Some Types of Contra- gp- Closed Functions in Topological Spaces

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Abstract

In this paper, we introduce a new type of contra- closed functions which is the contra-gp- closed functions, as well as we using this function to give and study other types of contra-gp- closed functions namely (contra-gp*- closed function, contra-gp**- closed function, contra-pre gp- closed function and contra-pre gp*- closed function) and find the relation between these functions. Also, we will proved several properties of these functions types.

1-Introduction

In 1996, Maki et al [11] introduced the notion of gp- closed sets and investigated their basic properties. They used these sets to define gp- closed functions and study some of their properties. Dontcher [5] introduced the notions of contra- continuity and strong S- closedness in topological spaces. He obtained very interesting and important results concerning contra-continuity. While the concepts (contra-closed functions and contra-preclosed functions) were discussed and introduced by (Baker. C.W (1997) in [1], Caldas. M (2004) in [4]) respectively.

In this paper , we introduce and study new type of contra-closed functions namely contra-gp- closed function in topological space and we use this function to give other types which are (contra-gp*- closed, contra-gp**- closed, contra-pre-gp- closed and contra-pre gp**- closed) functions. Moreover, show the relation between these types of functions. Furthermore, we study and proved some properties of these functions.

Throughout this paper (X,τ) and (Y,σ) (or simply X and Y) represents non-empty topological spaces, For a subset A of a space X. cl (A), int (A) and A^c denoted the closure of A, the interior of A and the complement of A in X respectively.

2-Preliminaries

Some definitions and basic concepts related to this paper.

Definition (2-1), [2]:

A subset **A** of a topological space (X,τ) is said to be **preopen** if A⊆ int(cl(A)) and **preclosed set** if cl(int(A)) ⊆A.

The **pre closure** of a subset A of (X,τ) is the intersection of all preclosed sets containing A and is denoted by **pcl (A).**

Definition (2-2), [8]:

A subset **A** of a topological space (X,τ) is said to be **generalized preclosed** (briefly, gp-closed) if pcl(A)⊆U if for some open set U of (X,τ) such that A⊆U .

The complement of gp-closed is called **gp-open set**.

Remark (2-3), [3], [8]:

In any topological space (X,τ), it is clear:

1. Every open (closed) set is preopen (preclosed) set, but the converse need not to be true in general.
2. Every preopen (preclosed) set is gp-open (gp-closed) set, but the converse need not to be true in general.

To illustrate the above remark, consider the following example:

Example (2-1):

Let X = {a,b,c,d} ,τ ={X,ϕ,{a},{b},{a,b},{a,b,c}}. Then the set A= {a,b,c} is preopen set and gp- open set in (X,τ), but is not open set in (X,τ). Also, A^c ={c} is a preclosed set and gp- closed set in (X,τ), but is not closed and let B={a,c} is gp- open set in (X,τ), but is not preopen, also B^c ={b,d} is a gp- closed set in (X,τ), but is not preclosed set in (X,τ).

Remark (2-4), [8], [9]:

In any topological spaces (X,τ)

1-The union of any collection of gp-closed sets in (X,τ) is gp-closed.

2-The intersection of two gp-closed sets in (X,τ) is not necessary to be gp-closed.

Definition (2-5), [6]:

The topological space (X,τ) is said to be **locally indiscrete** if every open subset in (X,τ) is a closed set.

Definition (2-6), [10]:

The topological space (X,τ) is said to be **submaximal** if every dense set of (X,τ) is an open set.

Theorem (2-7), [10]:

Let (X,τ) be a topological space, then (X,τ) is submaximal space if and only if every preopen set in (X,τ) is open.

Corollary (2-8), [10]:

Let (X,τ) be a submaximal space, then every preclosed set in (X,τ) is a closed set.

Remark (2-9):

Dontch [7] proved that a space (X,τ) is **semi-pre-** $T_{1/2}$ **- space** if and only if every gp-closed is preclosed set.

Definitions (2-10), [3], [9]:

A function f: $(X,\tau)\rightarrow (Y,\sigma)$ is said to be:

1. **closed** if f(A) is closed in (Y,σ) for every closed set A of (X,τ).
2. **preclosed** if f(A) is preclosed set in (Y,σ) for every closed set A of (X,τ).
3. **gp- closed** if f(A) is gp- closed in (Y,σ) for every closed set A of (X,τ) .
4. **Pre gp- closed** if f(A) is gp- closed set in (Y,σ) for every preclosed A set of (X,τ) .

