# Cassegrian Telescope Design Based on Root-Finding Optimization

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**Abstract**

A two-mirror Cassegrian telescope design is presented. The primary mirror is paraboloid with an aperture of 1m in diameter. The secondary mirror is hyperboloid with an aperture that makes a light obstruction of merely 0.25%. The telescope resolving power is twofold the one a spherical primary mirror produces with the same aperture. Aberration elimination is obtained by modifying the radius of curvature of the secondary mirror and also its asphericity factor. Achieving this work demanded constructing a programming code for tracing light rays. This code not only used to tracing the general type of light rays (skew ray) through all Cartesian surfaces but it is also used to exhibit the telescope performance when varying the secondary mirror radius of curvature and its asphericity factor.

Keywords***:*** aberrations, asphericity factor, quadric surfaces of revolution, ray-tracing.

## 1. Introduction

The powers of a telescope, the telescopes resolving power, the magnification power and the telescope light-gathering power /ability are the real goals for building telescopes. The telescopes resolving power for a spherical mirror is measured by the angular resolving power ($\alpha_O$) and expressed as [1]:

$$\alpha_o = 1.22\frac{\lambda}{D} \quad \text{......................................... (1.1)}$$

where $\lambda$ is the wavelength and $D$ is the aperture diameter of the primary mirror. If the resolving power (angular separation) of a two stars is $\alpha_o$ then the centers of the two diffraction patterns are separated by a distance $\alpha_o f$ , where $f$ is focal length. Hence, the linear separation between the centers of the two diffraction patterns is [1]:

$$linear\ separation = 1.22\frac{\lambda f}{D} \quad \text{.................. (1.2)}$$

Indeed, the latter equation is not very accurate, for it depends on the ratio of the focal length to the aperture diameter. For most telescopes, the radius of the primary mirror is small in comparison to the focal length, which means that the primary mirror is relatively flat and that's why it is just an approximation [2]. The latter reference introduced another expression derived for exact linear separation as [2]:

$$linear separation = 1.22\frac{\lambda}{n\sin(u)} \quad \text{........... (1.3)}$$

where $n$ is the surface refractive and index and $u$ is the extremely marginal ray convergence angle. Reference [2] predicts that in the case of paraboloid, primary mirror the focal length is nearly equal to the mirror diameter, a very small Airy disc which can be attained, of course, means exceptional resolution. In fact, it attains a resolution better than any conventional telescope. And, since the resolution is determined by the angle $u$ and not by the ratio of focal length to the diameter, as is implied by the approximation equation (1.3), this configuration (deep-dish mirrors) can be used for mirrors of any size, even very small sizes, while still remaining exceptional resolution [3].

The paper is organized as follows. Section(2) illustrate the ray tracing procedure. Section (3) shows computing the transverse aberrations of rays, which are considered as the measure of performance of the secondary mirror. Section (4) justifies the telescope configuration we use. Section (5) explains the root-finding optimization method. The results, telescope features, and conclusions are represented sections 6,7 and 8 respectively.

## 2. Skew Ray Tracing

Skew ray tracing equations are divided into two sets of equations, equations set of ray transfer between surfaces, and equations set of reflection or refraction.

### Transfer between Surfaces

It can be expressed by [3]:

$$\left.\begin{aligned} x_0 &= x_{-1} + \frac{L}{N}\left(d - z_{-1}\right) \\ y_0 &= y_{-1} + \frac{M}{N}\left(d - z_{-1}\right) \end{aligned}\right\} \quad \text{.............................. (2.1)}$$

*L, M,* and *N* are the direction cosines of the ray along x-axis, y-axis, and z-axis respectively; $x_0$, and $y_0$ are the coordinates of ray intersection with the tangent x-y plane; $x_{-1}$ and $y_{-1}$ are the coordinates of coming ray. The ray intersects the surface (mirror) in the coordinates are given by [4]:

$$\left.\begin{aligned} x &= x_0 + L\Delta \\ y &= y_0 + M\Delta \\ z &= N\Delta \end{aligned}\right\} \quad \text{........................................... (2.2)}$$

Where $\Delta = \dfrac{F}{G + \sqrt{G^2 - cF(1 + (\varepsilon - 1)N^2)}}$ ....... (2.3)

$$F = c\left(x_0^2 + y_0^2\right) \text{........................................... (2.4)}$$

$$G = N - c\left(Lx_0 + My_0\right) \text{ ................................. (2.5)}$$

where Δ represents the length segment from the tangent plane to the surface, *c* is the curvature of the mirror or the lens, and ε is the asphericity factor. The parameter ε determines the asphericity as follows [4]:

