image processing.
17	Sharma et al., 2017, [24]	A methodology for parameterizing the	To evaluate the efficacy of the proposed filter banks in	The efficacy of the optimal wavelet bases is assessed

		design of linear-phase biorthogonal filter banks.	relation to other widely recognized biorthogonal wavelet filter banks.	within the context of image encoding applications.
18	Quazi 2017, [58]	Three fundamental methodologies exist within the realm of image enhancement techniques: spatial domain approaches, frequency domain approaches, and hybrid techniques.	To enhance Image in the computer vision systems	Enhancements in image processing efficacy were observed through the calculation of Mean Squared Error (MSE) and Peak Signal-to-Noise Ratio (PSNR) values.
19	Qiao et al., 2017, [76]	improved histogram equalization and wavelet transform	To enhance the sea cucumber image quality.	In the context of underwater imagery of sea cucumbers, the implementation of advanced techniques enhances contrast, minimizes noise, and mitigates its impact on the enhanced regions of the resultant image. Furthermore, these methods contribute to the overall improvement of image quality and visual representation.
20	Yu et al., 2018, [75]	hue- preserving color image enhancement	to enhance the intensity of colors while maintaining hue fidelity	The suggested approach enhances the saturation levels of specific color images.
21	Alickovic et al., 2018, [13]	Freiburg (intracranial EEG) and CHB-MIT (scalp EEG)	Automated detection of seizure onset and prediction of seizure occurrences utilizing electroencephalography (EEG) measurements.	In the context of seizure onset prediction, there exists a significant distinction among interictal, preictal, and ictal electroencephalographic data, which can be classified with an impressive accuracy of 99.77%.
22	Canditiis et al., 2018, [20]	Optimal strategies for finite frames	To broaden the classical methodologies pertaining to orthonormal bases.	Deriving the optimal linear diagonal estimator and formulating a representation for an unbiased estimator of the smooth shrinkage rule.
23	Taujuddin et al, 2018, [42]	Transform-based compression	to provide a comprehensive overview of the principles of image compression, the methodologies employed in compression techniques, and the diverse array of thresholding algorithms available.	Selecting the appropriate image compression algorithms for the selected test image
24	Dolly and Raj, 2018, [55]	Numerous techniques have been advanced to facilitate data recovery from images, including color image enhancement strategies such as contrast stretching, histogram equalization and its various enhancements, homomorphic filtering, retinex algorithms (both single and multiple domain variants), and	The clarity of an image is significantly influenced by various factors, including environmental conditions, lighting, improper camera exposure, or aperture settings, as well as the presence of extensive elements within the scene. These variables contribute to a substantial loss of essential and fundamental information within the image.	The primary rationale for enhancing image quality is to obtain more precise visual details and to emphasize pertinent information.

		wavelet multiscale transformation.		
25	Venkateswarlu, 2018, [72]	Two-Way Arnold Transformation (TWAT) and 2-D Wavelet Transformation (WT)	A careful application of a comprehensive framework in the field of watermark-based secure transmission of medical images.	High-quality watermarked images were utilized, and upon the application of various attacks, the extracted watermarks exhibited minimal degradation.
26	Zeebaree et al., 2019, [12]	breast ROI extraction	reducing false positive cases (FP)	Attaining a success rate of approximately 95.4% in the extraction of breast contours.
27	Choi and Jeong, 2019, [25]	SRAD filtering Algorithm	To mitigate speckle noise while simultaneously preserving edge information within synthetic aperture radar (SAR) imagery.	Achieving effective removal of speckle noise while preserving edge details and ensuring minimal computational complexity is essential.
28	Raghavendra et al., 2019, [48]	Image compression techniques can be categorized into two primary classifications: transform-based methods, which include the Discrete Cosine Transform (DCT), Joint Photographic Experts Group (JPEG), Fast Fourier Transform (FFT), and wavelet transforms; and non-transform methods, which encompass Pulse Code Modulation (PCM) and Differential Pulse Code Modulation (DPCM).	Effective transmission and retention of visual representations.	Reducing the dimensions of an image.
29	Latha et al., 2019, [56]	Hybrid combination of Discrete Wavelet and Contourlet	In order to apply image fusion within the context of medical diagnosis, various imaging modalities, including X-ray, computed tomography (CT), and magnetic resonance imaging (MRI), are utilized.	Enhancing performance metrics such as Entropy, Peak Signal-to-Noise Ratio, and Mean Squared Error in relation to established methodologies.
30	Kaibou et al., 2019, [70]	The predominant methods within both spatial and transform domains are LSB, DCT, DWT, and RDWT.	The application of various chaos-based digital image watermarking methodologies is analyzed with respect to their imperceptibility, robustness, security, and complexity.	facilitating the adoption of optimal methodologies for specific applications, contingent upon the utilization of a sufficient number of test images.
31	Tuncer and Kaya, 2019, [73]	Center-Symmetric Local Binary Patterns (CSLBP) in integer wavelet domain	This study introduces an innovative watermarking technique that integrates Central-Symmetric Local Binary Patterns (CSLBP), Advanced Encryption Standard (AES), chaotic maps, and the Manhattan distance metric.	A semi-fragile watermarking technique characterized by superior visual quality.
32	Shaker, 2020, [22]	Wavelets transform techniques using	In order to conduct a comparative analysis of	The efficacy of biorthogonal wavelet filters surpasses that of orthogonal wavelet filters in the

