Al-Nahrain Journal of Science
ANJS, Vol. 28 (1), March, 2025, pp. 151 – 158

Solution of Oxygen Diffusion Moving Boundary Value Problem Based on Variational Iteration Least Square Methods

Asmahan I. Abdulhussein[1,2,*], Fadhel S. Fadhel[1]

[1]Department of Mathematics and Computer Applications, College of Sciences, Al-Nahrain University, Jadiriya, Baghdad, Iraq
[2]Baghdad Education Directorate, the Second Karkh Specialized Supervision Department, Baghdad, Iraq

Article's Information

Received: 22.09.2024
Accepted: 07.11.2024
Published: 15.03.2025

Keywords:
Variational iteration method
Modified variation iteration method
Least Square Method
One-phase Stefan problem
Oxygen diffusion problem

Abstract

This article aims to address and solve the oxygen diffusion problem which involves oxygen diffraction into a medium that absorbs and immobilizes the oxygen concentration at a constant rate. Such types of problems are difficult to solve analytically since it is a type of moving boundary value problem or one-phase Stefan problem, and such problems require us to determine the domain boundary as a part of the solution that is unknown or may vary with respect to time. The main governing equation as a partial differential equation used to approximately govern the oxygen diffusion in an absorbing medium is the heat equation with a moving boundary that varies with respect to time. Also, the presence of a moving boundary that indicates the farthest point where oxygen enters the medium, as well as, the initial distribution of oxygen across the medium will raise fundamental mathematical challenges in the solution. The approach followed for solving this problem is a semi-analytical method, which is a hybrid approach between the variational iteration method and the least squares method. These two methods are used to derive an efficient iterative approximate solution of the considered problem of this paper, because of their simplicity, efficiency and reliability in computational applications.

http://doi.org/10.22401/ANJS.28.1.17
*Corresponding author: asm.alkhaleeli@gmail.com

1. Introduction

Boundary-value problems (BVPs) governed by partial differential equations (PDEs) that satisfy certain conditions on a prescribed domain in addition to moving or free boundaries are called moving boundary value problems (MBVPs) which are important because they accurately describe systems that have two states of matter, namely, solid, liquid and gas. Practical applications include fluid flow in porous media, diffusion and heat flow, ice melting, laser ablation, chemical reactions, etc., in which such problems are often called Stefan problems [1, 2]. As an application to the above, the oxygen diffusion problem in a certain medium that consumes oxygen is typically divided into two sections, which are initially permitted to seep into a medium, where it is absorbed at a constant rate. Thus, the problem remains in its initial stage until it reaches a state of equilibrium that is characterized by the absence of oxygen infiltration into the medium. Next, the oxygen supply is terminated and hermetically seal the medium's surface to stop any further oxygen ingress or egress. In the steady state, the medium absorbs the oxygen that is already diffusing in it, causing the boundary indicating the depth of penetration to move back towards the sealed surface. The second aspect of this problem entails tracking the moving boundary and determining the temporal distribution of oxygen in the medium [1, 3]. In 1972, Crank and Radhey provided a description of obtaining the analytical or numerical approximation solutions for the MBVP that models the diffusion of oxygen in an absorbing medium [4]. Also, Crank and Gopta in 1972 proposed a new approach for solving BVPs using a grid system based on the finite difference method that moves with the boundary [5].