Definition (2-11), [1], [4]:

A function f: $(X,\tau)\rightarrow (Y,\sigma)$ is said to be:

1. **contra- closed** if f(A) is an open set in (Y,σ) for every closed set A of (X,τ).
2. **contra- preclosed** if f(A) is a preopen set in (Y,σ) for every closed set A of (X,τ).

Remark (2-12), [4]:

The following examples show that closed functions (resp. preclosed functions) are independent of contra- closed functions (resp. contra- preclosed functions).

Example (2-2):

Let $X=\{a,b,c\}$,$\tau=\{X,\phi,\{a\},\{a, c\}\}$. Define a function f: $(X,\tau)\rightarrow (X,\tau)$ by f(a)=a, f(b)=c and f(c)=b. It is easily that f is closed function and preclosed function but is not contra- closed and contra- preclosed function.

Example (2-3):

Let $X= \{a,b,c\}$,$\tau =\{X,\phi,\{a\},\{b\},\{a ,b\}\}$. Define a function f: $(X,\tau)\rightarrow (X,\tau)$, by f(a)= f(c)=b and f(b)=a. It is observe that f is contra- closed function and contra- preclosed function but is not closed function.

Remark (2-13), [4]:

Every contra- closed function is contra- preclosed, but the converse is not true. It is easy see that in the following example:

Example (2-4):

Let $X=\{a,b,c\}$, $\tau =\{X,\phi,\{a\},\{a,c\}\}$. Define a function f: $(X,\tau)\rightarrow (X,\tau)$, by f(a)= f(b)=a and f(c)=b .We observe that f is contra- preclosed function, which is not contra- closed.

3-Contra-gp- closed function types

In this section, a new type of contra- closed functions namely contra-gp- closed function will be given, and use this function to introduce and study some other types of functions which are (contra-gp^*- closed, contra-gp^{**}- closed, contra-pre gp- closed and contra-pre gp*- closed) functions, and study some of their properties and relations among them.

Now, the definition of contra-gp- closed function will be introduce, which is a modification to the definition of contra- closed function that appeared in [1].

Definition (3-1):

A function f: $(X,\tau)\rightarrow (Y,\sigma)$ is said to be **contra-gp- closed** if f(A) is a gp- open set in (Y,σ) for every closed set A in (X,τ).

Proposition (3-2):

Every contra- preclosed function is contra- gp- closed function

***Proof*:**

Let f: $(X,\tau)\rightarrow (Y,\sigma)$ be a contra- preclosed function and let A be a closed set in (X,τ). Thus, f(A) is a preopen set in (Y,σ). By using remark (2-3) we get f(A) is gp- open set in (Y,σ) .Hence, f: $(X,\tau)\rightarrow (Y,\sigma)$ is contra-gp- closed function.

The converse is not true in general, as the following example show:

Example(3-1):

Let X= {a,b,c},τ ={X,ϕ,{a},{a,c}}. Define a function f: (X,τ)→ (X,τ) by f(b)=c and f(a)=f(c)=a. It is easily see that f is a contra-gp- closed function, but is not contra-preclosed. Since for the closed set A={b} in (X,τ), f(A)= f{b}={c} is a gp- open set in (Y,τ), but is not preopen set in (X,τ).

Corollary (3-3):

Every contra- closed function is contra-gp-closed function.

***Proof*:**

It is clear.

The converse of the above corollary need not be true, in the above example (3-1) a function f is contra- gp- closed function, but not contra – closed .

Now, some other types of contra-gp-closed function are given, and start first by the following definition.

Definition (3-4):

A function f: (X,τ)→ (Y,σ) is said to be **contra-gp*- closed** if f(A) is an open set in (Y,σ) for every gp- closed set A in (X,τ).

Proposition (3-5):

Every contra-gp*-closed function is contra-closed function.

***Proof*:**

Let f: (X,τ)→ (Y,σ) be a contra-gp*-closed function and let A be a closed set in (X,τ). By remark (2-3), we get A is a gp-closed set in (Y,σ). Since f is contra–gp*-closed function. Thus, f(A) is an open set in (Y,σ). Hence, f: (X,τ)→ (Y,σ) is a contra-- closed function.

Corollary (3-6):

Every contra- gp*- closed function is contra- preclosed function.

***Proof*:**

It is clear .

Corollary (3-7):

Every contra-gp*- closed function is contra-gp- closed function.

***Proof*:**

It is clear that from corollary (3-6) and proposition (3-2).

Remark (3-8):

The converse of proposition (3-5), corollary (3-6) and corollary (3-7) respectively need not be true.