$\varepsilon < 0$ , hyperboloid
$\varepsilon = 0$ , paraboloid
$0 < \varepsilon < 1$ , prolate ellipsoid
$\varepsilon = 1$ , sphere
$\varepsilon > 1$ , oblate ellipsoid

### Reflection (or Refraction) Equations Set

To obtain the cosine of the incident angle (cos*I*), the components of the unit normal (α, β, γ) should be determined at the point of incidence. The direction cosines of the unit normal are given by [4]:

$$\left.\begin{aligned} \alpha &= \frac{-cx}{\sqrt{1 - 2c(\varepsilon-1)z + c^2\varepsilon(\varepsilon-1)z^2}} \\ \beta &= \frac{-cy}{\sqrt{1 - 2c(\varepsilon-1)z + c^2\varepsilon(\varepsilon-1)z^2}} \\ \gamma &= \frac{1 - c\varepsilon z}{\sqrt{1 - 2c(\varepsilon-1)z + c^2\varepsilon(\varepsilon-1)z^2}} \end{aligned}\right\} \quad \text{.............. (2.6)}$$

Then, the cosine of the angle of incidence cos *I* is obtained by [5]:

$$\cos I = \frac{N - c(Lx + My + N\varepsilon z)}{\sqrt{1 - 2c(\varepsilon-1)z + c^2\varepsilon(\varepsilon-1)z^2}} \text{ ............. (2.7)}$$

The cosine angle of reflection or refraction can be expressed by [4]:

$$n' \cos I' = \sqrt{(n')^2 - n^2(1 - \cos^2 I)} \text{ .................. (2.8)}$$

where the non-primed parameters are those of the previous surface.

The direction cosines of the ray, after reflection or refraction, are given by [4]:

$$\left.\begin{aligned} n'L' - nL &= k\alpha \\ n'M' - nM &= k\beta \\ n'N' - nN &= k\gamma \end{aligned}\right\} \text{ ....................................... (2.9)}$$

where

$$k = n'\cos I' - n\cos I \text{ ............................ (2.10)}$$

After each reflection (or refraction) process the direction cosines should be checked to assert the tracing validity. This can be done by [4]:

$$(L')^2 + (M')^2 + (N')^2 = 1 \text{ ............................ (2.11)}$$

### 3. Computing Transverse Aberrations (TA)

This section exhibits the expressions used to compute the results of TA. These values are computed by using one of the skew rays tracing equations equation (1.3) specifically

$$TA = y_{-1} + \frac{M}{N}\left(fl - z_{-1}\right) \text{ ........................... (3.1)}$$

where $y_{-1}$ is the incident ray height at the secondary mirror (*M2*), *fl* is the distance from *M2* to the image focal plane, and $z_{-1}$ is the length segment from the (x-y) plane tangent to the *M2* surface.

### 4. The Telescope Configuration

The primary mirror is paraboloid, with radius of curvature *R1*= -5m and aperture to meet the needs of two goals; first, eliminating spherical aberrations, and second improving the resolving power ensured by equation (1.3).

For the light gathering power importance, minimum light obstruction should be taken into account. The distance of separation (*d*) between the two mirrors is the key for this point. So, for 0.25% light obstruction, the secondary aperture diameter shouldn't exceed 5cm, i.e. ( $[5cm]^2/[1m]^2)\times 100\% = 0.25\%$ ).

This means that the height of the semi-aperture ray (marginal ray) at the secondary mirror is ≤ 2.5cm. To obtain such a light obstruction, the distance separating the two mirrors (*d1*=238cm) is determined by using the skew ray tracing code. Thus, the considered separating distance between the two mirrors is.

To obtain the shortest (most compact) design, the focus, the secondary mirror (*M2*) creates, must be very near to the focal length of the primary which is 2.5m. The key for this point is the selection of radius of curvature of the secondary mirror (*R2*). The proper determination of *R2* is restricted by the value of aberration (*TA*) yielded at the focal plane.