		MATLAB software package	Orthogonal and Bi-Orthogonal wavelets.	context of image processing applications.
33	Nisha and Raja, 2020, [26]	Basic shrinkage thresholding techniques	Medical image denoising	Attaining substantial advancements in denoising techniques for medical images utilizing Contourlet transforms.
34	Barina and Klima, 2020, [28]	JPEG 2000 format	To manage lossy compression, it is essential to regulate several parameters that influence the speed of compression and the trade-off between rate and distortion.	Functioning as a framework for the conservation of digital heritage within cultural heritage organizations, such as libraries, archives, and museums.
35	Bnou et al., 2020, [49]	K-Singular Value Decomposition (K-SVD) algorithm	To introduce an innovative wavelet denoising method founded on an unsupervised learning framework.	The proposed approach demonstrates exceptional denoising efficacy, surpassing the performance of contemporary state-of-the-art denoising techniques, particularly in terms of peak signal-to-noise ratio (PSNR) and the structural similarity index (SSIM).
36	Abdulazeez et al., 2020, [61]	Discrete Wavelet Transformation	An evaluation of the fundamental principles of wavelet transformation theory encompasses a range of applications pertaining to wavelet and multi-solution analysis, particularly in the domain of image processing.	The primary contribution of this study will provide valuable insights for researchers seeking to implement effective methodologies in image processing applications.
37	Arfaoui et al., 2020, [77]	Wavelets and Clifford algebras	The analysis of three-dimensional imagery is employed across various scientific domains, including medicine, computational physics, and informatics.	The principles of reconstruction formulas and the Fourier–Plancherel theorem have been established. Furthermore, practical computational examples are demonstrated through various illustrative cases that incorporate graphical representations of the Clifford mother wavelets.
38	Grant et al., 2022, [1]	Combining high-frequency bio sampling technologies with wearable devices	To investigate the complex, heterogeneous, temporal datasets and to discern wearable indicators that may serve as clinical surrogates for endocrine cycles.	Development of hormonal surrogates
39	Kwon et al, 2022 ,[16]	machine-learning-based prediction model for distillation process	Image Denoising by using wavelet transform	The application of the optimal wavelet transforms to the long short-term memory (LSTM) model resulted in a 10% enhancement in predictive performance.
40	Madhankumar et al., 2022, [62]	A low-complex 2D image compression method	To facilitate the efficient and effective retention of digital information.	This stacking approach facilitates the process of making estimations at multiple intermediate levels. These approximations occupy significantly less spatial capacity compared to the original data.

41	Shihab, 2023, [27]	Wavelets transform techniques using MATLAB software package	This study aims to conduct a comparative analysis of the Discrete Cosine Transform (DCT) and the Discrete Wavelet Transform (DWT) with respect to their effectiveness in compression methodologies.	DWT-based compression method is better than DCT.
42	Kamalin and MuthuLakshmi, 2023, [30]	combining the Discrete Wavelet Transform (DWT) with Adaptive Block Truncation Coding –Edge Quantization (ABTC- EQ)	To introduce an innovative methodology for the compression of general image data.	The efficacy of image compression demonstrated superior outcomes in comparison to alternative methodologies.
43	Kuo et al., 2023, [37]	fast Fourier transform (FFT)	The processes of recognizing and extracting repetitive patterns are crucial components in the production and analytical assessment of printed textiles.	The similarity demonstrated an accuracy of 98.0%, a sensitivity of 94.4%, and a specificity of 98.5%.
44	Stromvig and Ren, 2024, [64]	SPIHT (Set Partitioning in Hierarchical Trees) algorithm compression	To empirically assess the quality of compressed images, this study employs quantitative metrics including peak signal-to-noise ratio (PSNR), mean squared error (MSE), and compression ratio (CR).	The evaluation metrics of Mean Squared Error (MSE) and Peak Signal-to-Noise Ratio (PSNR) demonstrate improved outcomes when the transformation process employs the Bior 4.4 wavelet initially, followed by the Haar wavelet at subsequent levels, as recommended, compared to the application of a single, homogeneous wavelet for the transformation.

4. Conclusion

In summary, the Discrete Wavelet Transform (DWT) has proven to be an invaluable asset in the field of image processing. Its ability to perform multi-resolution analysis and provide both spatial and frequency domain information sets it apart from traditional Fourier Transform methods, making it especially suited for handling non-stationary signals prevalent in images. This review has highlighted the wide-ranging applications of DWT in image processing. From enhancing image compression techniques, such as those seen in the JPEG 2000 standard, to effectively de-noising images while preserving critical features, DWT demonstrates its versatility and efficiency. Moreover, the exploration of hybrid methods that integrate DWT with advanced computational techniques, including neural networks and deep learning, showcases the ongoing innovation and potential for even greater advancements in image processing. Despite its numerous advantages, DWT-based image processing does face challenges, such as computational complexity and the inherent trade-off between resolution and processing load. Addressing these issues remains a key area for future research and development. In conclusion, the continuous evolution of DWT, driven by technological advancements and innovative approaches, ensures its enduring relevance in the image processing domain. As researchers and practitioners continue to explore and refine DWT-based techniques, the transformative impact of this powerful tool will undoubtedly persist, paving the way for new breakthroughs and applications in image processing. This review aims to provide a solid foundation for understanding the principles, applications, and future potential of DWT in image processing, encouraging further research and development in this dynamic and rapidly evolving field.

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