

## Nucleon Momentum Distributions and Elastic Electron Scattering from $^{59}\text{Co}$ , $^{61}\text{Ni}$ , $^{63}\text{Cu}$ and $^{65}\text{Cu}$ Nuclei

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### Abstract

The nucleon momentum distributions (NMD) and the elastic electron scattering form factors  $F(q)$  of the ground state for some 1f-2p shell nuclei for  $^{59}\text{Co}$ ,  $^{61}\text{Ni}$ ,  $^{63}\text{Cu}$  and  $^{65}\text{Cu}$  are calculated using the Coherent Density Fluctuation Model (CDFM) and expressed in terms of the fluctuation function (weight function)  $|f(x)|^2$ . The fluctuation function has been related to the nucleon density distribution (NDD) of the nuclei and determined from the theory and experiment. The property of the long-tail behavior at high momentum region of the NMD has been obtained by both the theoretical and experimental fluctuation functions. The calculated form factors  $F(q)$  of all nuclei under study are in good agreement with those of experimental data throughout all values of momentum transfer  $q$ .

Keyword: density distributions; elastic electron scattering form factors; momentum distributions; 2p-1f shell nuclei; Coherent Density Fluctuation Model.

### Introduction

There is no method for directly measuring the nucleon momentum distribution (NMD) in nuclei. The quantities that are measured by particle-nucleus and nucleus-nucleus collisions are the cross sections of different reactions, and these contain information on the (NMD) of target nucleons. The experimental evidence obtained from inclusive and exclusive electron scattering on nuclei establish the existence of long-tail behavior of the (NMD) at high momentum region ( $k \geq 2\text{fm}^{-1}$ ) [1-3]. In principle, the mean field theories cannot describe correctly the form factors  $F(q)$  and the (NMD) simultaneously [4] and they exhibit a steep-slope behavior of the (NMD) at high momentum region. In fact, the (NMD) depends a little on the effective mean field considered due to its sensitivity to the short-range and tensor nucleon-nucleon correlations [4, 5] which are not included in the mean field theories.

There are several theoretical methods used to study elastic electron-nucleus scattering, such as the plan-wave Born approximation (PWBA), the eikonal approximation and the phase-shift analysis method [6-10]. The (PWBA) method can give qualitative results and has been used widely for its simplicity. To include the Coulomb distortion effect, which is neglected in (PWBA), the other two methods

may be used. In the last few years, some theoretical studies of elastic electron scattering off exotic nuclei have been performed. Wang et al. [6, 7] studied such scattering along some isotopic and isotonic chains by combining the eikonal approximation with the relativistic mean field theory. Roca-Mazza et al, [8] systematically investigated elastic electron scattering off both stable and exotic nuclei with the phase-shift analysis method. Karataglidis and Amos [9] have studied the elastic electron scattering form factors, longitudinal and transverse, from exotic (*He* and *Li*) isotopes and from  $^8\text{B}$  nucleus using large space shell models. Chu et al. [10] have studied the elastic electron scattering along *O* and *S* isotopic chains and showed that the phase-shift analysis method can reproduce the experimental data very well for both light and heavy nuclei. Al-Rahmani and Hussein [11] have studied the (CDD) and elastic electron scattering form factors of some  $2s-1d$  shell nuclei using the (PWBA) and demonstrated that the inclusion of the higher 1f-2p shell in the calculations leads to produce a good results in comparison with those of the experimental data.

In the coherent density fluctuation model (CDFM), which is exemplified by the work of Antonov et al. [12], the local nucleon density distribution (NDD) and the (NMD) are simply

related and expressed in terms of an experimentally obtainable fluctuation function (weight function)  $|f(x)|^2$ . They [12] investigated the (NMD) of ( ${}^4\text{He}$  and  ${}^{16}\text{O}$ ),  ${}^{12}\text{C}$  and ( ${}^{39}\text{K}$ ,  ${}^{40}\text{Ca}$  and  ${}^{48}\text{Ca}$ ) nuclei using weight functions  $|f(x)|^2$  expressed in terms of, respectively, the two parameter Fermi (2PF) (NDD) [13], the experiment (NDD) of (3PF) and the experimental (NDD) model-independent Fourier- Bessel analysis or sum of Gaussians [14]. It is important to point out that all above calculations obtained in the framework of the (CDFM) proved a high momentum tail in the (NMD). Elastic electron scattering from  ${}^{40}\text{Ca}$  nucleus was also investigated in Ref. [12], where the calculated elastic differential cross sections ( $d\sigma/d\Omega$ ) were found to be in good agreement with those of 2PF [13].