In addition, a study given by Gupta and Kumar in 1981 which addresses the oxygen diffusion problem in absorbing tissue using a variable-time step method unlike other techniques, this method includes the time for complete absorption and requires fewer time steps. The numerical results are reasonably comparable to those obtained by earlier authors [6]. Moreover, Liapis et al. in 1982 presented a mathematical model formulating the oxygen diffusion problem in absorbing tissues incorporating a moving boundary for the furthest oxygen penetration [7]. In 1999, Ahmed presented a paper concerning a new semi-analytical method for solving the oxygen diffusion problem, which starts with a polynomial representing the oxygen concentration profile and then obtains a system of linear equations, relating the moving boundary and velocity to unknown functions [8]. Later on, in 2006, the same author investigated the oxygen diffusion problem in sick cells with simultaneous absorption, in which a proposed numerical method was developed to trace the moving boundary in a sick cell and determine the concentration at the fixed surface using a double-linear system of equations [9]. Also, Boureghda's work in 2006, studied the oxygen diffusion problem in a tissue using various methods that provide analytical and numerical solutions, revealing various conditions affecting the moving boundary, [10]. Later on, Bougoffa's paper in 2014, presents results from the Crank-Gupta and Gupta-Banik methods for the oxygen diffusion problem in tissue and proposes a new resolution method using the domain decomposition method, which gives an accurate and efficient approximate result of the solution [11]. In 2015, González et al. investigated the free boundary value problem of oxygen diffusion in a spherical coordinated system, identifying the mistakes provided in the previous solutions and finding a correct solution [12], where a free boundary means a movable boundary which do not vary with respect to time. Gülkaç in 2016 extended the homotopy perturbation method with time-fractional derivatives to evaluate a convergent series as an approximate solution for some important medical applications, which is the oxygen transport into cells with simultaneous absorption [13]. Nama and Fadhel in 2022 analyzed the oxygen diffusion problem as a MBVP with a time-fractional order derivative through applying the modified Magri's approach used in calculus of variation and finds an approximate solution of the problem which is converge to the exact solution of the problem [14]. The observed fractional order model can be improved to include fractional order derivatives with respect to time and/or space variable and with higher orders. Miklavčič (2023) presents a numerical method for evaluating an accurate solution of MBVPs, modeled by oxygen diffusion in a medium, applied to classic and permeable surfaces, and examining approximation errors [15]. In this paper, the oxygen diffusion problem as a MBVP will be solved using a hybrid approach between the VIM and the least squares method (LSM). Such equations are so difficult to solve analytically, however, numerical and approximate methods seem to be necessary and reliable to give resolvable accurate results. The structure of this paper is as follows. In section 1, present a historical background and literature survey of the oxygen diffusion problem. In section 2, some basic concepts concerning the approximation methods used in this work, which are combined together to give the proposed hybrid approach. The mathematical formulation of the oxygen diffusion problem as a MBVP is illustrated and presented in section 3. The obtained approximate-numerical results are presented and illustrated through figures and scheduled in tables in section 4, which shows the applicability and reliability of the followed approach in this paper. Finally, some conclusions and recommendations for further studies are given, which are related to the topics connected with this work.

2. Basic Concepts

The VIM is used as a straightforward and efficient approach to solving both linear and nonlinear problems. The primary approximations of the VIM utilize a broad strategy for solving large-scale problems expressed in operator form:

$$L[u(x)] + N[u(x)] = g(x), \quad x \geq 0$$

and when starting with the initial guess solution $u_0(x)$, then the next iterative solutions are achieved using the following recursive approximation formula:

$$u_{n+1}(x) = u_n(x) + \int_0^x \lambda(x,s)\{L[u_n(s)] + N[u_n(s)] - g(s)\}ds$$

where λ stands for the general Lagrange multiplier, which is derived and obtained optimally via variational theory [16, 17]. In connection with the VIM, a modified VIM that uses restricted variations entails incorporating unknown parameters into the correction function. The main benefit of this approach is its ability to circumvent uncontrollability issues problems that arise, from nonzero endpoint conditions a common challenge in the conventional VIM. We also apply the method to many different nonlinear equations, and the numerical results demonstrate that the modified method is accurate and helpful in solving a wide range of nonlinear differential equations. Moreover, a modified VIM, which will abbreviated as MVIM, is applied when the obtained sequence of iterated solutions do not satisfy some of the initial or boundary conditions and therefore they are scaled in order to satisfy such conditions [18]. In some cases, the proposed modification is made by introducing He's polynomials in the correction function of the VIM [19]. In relation to this work, the LSM is a mathematical approach which used to find functions that best fit a set of data points by minimizing the sum of squared residuals. There are two types of LSMs, namely discrete and continuous LSMs. In the discrete case, the goal is to find a polynomial p function of degree n that can approximate a given function f at a set of specific locations $(x_0, x_1, \dots, x_m)$. Here, m is greater than or equal to n. This motivates the investigation of discrete LSM. For the social type, it is important to accurately represent the function f over the entire interval $[a, b]$. An optimal strategy for selecting the node points is to evenly distribute them across the interval. The LSM error can be expressed as a Riemann sum that approximates an integral as the number of terms, m, tends towards infinity [20-21]. The direct Stefan problem consists of finding a function to describe a solution $u(x,t)$ of certain PDE in domain D and a function $g(t)$ describing the position of the moving interface that will satisfy the conditions of the equation. In the direct Stefan problem, the function $u(x,t)$ relies on an unknown function $g(t)$. The study uses a combination of the VIM and the LSM to analyze the data. The proposed method effectively solves problems by obtaining interface position and solution in continuous functions, eliminating the discretization of regions like classical methods. It produces satisfactory results in one or two iterations, compared to classical methods that require dense lattices for similar accuracy [22-24].