Example (3-2):

Let X=Y={a,b,c}, τ ={X,ϕ} and σ={Y,ϕ,{a},{a, c}} and let f:(X,τ)→ (Y,σ) be the identity function. It is observe that f is contra- closed (contra- pre- closed and contra-gp-closed) function but is not contra-gp*-closed function. Since for gp- closed set A={c} in (X,τ), we see that f(A)= f({c})={c} is not open set in (Y,σ).

Definition (3-9):

A function f:(X,τ)→ (Y,σ) is said to be **contra-gp**- closed function** if f(A) is a gp- open set in (Y,σ) for every gp- closed set in (X,τ).

Proposition (3-10):

Every contra-gp**-closed function is contra-gp- closed function.

***Proof*:**

Let f: (X,τ)→ (Y,σ) be a contra-gp*-closed function and let A be a closed set in (X,τ), by remark (2-3), we get, A is a gp-closed set in (X,τ).Since f is contra-gp**-closed function. Then f(A) is a gp- open set in (Y,σ).Hence, f is contra-gp-closed.

The converse of the proposition (3-10) is not true.

Example (3-3):

Let X= {a,b,c}, τ ={X,ϕ,{a},{a,c}}. Define a function f:(X,τ)→ (X,τ) by f (a)=b, f (b)=c and f(c)= a. Then f is contra-gp- closed but is not contra-gp**- closed function. Since for a gp- closed set A={a ,b} in (X,τ), we see f(A)= f({a ,b)}= {b ,c} is not gp- open set in (X,τ).

Now, the following proposition show the relation between contra-gp**- closed function and contra-gp*- closed function**.**

Proposition (3-11):

Every contra-gp*-closed function is contra-gp**- closed function.

***Proof*:**

Let f: (X,τ)→ (Y,σ) by a contra-gp*-closed function and let A be a gp-closed set in (X,τ). Thus, f (A) is an open set in (Y,σ), and by using remark (3-2), we get f (A) is a gp-open set in (Y,σ) .Hence, f: (X,τ)→ Y,σ) is contra-gp**- closed function.

The converse of the above proposition need not be true.

Example (3-4):

Let X={a,b,c},τ={X,ϕ,{a},{a,c}}. Define a function f: (X,τ)→ (X,τ) by f(a)=b, f(b)=a and f(c)= c. It is observe that f is a contra-gp**- closed function, but is not contra-gp**- closed function. Since for a gp- closed set A={c} in (X,τ), we see f(A)=f({c})={c} is not open set in (X,τ).

Remark (3-12):

The concepts of(contra–closed and contra- preclosed) functions are independent to contra-gp**- closed functions. As shown in the following example.

Example (3-5):

(i) From example (3-2). It is observe that f is contra- closed and contra- preclosed, but is not contra-gp**- closed function. Since for a gp-closed set A={b} in (X,τ), we see, f(A) = f({b})={b}is not gp-closed set in (Y,σ) .

(ii) Let X={a,b,c},τ={X,ϕ,{a},{a,c}}. Define a function f: (X,τ)→ (Y,σ) by f(a)= f(c)=a and f(b)=c. It is observe that f is a contra-gp**- closed function, but is not contra- closed and contra-preclosed. Since for a closed set A={b}, then f(A)= f({b})= {c} is not open and preopen set in (X,τ).

Now, we give another types of contra-gp- closed function is called contra- pre gp- closed function.

Definition (3-13):

A function f:(X,τ)→ (Y,σ) is said to be **contra- pre gp- closed** if f(A) is a gp- open set in (Y,σ) for every preclosed set in (X,τ).

Proposition (3-14):

Every contra-pre gp- closed function is contra-gp- closed.

***Proof*:**

Let f: (X,τ)→ (Y,σ) by a contra-pre gp- closed function and let A be a closed set in X, by using remark (2-3) ,we get A is a preclosed set in (X,τ). Since f is contra- pre gp- closed function. Thus, f(A) is a gp- open set in (Y,σ). Therefore, f is a contra-gp- closed function.

The following example show that the converse of proposition (3-14) need not true in genera.

Example (3-6):

Let X={a,b,c},τ ={X,ϕ,{a},{a c}}. Define a function f: (X,τ)→ (X,τ) by f(a)= f(c)=b and f(b)= a. It is easily see that f is a contra-gp- closed but is not contra-pre gp- closed function. Since for the preclosed set A={c} in (X,τ), but f(A)= f({c})= {b} is not gp- open set in (X,τ).

The following preposition give the condition to make the converse of a proposition (3-14) is true.

Proposition (3-15):

If f: (X,τ)→ (Y,σ) is contra-gp- closed function and a space (X,τ) is a submaximal space. Then f is contra- pre gp- closed function.