The aim of the present work is to derive an analytical expressions for the (NDD) applicable throughout all 1f-2p shell nuclei based on the use of the single particle harmonic oscillator wave functions and the occupation numbers of the states. The derived form of the (NDD) is employed in determining the theoretical weight function  $|f(x)|^2$  which is then used in the (CDFM) to study the (NMD) and the elastic electron scattering form factors  $F(q)$  for some 2p-1f shell nuclei for  ${}^{59}\text{Co}$ ,  ${}^{61}\text{Ni}$ ,  ${}^{63}\text{Cu}$  and  ${}^{65}\text{Cu}$  nuclei. We shall see later that

$$\rho(r) = \frac{e^{-r^2/b^2}}{\pi^{3/2} b^3} \left\{ \left(10 - \frac{3}{2}\beta_1\right) + \left[\frac{11}{3}\beta_1 + \frac{5}{3}(A - 40 - \beta_2)\right] \left(\frac{r}{b}\right)^2 + \left[8 - 2\beta_1 - \frac{4}{3}(A - 40 - \beta_2)\right] \left(\frac{r}{b}\right)^4 + \left[\frac{8}{105}\beta_2 + \frac{4}{15}(A - 40 - \beta_2) + \frac{4}{15}\beta_1\right] \left(\frac{r}{b}\right)^6 \right\}, \dots (2a)$$

for  ${}^{59}\text{Co}$  and

$$\rho(r) = \frac{e^{-r^2/b^2}}{\pi^{3/2} b^3} \left\{ \left(10 - \frac{3}{2}\beta_1\right) + \left[\frac{11}{3}\beta_1 + \frac{5}{3}\beta_2\right] \left(\frac{r}{b}\right)^2 + \left[8 - 2\beta_1 - \frac{4}{3}\beta_2\right] \left(\frac{r}{b}\right)^4 + \left[\frac{20}{105}\beta_2 + \frac{8}{105}(A - 40) + \frac{4}{15}\beta_1\right] \left(\frac{r}{b}\right)^6 \right\}, \dots (2b)$$

for  ${}^{61}\text{Ni}$ ,  ${}^{63}\text{Cu}$  and  ${}^{65}\text{Cu}$  where  $A$  is the nuclear mass number. The parameter  $\beta_1$  characterizes the deviation of the nucleon occupation numbers from the prediction of the

theoretical  $|f(x)|^2$ , based on the derived (NDD), is capable of giving information about the (NMD) as does that of the experimental 2PF and 3PF.

**Theory**

The (NDD) of the one-body operator can be written, as [15]:

$$\rho(r) = \frac{1}{4\pi} \sum_{nl} \eta_{nl} 4(2l+1) |R_{nl}(r)|^2 \dots (1)$$

Where  $\rho(r)$  is the (NDD) of nucleus,  $\eta_{nl}$  is the nucleon occupation probability of the state  $nl$  ( $\eta_{nl} = 0$  or  $1$  for closed shell nuclei and

$0 < \eta_{nl} < 1$  for open shell nuclei) and  $R_{nl}(r)$  is the radial part of the single-particle harmonic oscillator wave function. To derive an explicit form for the NDD of 1f-2p shell nuclei, we assume that there is a core of filled  $1s$ ,  $1p$  and  $1d$  shells and the nucleon occupation numbers in  $2s$ ,  $1f$  and  $2p$  shells are equal to  $(4 - \beta_1)$ ,  $\beta_2$  and  $[(A - 40 - \beta_2 + \beta_1)]$  (for  ${}^{59}\text{Co}$ ), and  $(4 - \beta_1)$ ,  $[(A - 40 - \beta_2)]$  and  $(\beta_1 + \beta_2)$  (for  ${}^{61}\text{Ni}$ ,  ${}^{63}\text{Cu}$  and  ${}^{65}\text{Cu}$ ) respectively, but not to  $4$ ,  $(A - 40)$  and  $0$  as in simple shell model. Using this assumption in eq. (1), an analytical form for the ground state of (NDD) the 1f-2p shell nuclei is obtained as:

simple shell model ( $\beta_1 = 0$ ), the parameter  $\beta_2$  is a assumed as a free parameter to be adjusted to obtain agreement with the experimental (NDD) and  $b$  is the harmonic oscillator size parameter.