3. Formulation of Oxygen Diffusion Problem

Crank and Gupta studied the oxygen diffusion problem, which involves oxygen diffraction into a medium that absorbs and immobilizes oxygen at a constant rate. This phase change will continue until a steady state is reached, where oxygen doesn't penetrate further. The sealed surface of a medium prevents oxygen from entering or leaving, causing the boundary marking penetration depth to recede. The major problem consists of tracing this boundary movement and determining oxygen distribution over time [1][4]. The moving boundary of oxygen diffusion in the medium is sketched in Figure 1 [10].

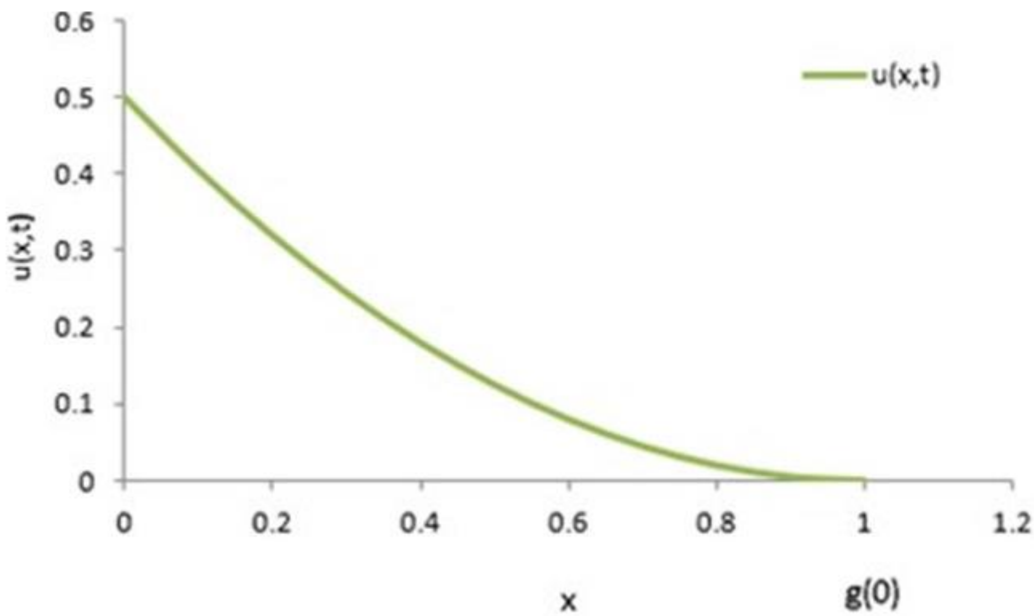

Figure 1. Concentration distributions for steady-state at time $t = 0$.

The concentration distributions domain of Figure 1 related to the governing mathematical model of the problem is given mathematically by:

$$R = \{(x,t): 0 \le x \le g(t) \text{ and } t \ge 0\}$$

where, $u(x,t)$ denotes the oxygen concentration at time t and state x, whereas $g(t)$ refers to the position of the moving boundary. Now, expressing the problem in non-dimensional form, the mathematical model that governs the problem under concentration is given by [4, 6]:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} - 1 \quad \dots (1)$$

where $0 \le x \le g(t)$ and $t \ge 0$ subject to the initial condition:

$$u(x,0) = \frac{1}{2}(1-x)^2, \qquad 0 \le x \le 1 \quad \dots (2)$$

whilst the boundary conditions are:

$$\frac{\partial u(x,t)}{\partial x} = 0 \quad \dots (3)$$

at the moving boundary $x = \mathrm{g}(t)$. And

$$u(\mathrm{x},\mathrm{t}) = 0 \quad \dots (4)$$

at the moving boundary $x = \mathrm{g}(t)$

Also, an additional condition is needed for evaluating the moving boundary in order to make the problem well-posed, which is:

$$g(0) = 1 \quad \dots (5)$$

4. Approximate Solution of the Problem

The function of oxygen concentration is $u(x,t)$ and $g(t)$ is the position of the moving boundary. Findings have been calculated using the PTC Mathcad Prime 6.0.0.0 and Mathcad 15 computer software's. The findings have been systematically arranged in tables and visually shown in figures. By selecting the initial approximations of $u(x,t)$ and $g(t)$ as follows [14]:

$$u_0(x,t) = \frac{1}{2}(1 + bt + ct^2)(g(t) - x)^2 \quad \dots (6)$$

in association with the moving boundary $g(t)$, which satisfies the desired condition on the boundary and is taken based on the medical phenomena of the problem to be more conformable with real-life situations, we may define:

$$g(t) = 1 - at^2 \quad \dots (7)$$

Also, a, b, and c are parameters to be determined. After substituting Eq. (7) back into Eq. (6), we obtain:

$$u_0(x,t) = \frac{(at^2 + x - 1)^2(ct^2 + bt + 1)}{2} \quad \dots (8)$$

By using the following variational iteration formula with respect to x:

$$u_{n+1}(x,t) = u_n(x,t) + \int_0^x (w - x)\left(\frac{\partial^2}{\partial w^2}u_n(w,t) - \frac{\partial}{\partial t}u_n(w,t) - 1\right)\mathrm{d}w \quad \dots (9)$$

We can assess the first approximate solution as follows:

$$u_1(x,t) = u_0(x,t) + \int_0^x (w-x)\left(\frac{\partial^2}{\partial w^2}u_0(w,t) - \frac{\partial}{\partial t}u_0(w,t) - 1\right)\mathrm{d}w = \frac{x^2}{24}(6b - 12ct^2 + bx^2 + 24a^2t^3 - 24at - 12bt + 12ct - 4bx + 30a^2bt^4 + 36a^2ct^5 + 8atx - 8ctx - 36abt^2 - 48act^3 + 2ctx^2 + 12abt^2x + 16act^3x) + \left(\frac{ct^2}{2} + \frac{bt}{2} + \frac{1}{2}\right)(at^2 + x - 1)^2 \quad \dots (10)$$

where a, b and c are real numbers to be determined, which are chosen to minimize the next functional. We are aiming to identify the smallest possible value of the following functional [22]:

$$J(a,b,c) = \int_0^{t^*} (u_1(g(t),t) - u^*(g(t),t))^2\,\mathrm{d}t + \int_0^{g(0)} (u_1(x,0) - \varphi(x))^2\,\mathrm{d}x \quad \dots (11)$$

Now, considering the oxygen diffusion conditions outlined in Eqs. (2), (4) and (5), and selecting a value of $t = 0.1$, then Eq. (11) can be reformulated as:

$$J(a,b,c) = \int_0^{0.1} (u_1(g(t),t) - 0)^2\,\mathrm{d}t + \int_0^1 \left(u_1(x,0) - \frac{1}{2}(1-x)^2\right)^2 \mathrm{d}x = -6.381 \times 10^{-5}ab^2 + 4.472 \times 10^{-5}a^2b - 1.377 \times 10^{-6}a^3b + 2.322 \times 10^{-8}a^4b - 2.331 \times 10^{-10}a^5b + 1.398 \times 10^{-12}a^6b - 4.643 \times 10^{-15}a^7b - 1.32 \times 10^{-6}ac^2 + 5.889 \times 10^{-6}a^2c - 1.679 \times 10^{-7}a^3c + 2.711 \times 10^{-9}a^4c - 2.649 \times 10^{-11}a^5c + 1.559 \times 10^{-13}a^6c + 1.481 \times 10^{-4}a^2 - 5.333 \times 10^{-6}a^3 + 9.524 \times 10^{-8}a^4 - 9.877 \times 10^{-10}a^5 + 6.061 \times 10^{-12}a^6 - 2.051 \times 10^{-14}a^7 + 1.508 \times 10^{-5}c^2 + 3.251 \times 10^{-6}a^2b^2 - 8.925 \times 10^{-8}a^3b^2 + 1.425 \times 10^{-9}a^4b^2 - 1.384 \times 10^{-11}a^5b^2 + 8.101 \times 10^{-14}a^6b^2 + 5.599 \times 10^{-8}a^2c^2 - 1.347 \times 10^{-9}a^3c^2 + 1.978 \times 10^{-11}a^4c^2 - 1.816 \times 10^{-13}a^5c^2 - 1.022 \times 10^{-15}a^6c^2 - 6 \cdot 111 \times 10^{-4}ab - 9 \cdot 444 \times 10^{-5}ac + 2 \times 10^{-4}bc + 1.817 \times 10^{-5}abc + 8.513 \times 10^{-7}a^2bc - 2.187 \times 10^{-8}a^3bc + 3.349 \times 10^{-10}a^4bc - 3.163 \times 10^{-12}a^5bc + 1.816 \times 10^{-14}a^6bc$$

and upon minimizing Eq. (12), we get:

Al-Nahrain Journal of Science
ANJS, Vol. 28 (1), March, 2025, pp. 151 – 158

$a = 0.061$, $\quad b = 1.6 \times 10^{-4}$ and $c = 0.18$

Hence, after substituting the values of a, b and the values of c, we get:

$g(t) = -0.061t^2 + 1$

Therefore:

$$u_0(x,t) = (1 - x - 0.061t^2)^2(0.09t^2 + 8 \times 10^{-5}t + 0.5)$$

and:

$$u_1(x,t) = (8t \times 10^{-5} + 0.09t^2 + 0.5)(x + 0.061t^2 - 1)^2 + 0.042x^2(0.694t - 6.4x \times 10^{-4} + 0.36tx^2 + 1.171t^2x \times 10^{-4} + 0.176t^3x - 2.16t^2 - 0.438t^3 + 1.786t^4 \times 10^{-5} + 0.024t^5 + 1.6x^2 \times 10^{-4} - 0.952tx + 9.6 \times 10^{-4})$$

The second approximate solution is:

$$u_2(x,t) = u_1(x,t) + \int_0^x (w - x)\left(\frac{\partial^2}{\partial w^2}u_1(w,t) - \frac{\partial}{\partial t}u_1(w,t) - 1\right)\mathrm{d}w \quad \ldots(13)$$

$$= x^4(-0.015t + -1.983 \times 10^{-3}x + 1.098 \times 10^{-3}t^2x - 4.56 \times 10^{-3}t^2 + 2.481 \times 10^{-7}t^3 + 4.186 \times 10^{-4}t^4 + 5 \times 10^{-4}x^2 + 4.88 \times 10^{-7}t\,x + 2.41 \times 10^{-3}) + (8 \times 10^{-5}t + 0.09t^2 + 0.5)(x + 0.061t^2 - 1)^2 + 0.042x^2(0.694t + -6.4 \times 10^{-4}x + 0.36tx^2 + 1.171 \times 10^{-4}t^2x + 0.17t^3x - 2.16t^2 + -0.438t^3 + 1.786 \times 10^{-5}t^4 + 0.024t^5 + 1.6 \times 10^{-4}x^2 - 0.952tx + 9.6 \times 10^{-4})$$

Through the use of similar manipulation, the third approximate solution is obtained as:

$$u_3(x,t) = u_2(x,t) + \int_0^x (w - x)\left(\frac{\partial^2}{\partial w^2}u_2(w,t) - \frac{\partial}{\partial t}u_2(w,t) - 1\right)\mathrm{d}w \quad \ldots(14)$$

$$= 2x^4(-0.015t + -1.983 \times 10^{-3}x + 1.098 \times 10^{-3}t^2x - 4.56 \times 10^{-3}t^2 + 2.481 \times 10^{-7}t^3 + 4.186 \times 10^{-4}t^4 + 5 \times 10^{-4} \cdot x^2 + 4.88 \times 10^{-7}t\,x + 2.41 \times 10^{-3}) + (8 \times 10^{-5}t + 0.09t^2 + 0.5)(x + 0.061t^2 - 1)^2 + 0.042x^2(0.694t + -6.4 \times 10^{-4}x + 0.36tx^2 + 1.171 \times 10^{-4}t^2x + 0.17t^3x - 2.16t^2 - 0.438t^3 + 1.786 \times 10^{-5}t^4 + 0.024t^5 + 1.6 \times 10^{-4}x^2 - 0.952tx + 9.6 \times 10^{-4})$$

Since $u_3(x,t)$ does not satisfy Eq. (2) as an initial condition, one may modify the foregoing VIM's results by adding the unknown parameter, namely the r parameter, embedded into Eq. (14) in the following form:

$$u_3(x,0) = \frac{1}{2}(1 - \mathrm{x})^2 + r \quad \ldots(15)$$

and so when letting:

$$u_3^*(x,t) = u_3(x,t) - r \quad \ldots(16)$$

where r also represents the remaining part of the equation u_3.