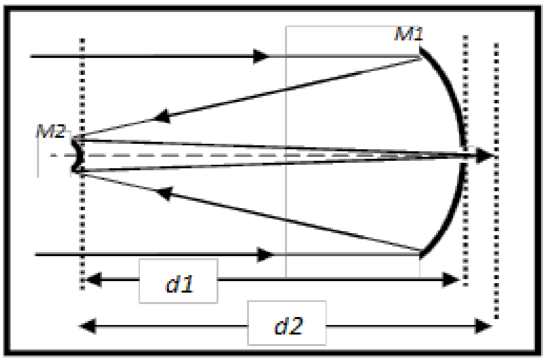


***Fig.(1): Telescope configuration.***

## 5 The Root–Finding Optimization Method

The Root-Finding Method is the one that be used to solve optimization problems. It consists of finding values of the variable $x$ that satisfy *the condition* $f(x)=0$ for given function. The solution of such problem is called the root of $f(x)$ [5]. Practically and numerically, the root which is being searched is not necessarily yields $f(x)$ =0, but the one which gives acceptable function value. It is known that at the proposed root, the function values witness change in its values either from positive values to negative or vice versa.

This method is used to examine the telescope performance via the transverse aberration (TA) of light rays. For this purpose, two different rays' heights are used; the paraxial ray of height 2.5cm and the marginal of 50cm height.

**The procedure steps are**

1) Initially, the secondary mirror surface has considered as spherical one (ε2=1). The search of the proper *R2* is based upon observing the *TA* values of *R2* and resuming in the direction that shows aberration reduction at the image focal plane.
2) The search goes on until the *TA* witnesses a change in its values. The root is the optimum radius of the second mirror (*R2*) when it gives *TA* values < $\lambda$.
3) This procedure is resumed for the optimum *R2* but for a conic or Cartesian surface, i.e. (ε2≠1). The search of the proper ε*2* is based upon observing the *TA* values of ε*2* and resuming in the direction that shows aberration reduction at the image focal plane. The root is the optimum ε*2* when it gives *TA* values < $\lambda$.

## 6. Results

Fig.(2) shows the TA values as functions of the *R*2. It also shows that zero TA-values are near *R*2= -25cm. Fig.(3) shows the TA values for the paraxial ray is zero for *R*2= -25.21cm while Fig.(4) shows that the TA value of the marginal ray is still appreciable when *R*2=-25.21cm. Fig.(5) shows that varying the asphericity factor ε2 of the second mirror leads to get zero TA for the marginal ray when ε2= -0.21.