The normalization condition of the  $\rho(r)$  is given by [4]

$$A = 4\pi \int_0^\infty \rho(r)r^2 dr. \dots\dots\dots (3)$$

The mean square radius (MSR) of the considered 1f-2p shell nuclei is given by:

$$\langle r^2 \rangle = \frac{4\pi}{A} \int_0^\infty \rho(r)r^4 dr. \dots\dots\dots (4)$$

The central  $\rho(r=0)$  is obtained from eq. (2) as:

$$\rho(0) = \frac{1}{\pi^{3/2}b^3} \left\{ 10 - \frac{3}{2} \beta_1 \right\}, \dots\dots\dots (5)$$

then  $\beta_1$  is obtained from eq. (5) as:

$$\beta_1 = \frac{2}{3} \left\{ 10 - \pi^{3/2}b^3 \rho(0) \right\}. \dots\dots\dots (6)$$

The (NMD) for the 1f-2p shell nuclei is studied using two distinct methods. In the first method, it is determined by the shell model using the single-particle harmonic oscillator wave functions in momentum representation and expressed as:

$$n(k) = \frac{b^3}{\pi^{3/2}} \exp\left(-\frac{b^2}{k^2}\right) \times \left[ 10 + 8(bk)^4 + 8 \frac{(A-40)}{105} (bk)^6 \right] \dots\dots\dots (7)$$

$k$  is the momentum of the particle. In the second method, the  $n(k)$  is determined by the (CDFM), where the mixed density is given by [12]:

$$\rho(r,r') = \int_0^\infty |f(x)|^2 \rho_x(r,r') dx \dots\dots\dots (8)$$

Since

$$\rho_x(r,r') = 3\rho_0^g(x) \frac{j_1(k_F(x)|\vec{r}-\vec{r}'|)}{k_F(x)|\vec{r}-\vec{r}'|} \times \theta\left(\bar{x} - \frac{|\vec{r}+\vec{r}'|}{2}\right) \dots\dots\dots (9)$$

is the density matrix for  $A$  nucleons uniformly distributed in the sphere with radius  $x$  and density  $\rho_0(x) = 3A/4\pi x^3$ . The Fermi momentum is defined as [12]:

$$k_F(x) = \left(\frac{3\pi^2}{2} \rho_0(x)\right)^{1/3} \equiv \frac{\eta}{x}; \quad \eta = \left(\frac{9\pi A}{8}\right)^{1/3} \dots\dots\dots (10)$$

and the step function  $\theta$ , in eq. (9), is defined by:

$$\theta(y) = \begin{cases} 1, & y \geq 0 \\ 0, & y < 0 \end{cases} \dots\dots\dots (11)$$

According to the density matrix definition of eq.(8), one-particle density  $\rho(r)$  is given by its diagonal element as [12]:

$$\rho(r) = \rho(r,r')|_{r=r'} = \int_0^\infty |f(x)|^2 \rho_x(r) dx, \dots\dots\dots (12)$$

In eq. (12),  $\rho_x(r)$  and  $|f(x)|^2$  have the following forms [12]:

$$\rho_x(r) = \rho_0(x) \theta(x - |\vec{r}|) \dots\dots\dots (13)$$

$$|f(x)|^2 = \frac{-1}{\rho_0(x)} \frac{d\rho(r)}{dr} \Big|_{r=x}. \dots\dots\dots (14)$$

The weight function  $|f(x)|^2$  of eq. (14), determined in terms of the ground state  $\rho(r)$ , satisfies the following normalization condition [12]:

$$\int_0^\infty |f(x)|^2 dx = 1, \dots\dots\dots (15)$$

and holds only for monotonically decreasing  $\rho_g(r)$ , i.e.  $\frac{d\rho(r)}{dr} < 0$ .

On the basis of eq. (12), the NMD,  $n(k)$ , is given by [12]:

$$n(k) = \int_0^\infty |f(x)|^2 n_x(k) dx, \dots\dots\dots (16)$$

where

$$n_x(k) = \frac{4}{3} \pi x^3 \theta(k_F(x) - |\vec{k}|), \dots\dots\dots (17)$$

is the Fermi-momentum distribution of the system with density  $\rho_0(x)$ . By means of (14), (16) and (17), an explicit form for the NMD is expressed in terms of  $\rho(r)$  as:

$$n^{CDFM}(k;[\rho]) = \left(\frac{4\pi}{3}\right)^2 \frac{4}{A} \times \left[ 6 \int_0^{\eta/k} \rho(x)x^5 dx - \left(\frac{\eta}{k}\right)^6 \rho\left(\frac{\eta}{k}\right) \right], \dots\dots\dots (18)$$

with normalization condition

$$A = \int n^{CDFM}(k) \frac{d^3k}{(2\pi)^3} \dots\dots\dots (19)$$

The elastic monopole charge form factors  $F_{C0}(q)$  of the target nucleus is also expressed in the (CDFM) as [12]:

$$F_{C0}(q) = \frac{1}{A} \int_0^\infty |f(x)|^2 F(q, x) dx, \dots\dots\dots (20)$$

where the form factor of uniform charge density distribution is given by:

$$F(q, x) = \frac{3A}{(qx)^2} \left[ \frac{\sin(qx)}{(qx)} - \cos(qx) \right] \dots\dots\dots (21)$$