After that, we get, $u_3^*(x,t)$ is satisfied with the condition (2), then $u_3^*(x,t)$ became:

$$u_3^*(x,t) = -2x^4(-1.983 \times 10^{-3}x + 5 \times 10^{-4}x^2 + 2.41 \times 10^{-3}) + 2\boldsymbol{x}^4(-0.015t - 1.983 \times 10^{-3}x + 1.098 \times 10^{-3}t^2x - 4.56 \times 10^{-3}t^2 + 2.481 \times 10^{-7}t^3 + 4.186 \times 10^{-4}t^4 + 5 \times 10^{-4}x^2 + 4.88 \times 10^{-7}tx + 2.41 \times 10^{-3}) + (8 \times 10^{-5}t + 0.09t^2 + 0.5)(x + 0.061t^2 - 1)^2 + 0.042x^2(0.694t - 6.4 \times 10^{-4}x + 0.36tx^2 + 1.171 \times 10^{-4}t^2x + 0.176t^3x - 2.16t^2 - 0.438t^3 + 1.786 \times 10^{-5}t^4 + 0.024t^5 + 1.6 \times 10^{-4}x^2 - 0.952tx + 9.6 \times 10^{-4}) - 0.042x^2(9.6 \times 10^{-4} - 6.4 \times 10^{-4}x + 1.6 \times 10^{-4}x^2)$$

Hence, when substituting different values for t in the equation of the final form of $u_3^*(x,t)$, it is notable that all results are equal, as shown in Figure 2 and Table 1, and this is contrary to the medical phenomena, since there is a very small change in the oxygen concentration for different time levels, and so not reliable phenomena [4][10][14].

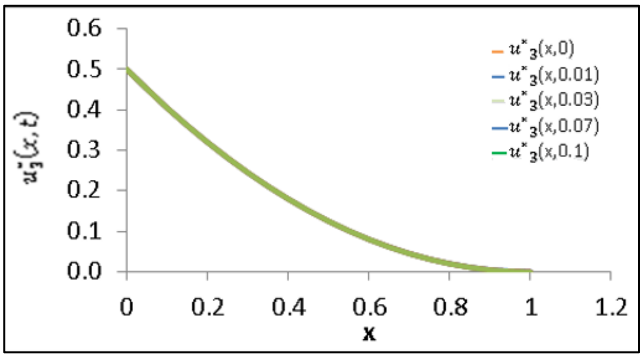

Figure 2. Plot of $u_3^*(x,t)$ of versus x and t.

Therefore, the final form of $u(x,t)$ will be assumed to be as follows [18]:

$$u(x,t) = u_3^*(x,t)(1 - 7t) \quad \ldots(17)$$

The conclusive formula for the oxygen diffusion equation is presented in Eq. (17), which satisfies is to the initial and boundary conditions and is consistent with the medical phenomena of the problem. The obtained results of the moving boundary $g(t)$ and the concentration distributions $u(x,t)$ are sketched in Figures 3 and 4, respectively.

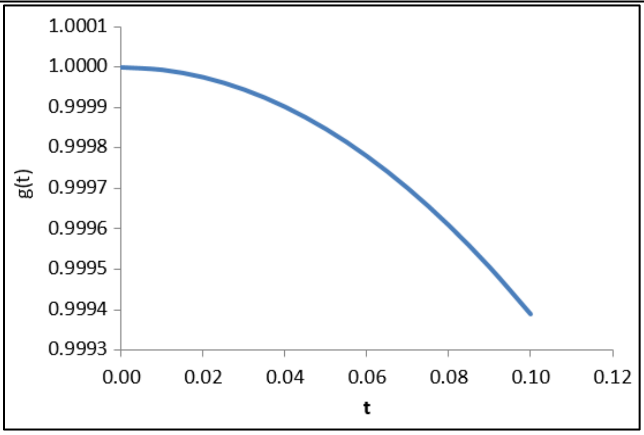

Figure 3. The moving boundary with respect to t.