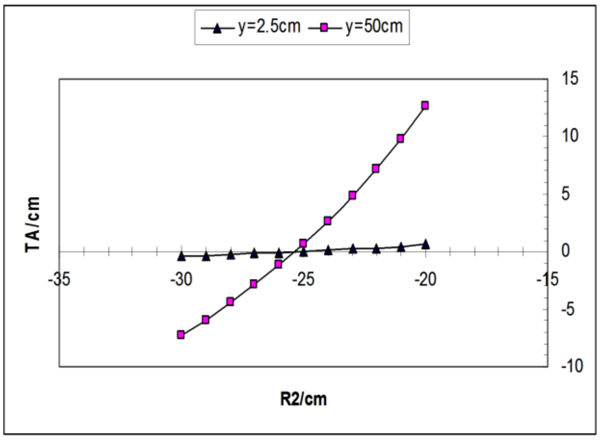


***Fig.(2): TA of the Paraxial and the marginal rays Vs. R2 for the initial suitable R2 determination.***

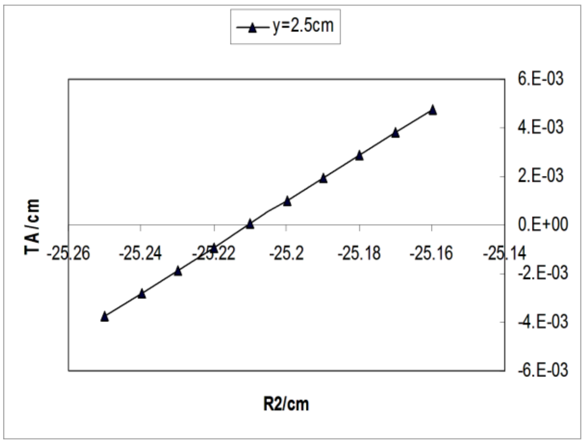


***Fig.(3): TA of the paraxial rays Vs. R2 for accurate R2 determination.***

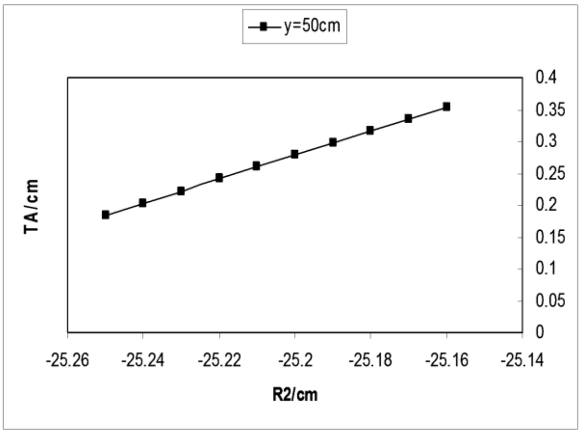


***Fig.(4): TA of the marginal Vs. R2 for accurate R2 determination.***

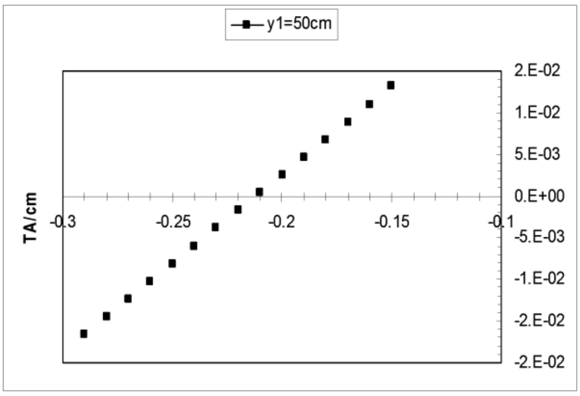


***Fig.(5): TA of the marginal Vs. ε2 for accurate ε2 determination.***

## 7. Conclusions

A two-mirror Cassegrian telescope is presented. The paraboloid mirror shows improvement in resolution over that of spherical surfaces, The design improved the light gathering power of 1m aperture telescope for light obstruction = 0.25% (less than 1% of the surface area of the primary mirror). The resolving power is also enhanced to 3.388μm.

This is because the linear separation for the paraboloid mirror, by using equation (1.3), is:

$$\frac{550nm}{n\sin(2\times 5.710623)} = 3.388\mu m$$

while the linear separation for a spherical mirror with the same aperture, by using equation (1.2), can be determined as:

$$1.22\frac{550nm}{1m}\times 2.5m = 1.677\mu m$$

The transverse ray aberration of the yielded design is also investigated for aberration elimination via the variation of both the radius of curvature of the secondary mirror R2 and its aspherisity factor ε2.

The root-finding optimization method proved its utility in spherical aberration reduction with varying the radius of curvature and the asphericity factor of the secondary mirror. The procedure used in aberration elimination showed excellent result in aberrations reduction and it may be, also, able to eliminate the aberration completely, if the distance d1, separating the two mirrors, would be involved in the optimization procedure.

## References

[1] Smith F. G. & Thomson J. H., Optics 2nd edition, John Wiley & Sons, 1988.

[2]http://www.thunderbirdtechnology.com/Other%20Pages/ Airy Disc and Angular Resolution.htm.

[3] Welford W. T., Aberrations of the Symmetrical Optical Systems, London, Academic Press, 1974.

[4] Abrahim A. Sadiq, (JUNS) 15(4), 157, 2012.

[5] Richard L. Burden, and J. Douglas Faires, Numerical Analysis 3rd edition, Prindle, Weber & Schmidt, Boston, 1985.

**الخلاصة**

إنّ التلسكوب المنجز تصميمه في هذا العمل هو منظومة المرآتين نوع كاسجرين. المرآة الأولية ذات قطع مكافئ ولها قطر ١ متر، أمّا المرآة الثانوية فهي ذات قطع زائد وبقطر يحقق نسبة حجب لا تزيد عن 0.25% لمقدار الضوء المنعكس من المرآة الأولية. كما أنّ التصميم قد نجح في تحسين قدرة التلسكوب في الفصل بين نقطتين متجاورتين الى الضعف. إنّ إزالة الزيوغ قد تم من خلال تغيير نصف قطر تكور المرآة الثانوية و عامل اللاتكور للمرآة الثانوية. إنّ إنجاز هذا العمل قد تطلّب بناء شفرة برمجية لأقتفاء أثر الأشعة البصرية قادرة على إقتفاء أثر كل انواع الاشعة البصرية وعبر كل انواع السطوح المخروطية (الكارتيزية). كما وأستخدمت هذه الشفرة البرمجية لأظهار ادآء التلسكوب عند تغيير قيم كل من نصف قطر تكور المرآة الثانوية وعامل اللاتكور للمرآة الثانوية من خلال زيغ الاشعة البصرية.