Inclusion of the correction due to the finite nucleon size  $f_{fs}(q)$  and the center of mass correction  $f_{cm}(q)$  in the calculations requires multiplying the form factor of eq. (20) by these corrections. Here,  $f_{fs}(q)$  is considered as a free nucleon form factor which is assumed to be the same for protons and neutrons [16]:

$$f_{fs}(q) = \exp\left[-\frac{0.43q^2}{4}\right] \dots\dots\dots (22)$$

The correction  $f_{cm}(q)$  removes the spurious state arising from the motion of the center of mass when shell model wave functions are used and is given by [16]

$$|f(x)|^2 = \frac{8\pi x^4}{3Ab^2} \rho(x) - \frac{16x^4 e^{-x^2/b^2}}{3Ab^5 \pi^{1/2}} \left\{ \left[ \frac{11}{6} \beta_1 + \frac{5}{6} (A-40-\beta_2) \right] + \left[ 8-2\beta_1 - \frac{4}{3} (A-40-\beta_2) \right] \left( \frac{x}{b} \right)^2 + \left[ \frac{4}{35} \beta_2 + \frac{2}{5} (A-40-\beta_2) + \frac{2}{5} \beta_1 \right] \left( \frac{x}{b} \right)^4 \right\} \dots\dots\dots (25a)$$

for  $^{59}\text{Co}$  and,

$$|f(x)|^2 = \frac{8\pi x^4}{3Ab^2} \rho(x) - \frac{16x^4 e^{-x^2/b^2}}{3Ab^5 \pi^{1/2}} \left\{ \left( \frac{11}{6} \beta_1 + \frac{5}{6} \beta_2 \right) + \left[ 8-2\beta_1 - \frac{4}{3} \beta_2 \right] \left( \frac{x}{b} \right)^2 + \left[ \frac{10}{35} \beta_2 + \frac{4}{35} (A-40) + \frac{2}{5} \beta_1 \right] \left( \frac{x}{b} \right)^4 \right\} \dots\dots\dots (25b)$$

for  $^{61}\text{Ni}$ ,  $^{63}\text{Cu}$ ,  $^{65}\text{Cu}$

While the experimental weight function is determined by substituting the NDD of 2PF or 3PF, into eq. (14), thus, yields respectively

$$|f(x)|_{2PF}^2 = \frac{4\pi x^3 \rho_0}{3Az} \left( 1 + e^{\frac{x-c}{z}} \right)^{-2} \exp\left(\frac{x-c}{z}\right) \dots\dots\dots (26)$$

$$f_{cm}(q) = \exp\left[\frac{q^2 b^2}{4A}\right] \dots\dots\dots (23)$$

Multiplying eq. (20) by these corrections, yields:

$$F_{C0}(q) = \frac{1}{A} \int_0^\infty |f(x)|^2 F(q, x) dx f_{fs}(q) f_{cm}(q) \dots\dots\dots (24)$$

It is important to point out that all physical quantities studied above in the framework of the CDFM such as  $n(k)$  and  $F_{C0}(q)$ , are expressed in terms of the weight function  $|f(x)|^2$ . In the work of refs. [12], the weight function was obtained from the NDD of the 2PF extracted by analyzing elastic electron-nuclei scattering experiments. In the present work, the theoretical weight function  $|f(x)|^2$  is expressed by introducing the derived density of eq. (2) into eq. (14), as:

and,

$$|f(x)|_{3PF}^2 = \frac{4\pi x^3 \rho_0}{3A} \left( \frac{\left( 1 + \frac{wx^2}{c^2} \right) \left( 1 + e^{\frac{x-c}{z}} \right)^{-2} e^{\frac{x-c}{z}} - 2wx \left( 1 + e^{\frac{x-c}{z}} \right)^{-1}}{z} \right) \dots\dots\dots (27)$$