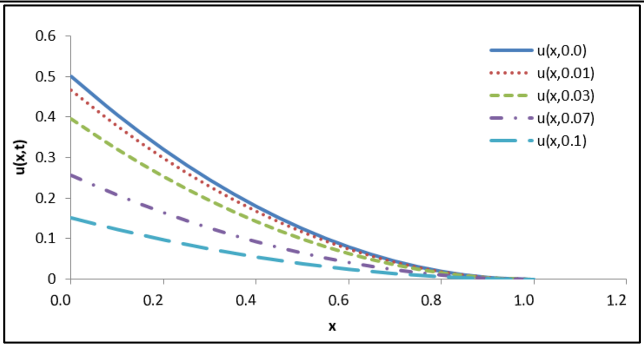

Figure 4. Plot of $u(x,t)$ of versus x and t.

Also, Table 2 presents the iterative approximate solutions up to the 3rd iteration of the oxygen concentration distributions $u(x,t)$.

Table 1. Results solutions of the oxygen concentration $u_3^*(x,t)$ versus x with different time t.

x	$u_3^*(x,0.0)$	$u_3^*(x,0.01)$	$u_3^*(x,0.03)$	$u_3^*(x,0.07)$	$u_3^*(x,0.1)$
0.0	0.5000000	0.5000037	0.5000285	0.5001475	0.5002971
0.1	0.4050000	0.4050048	0.4050248	0.4051054	0.4052013
0.2	0.3200000	0.3200092	0.3200306	0.3200857	0.3201371
0.3	0.2450000	0.2450138	0.2450373	0.2450676	0.2450752
0.4	0.1800000	0.1800154	0.180035	0.1800282	0.1799822
0.5	0.1250000	0.1250104	0.1250126	0.1249415	0.1248213
0.6	0.0800000	0.0799948	0.0799581	0.0797793	0.0795518
0.7	0.0450000	0.0449641	0.0448584	0.0445106	0.0441292
0.8	0.0200000	0.0199136	0.0196992	0.0191021	0.0185056
0.9	0.0050000	0.0048384	0.0044652	0.0035177	0.0026294
1.0	0.0000000	−0.0002672	−0.0008600	−0.0022810	−0.0035547

Table 1. Results solutions of the oxygen concentration $u(x,t)$ versus x with different time t.

x	$u(x,0.0)$	$u(x,0.01)$	$u(x,0.03)$	$u(x,0.07)$	$u(x,0.1)$
0.0	0.5000000	0.4650034	0.3950225	0.2550752	0.1500891
0.1	0.4050000	0.3766545	0.3199696	0.2066038	0.1215604
0.2	0.3200000	0.2976086	0.2528243	0.1632439	0.0960413
0.3	0.2450000	0.2278630	0.1935798	0.1249848	0.0735228
0.4	0.1800000	0.1674145	0.1422280	0.0918148	0.0539949
0.5	0.1250000	0.1162599	0.0987604	0.0637206	0.0374466
0.6	0.0800000	0.0743954	0.0631674	0.0406879	0.0238656
0.7	0.0450000	0.0418169	0.0354387	0.0227007	0.0132387
0.8	0.0200000	0.0185200	0.0155629	0.0097422	0.0055513
0.9	0.0050000	0.0044999	0.0035279	0.0017938	0.0007881
1.0	0.0000000	−0.0002482	−0.0006791	−0.0011639	−0.0010676

5. conclusions

This work investigates a mathematical model proposed recently in literatures to address the challenge of the moving boundary problem that arises in the oxygen diffusion process. We can derive the analytical approximation solution of the problem through multiple iterations by combining the VIM and LSM methods. The proposed methodology is very reliable, easy to use and accurate for resolving the changing boundary problem. This approach is ideal for calculating solutions to various problems and it differs from the previous methods, which are considered to be difficult and complicated with the expectation that the outcomes will provide significant support to experts in the relevant field in comparison with those results used the same problem of this paper [3, 4, 6, 10, 11]. It is evident that the only limitation of the technique used in the present work is the difficulty in assessing additional recursive iterations of the approximate solution, since the approximate solution may be more complicated, making the computation using the correction functional (9) more difficult.

Acknowledgments:
The authors would like to thank the Department of Mathematics and Computer Applications at Al-Nahrain University for providing facilities.

Funding Statement:
This work has not received any funding from any organization.

Conflicts of Interest:
The authors declare, there is no conflict of interest.