## Results and Discussion

The nucleon momentum distributions  $n(k)$  and elastic form factors for some 1f-2p shell nuclei are studied by means of the CDFM. The distribution  $n(k)$  of eq. (18) is evaluated in terms of the NDD obtained firstly from theoretical consideration as in eq. (2) and secondly from experiments, such as 2PF and 3PF [13]. The harmonic oscillator size parameters  $b$  are selected in such a way as to reproduce the experimental root mean square radii (rms) of nuclei. The parameters  $\beta_1$  is evaluated by eq. (6). In table 1, we display the values of the parameters  $b$ ,  $\beta_1$  and  $\beta_2$  together with the experimental values of  $c$ ,  $z$ ,  $w$ ,  $\rho_{\text{exp}}(0)$  and  $\langle r^2 \rangle_{\text{exp}}^{1/2}$  utilized in this study for  $^{59}\text{Co}$ ,  $^{61}\text{Ni}$ ,  $^{63}\text{Cu}$  and  $^{65}\text{Cu}$  nuclei.

Fig.(1) shows the dependence of the (NDD) (in  $\text{fm}^{-3}$ ) on  $r$  (in  $\text{fm}$ ) for  $^{59}\text{Co}$ ,  $^{61}\text{Ni}$ ,  $^{63}\text{Cu}$  and  $^{65}\text{Cu}$  nuclei. The long dashed and solid distributions are the calculated (NDD), using eq. (2) with  $(\beta_1, \beta_2 = 0)$  and  $(\beta_1, \beta_2 \neq 0)$ , respectively. The experimental data [13], designated by the dotted symbols, are also displayed in this figure for comparison. It is obvious that the long dashed distributions are not in good accordance with the experimental data, especially for small  $r$ . Introducing the parameters  $\beta_1$  and  $\beta_2$  (i.e., taking into account the higher orbitals) into our calculations leads to a good agreement with the experimental data as displayed by the solid curves.

Fig.(2) shows the dependence of the  $n(k)$  (in  $\text{fm}^3$ ) on  $k$  (in  $\text{fm}^{-1}$ ) for  $^{59}\text{Co}$ ,  $^{61}\text{Ni}$ ,  $^{63}\text{Cu}$  and  $^{65}\text{Cu}$  nuclei. The long-dashed curves distributions are the (NMD) of eq. (7) calculated by the shell model utilizing the single particle harmonic oscillator wave functions in the momentum space. The dotted symbols and solid distributions are the (NMD) obtained by the (CDFM) of eq. (18) utilizing the experimental and theoretical (NDD), respectively. It is clear that the performance of the long-dashed curves distributions calculated by the shell model are in dissimilarity with the distributions reproduced by the (CDFM). The important property of the long-dashed curves distributions is the steep slope behavior when  $k$  increases. This performance is in

disagreement with the studies [5, 12, 18, 19] and it is attributed to the fact that the ground state shell model wave functions given in terms of a Slater determinant do not take into account the important effects of the short range dynamical correlation functions. Hence, the short-range repulsive features of the nucleon-nucleon forces are responsible for the high momentum behavior of the (NMD) [18, 19]. It is noted that the general structure of the dotted and solid distributions at the region of high momentum components is almost the same for  $^{59}\text{Co}$ ,  $^{61}\text{Ni}$ ,  $^{63}\text{Cu}$  and  $^{65}\text{Cu}$  nuclei, where these distributions have the property of long-tail behavior in the momentum region  $k \geq 2\text{fm}^{-1}$ . The property of long-tail behavior obtained by the CDFM, which is in agreement with the studies [5, 12, 18,19], is connected to the presence of high densities  $\rho_x(r)$  in the decomposition of eq. (12), though their fluctuation functions  $|f(x)|^2$  are small.

The elastic electron scattering form factors for the considered nuclei are calculated in the (PWBA) in the framework of the (CDFM) and by introducing the theoretical weight functions  $|f_c(x)|^2$  of the eq. (25) into eq. (24). The calculated form factors are plotted versus  $q$  as shown in figure 3 for  $^{59}\text{Co}$ ,  $^{61}\text{Ni}$ ,  $^{63}\text{Cu}$  and  $^{65}\text{Cu}$  nuclei where the dotted symbols are the experimental data [21, 13]. As there is no data available for the  $^{61}\text{Ni}$  nucleus, we have compared the calculated form factors of this nucleus with those obtained by the Fourier transform of the 3PF density. Fig.(3) shows that the diffraction minima and maxima of the considered nuclei are reproduced in the correct places. Both the behavior and the magnitudes of the calculated form factors of these nuclei are in reasonable agreement with those of the experimental data.

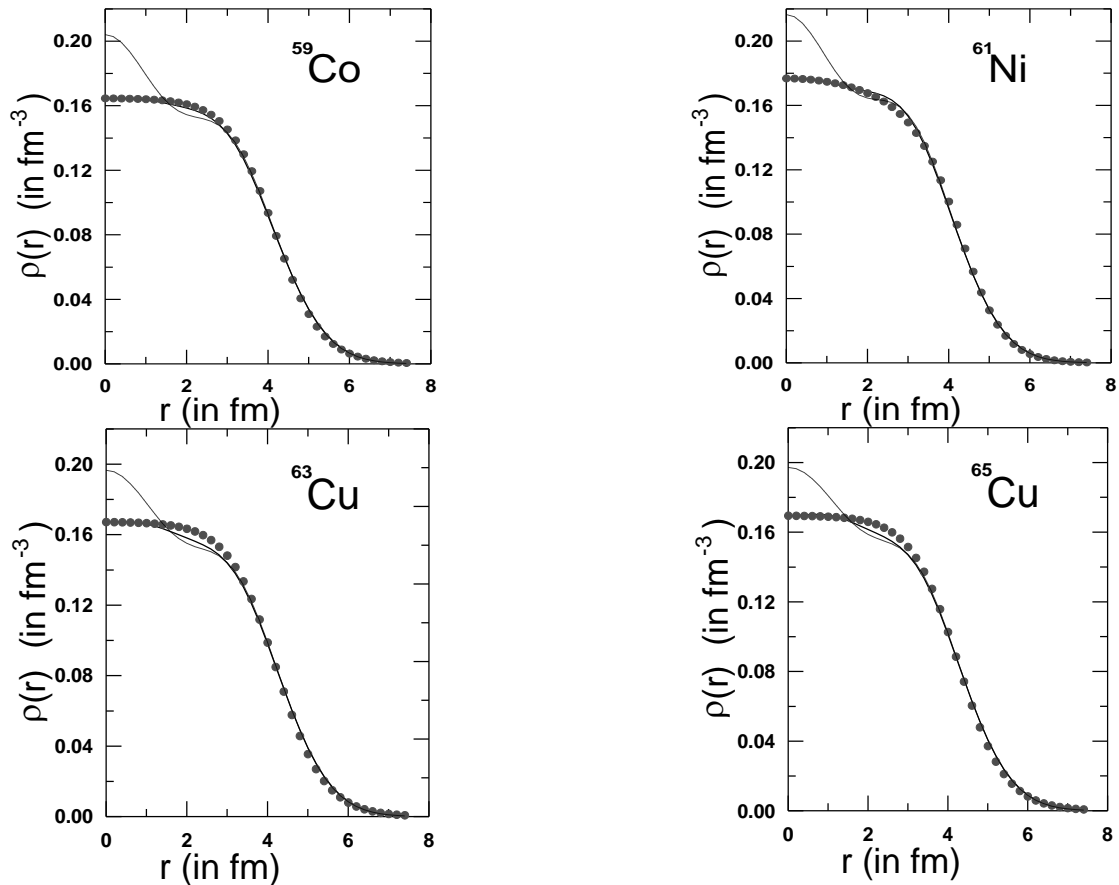
## Conclusions

The (NMD) and elastic electron scattering form factors, calculated in the framework of the (CDFM), are expressed by means of the weight function  $|f(x)|^2$ . The weight function, which is connected with the local density  $\rho(r)$ , was determined from experiment and from theory. The feature of the long-tail behavior of the (NMD) is obtained by both