References

[1] Crank G. and Makin M. I., "Oxidations of aromatic amines by superoxide ion". Aust. J. Chem, 37(4): 845-855, 1984.
[2] Kim K., Mueller C. and Sowers R. B. "A stochastic moving boundary value problem". Ill. J. Math., 54(3): 927-962, 2010.
[3] Galib Jr T. A., Bruch Jr J. C. and Sloss J. M. "Solution of an oxygen diffusion-absorption problem". Int. J. Bio-Med. Comput., 12(2) :157-180, 1981.
[4] Crank J. and Gupta R. S. "A moving boundary problem arising from the diffusion of oxygen in absorbing tissue". IMA. J. Appl. Math., 10(1): 19-33, 1972.
[5] Crank J., and Gupta R. S. "A method for solving moving boundary problems in heat flow using cubic splines or polynomials". J. Inst. Math. Its Appl., 10(3): 296-304, 1972.
[6] Gupta R. S. and Kumar D. "Complete numerical solution of the oxygen diffusion problem involving a moving boundary". Comput. Methods Exp. Appl. Mech. Eng., 29(2): 233-239, 1981.
[7] Liapis A. I., Lipscomb G. G., Crosser O. K. and Tsiroyianni-Liapis E. "A model of oxygen diffusion in absorbing tissue". Math. Modell., 3(1): 83-92, 1982.
[8] Ahmed S. G. "An approximate method for oxygen diffusion in a sphere with simultaneous absorption". Int. J. Numer. Methods Heat Fluid Flow, 9(6): 631-643, 1999.
[9] Ahmed S. G. "A numerical method for oxygen diffu5z2sion and absorption in a sike cell". Appl. math. comput., 173(1): 668-682, 2006.
[10] Boureghda A. "Numerical solution of the oxygen diffusion in absorbing tissue with a moving boundary". Commun. Numer. Methods Eng., 22(9): 933-942, 2006.
[11] Bougoffa L. "The Adomian decomposition method for solving a moving boundary problem arising from the diffusion of oxygen in absorbing tissue". Sci. World. J., 2014(1), 579628, 2014.
[12] González A. M., Reginato J. C. and Tarzia D. A. "A free boundary model for oxygen diffusion in a spherical medium". J. Biol. Syst., 23(supp01): S67-S76, 2015.
[13] Gülkaç V. "The new approximate analytic solution for oxygen diffusion problem with time-fractional derivative". Math. Probl. Eng., 2016(1), 8409839, 2016.
[14] Nama A. Y. and Fadhel F. S. "Fractional variational iteration method for solving two-dimensional Stefan problem with fractional order derivative". Int. J. of Nonlinear Anal. Appl., 13(2): 2167-2174, 2022.
[15] Miklavčič M. "Analytic and numeric solutions of moving boundary problems". J. Comput. Appl. Math., 431, 115270, 2023.
[16] Momani S., Abuasad S. and Odibat Z. "Variational iteration method for solving nonlinear boundary value problems". Appl. Math. Comput., 183(2): 1351-1358, 2006.
[17] Jawad U. M. and Fadhel F. S. "Modified variational iteration method for solving sedimentary ocean basin moving boundary value problem". Al-Nahrain. J. Sci., 25(3): 33-39, 2022.
[18] Tari H. "Modified variational iteration method". Phys. Lett. A, 369(4): 290-293, 2007.
[19] Noor M. A. and Mohyud-Din S. T. "Modified variational iteration method for Goursat and Laplace problems". World Appl. Sci. J., 4(4): 487-498, 2008.
[20] Embree M. "Numerical analysis I". Lect. Notes, Rice Univ., 1-207, 2010.

[21] Loach P. D. and Wathen A. J. "On the best least squares approximation of continuous functions using linear splines with free knots". IMA. J. Appl. Math., 11(3): 393-409, 1991.
[22] Słota D. "Direct and inverse one-phase Stefan problem solved by the variational iteration method". Comput. Math. Appl., 54(7-8): 1139-1146, 2007.
[23] Słota D. and Zielonka A. "A new application of He's variational iteration method for the solution of the one-phase Stefan problem". Comput. Math. Appl., 58(11-12): 2489-2494, 2009.
[24] Abdulhussein, A. I. and Fadhel F. S. "Solution of shoreline sedimentary ocean basin moving boundary value problem based on variational iteration-least square methods", 3rd International Conference on Mathematics, AI, Information And Communication Technologies (ICMAICT-2024), Bayan University, Erbil, Iraq in 27-28 April, 2024