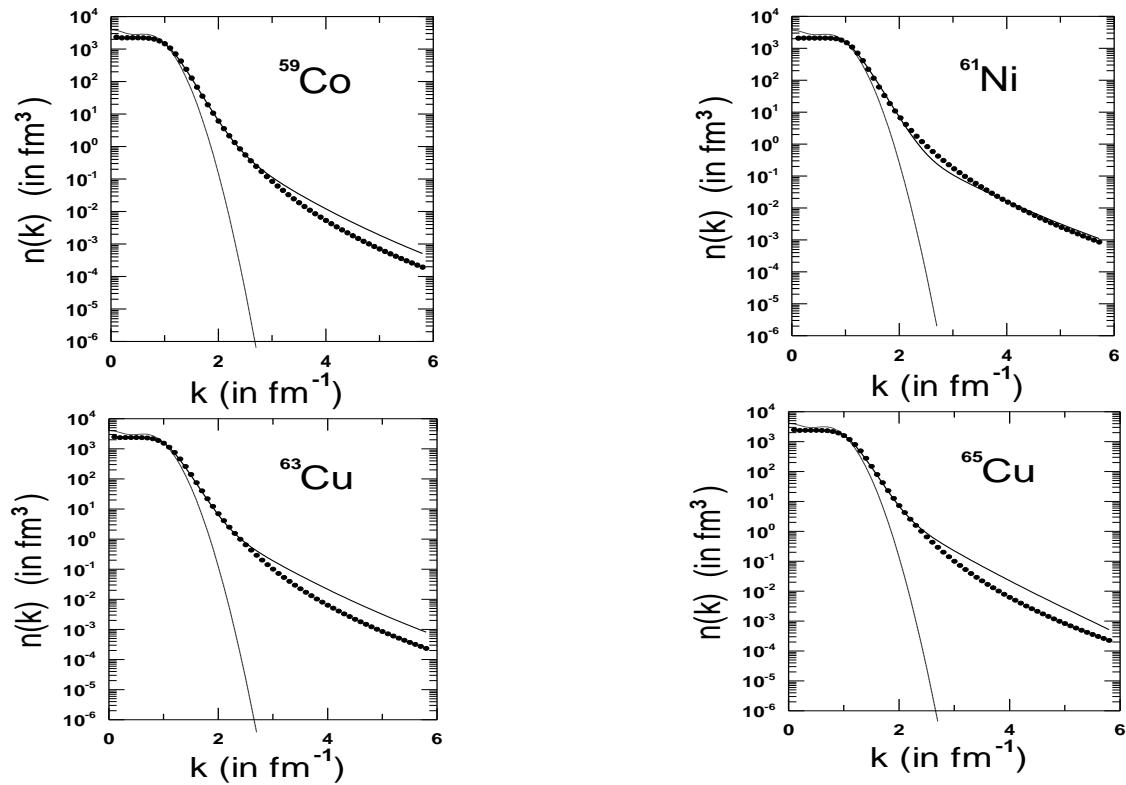
theoretical and experimental weight functions and is related to the existence of high densities  $\rho_x(r)$  in the decomposition of eq. (12), though their weight functions are small. The experimental form factors for elastic electron scattering from  $^{59}\text{Co}$ ,  $^{61}\text{Ni}$ ,  $^{63}\text{Cu}$  and  $^{65}\text{Cu}$  nuclei are well reproduced by the monopole form factors. It is noted that the theoretical ( $NDD$ ) of eq. (2) employed in the determination of the theoretical weight function of eq. (25) is capable of reproducing information about the ( $NMD$ ) and elastic form factors.

**Table (1)**  
*The Values of various parameters employed in the present calculations together with  $\rho_{\text{exp}}(0)$  and  $\langle r^2 \rangle_{\text{exp}}^{1/2}$ .*

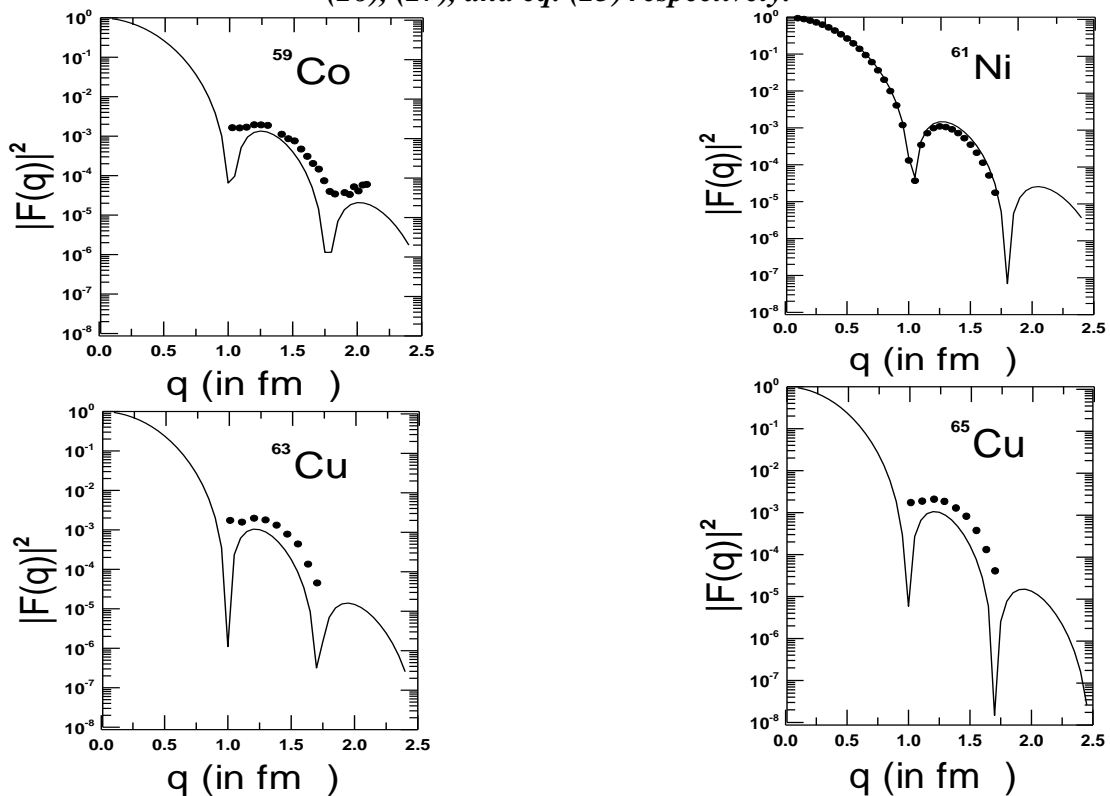
Nuclei	Model	$c$ (fm) [13]	$z$ (fm) [13]	$w$ (fm) [13]	$\rho_{\text{exp}}(0)$ ( $\text{fm}^{-3}$ ) [13]	$\langle r^2 \rangle_{\text{exp}}^{1/2}$ (fm) [13]	$b$ (fm)	$\beta_1$	$\beta_2$
$^{59}\text{Co}$	2PF	4.158	0.575		0.1646183	3.864	2.0640	1.2901	17
$^{61}\text{Ni}$	3PF	4.402	0.540	0.1983	0.1768291	3.806	2.0240	1.2206	2
$^{63}\text{Cu}$	2PF	4.216	0.596		0.1671940	3.947	2.0903	0.9946	3
$^{65}\text{Cu}$	2PF	4.252	0.589		0.1695018	3.954	2.0879	0.9360	3.5



**Fig.(1):** The dependence of the  $NDD$  on  $r$  for  $^{59}\text{Co}$ ,  $^{61}\text{Ni}$ ,  $^{63}\text{Cu}$  and  $^{65}\text{Cu}$  nuclei. The long-dashed and solid curves are the calculated  $NDD$  of eq. (2) when  $\beta_1 = \beta_2 = 0$  and  $\beta_1 \neq \beta_2 \neq 0$ , respectively. The dotted symbols are those fitted to the experimental data [13].



**Fig.(2):** The dependence of nucleon momentum distributions (NMD) on  $k$  for  $^{59}\text{Co}$ ,  $^{61}\text{Ni}$ ,  $^{63}\text{Cu}$  and  $^{65}\text{Cu}$  nuclei. The long-dashed distributions are the results obtained by the shell model calculation of eq. (7) using the single particle harmonic oscillator wave functions in the momentum representation. The dotted symbols and solid distributions are the calculated results expressed by the CFM of eq.(18) using the experimental and theoretical weight functions of eqs. (26), (27), and eq. (25) respectively.



**Fig.(3):** The dependence of form factors on  $q$  for  $^{59}\text{Co}$ ,  $^{61}\text{Ni}$ ,  $^{63}\text{Cu}$  and  $^{65}\text{Cu}$  nuclei. The solid curves are the form factors calculated using eq. (20). The dotted symbols are the experimental data, taken from refs. [20]. The form factor of  $^{61}\text{Ni}$  is obtained by the Fourier transform of  $\rho_{3PF}(r)$ .

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### الخلاصة

تم حساب كل من توزيعات زخم النيكلون وعوامل التشكل للاستطارة الالكترونية المرنة للحالة الارضية لبعض النوى الواقعة ضمن القشرة النووية  $^{59}\text{Co}$ ,  $^{61}\text{Ni}$ ,  $^{63}\text{Cu}$ ,  $^{65}\text{Cu}$  1f-2p حيث تم التعبير عنها بدلالة دالة التموج. ترتبط دالة التموج مع توزيعات كثافة النيكلون وتم حسابها من النتائج النظرية والعملية لتوزيعات كثافة النيوكليون. وقد تميزت نتائج توزيعات زخم النيكلون المعتمدة على دالة التموج النظرية والعملية بصفة الذيل الطويل عند منطقة الزخم العالي. اظهرت هذه الدراسة بان النتائج النظرية لعوامل التشكل للاستطارة الالكترونية المرنة للنوى  $^{59}\text{Co}$ ,  $^{61}\text{Ni}$ ,  $^{63}\text{Cu}$ ,  $^{65}\text{Cu}$  والمحسوبة بانموذج التموج المتشابه توافق جيد مع النتائج العملية و لكل قيم الزخم المنتقل.