***Proof*:**

Let A be a preclosed set in (X,τ) and since (X,τ) is a submaximal space, then by using corollary (2-8) ,we get A is a closed set in (X,τ). Since f is a contra-gp- closed function. Thus, f(A) is a gp- open set in (Y,σ). Hence, f: (X,τ)→ (Y,σ) is contra-pre gp- closed function.

Remark (3-16):

It is clear that the concepts contra- closed (contra- preclosed) function are independent to contra- pre gp- closed function. In the example (3-2) a function f is contra- closed (and contra- preclosed) function but it is not contra- pre gp- closed function. Since for a preclosed set A={b} in (X,τ), we see f(A)= f({b})= {b} is not gp- open set in (Y,σ). Also in example (3-3) it is clearly that f is contra- pre gp- closed function, but it is not contra- closed (and contra-preclosed) function. Since for a closed set A={b}in (X,τ), then f(A)= f({b}) ={c} is not open set and preopen set in (X,τ).

Now, in the following result given the condition to make every contra- closed (and contra- preclosed) function is contra- pre gp- closed function**.**

Corollary (3-17):

If f: (X,τ)→ (Y,σ) is contra- closed or contra- function and (X,τ) is a submaximal space. Then f is contra- pre gp-closed function.

***Proof*:**

Let A be a preclosed set in (X,τ). Since (X,τ) is a submaximal space and by using corollary (2-8), we get A is a closed set in (X,τ). Since f is a contra-gp- closed function. Thus, f(A) is an open set in (Y,σ) and by remark(2-3) we have f(A) is gp- open set in (Y,σ). Hence, f: (X,τ)→ (Y,σ) is contra-pre gp- closed function.

Similarly, we prove the following corollary:

Corollary (3-18):

If f: (X,τ)→ (Y,σ) is contra- preclosed closed function and let (X,τ) is a submaximal space, then f is contra- pre gp- closed function.

The following proposition show the relation between contra- pre gp-closed function and contra- gp^*- closed (contra- gp^{**}- closed) function respectively.

Proposition (3-19):

Every contra-gp^{**}- closed function is contra- pre gp- closed function.

***Proof*:**

Let f: (X,τ)→ (Y,σ) be a contra-gp^{**}- closed function and let A be a precluded set in (X,τ), and by using remark (2-3) we get A is a gp- closed set in (X,τ). Since f is contra-gp^{**}- closed function. Then f(A) is a gp- open set in (Y,σ). Hence, f: (X,τ)→ (Y,σ) is contra-pre gp- closed function.

The converse of proposition (3-19) need not be true in general, as the following example show:

Example(3-7):

Let X={1,2,3}, τ={X,ϕ,{1},{1,3}}. Define a function f:(X,τ)→ (X,τ) by f(1)=2, f(2)=3 and f(3)=1. It is observe that f is contra- pre gp-closed function, but its not contra- gp^{**}- closed function.

Since for a gp- closed set A={1,2}in (X,τ), we see that f(A)=f({1,2)}={2,3} is not gp- open set in (X,τ).

From proposition (3-11) and (3-19) we get the following corollary and it is prove easy. Thus, we omitted its proof:

Corollary (3-20):

Every contra-gp^{**}-closed function is contra- pre gp- closed function

Remark (3-21):

The converse of corollary (3-20) is not necessarily true. In example (3-7) a function f is contra- pre gp- closed function, but is not contra- gp^{**}- closed function. Since for a gp- closed set A={2}, it is observe that f(A)= f({2)}={3} is not open set in (X,τ).

The following proposition addition the condition in order to the converse of proposition (3-19) is true**:**

Proposition (3-22):

If f: (X,τ)→ (Y,σ) is contra- pre gp-closed function and let (X,τ) is a semi- pre $T_{1/2}$- space. Then f is contra- gp^{**}- closed function.

***Proof*:**

Let A be a gp- closed set in (X,τ). Since (X,τ) is semi- pre $T_{1/2}$ - space and by using remark (2-9) we get A is a preclosed set in (X,τ). Since f is contra-pre gp- closed function. Thus, f(A) is a gp- open set in (Y,σ). Therefore, a function f: (X,τ)→ (Y,σ) is contra- gp^{**}- closed.

Now, will be given another type of contra-pre gp- closed function is called contra-pre gp^*- closed function**.**

Definition (3-23):

A function f:(X,τ)→ (Y,σ) is said to be **contra- pre gp^*- closed** if f(A) is a preopen set in (Y,σ) for every gp- closed set A in (X,τ).

Proposition (3-24):

Every contra-pre gp^*- closed function is contra-preclosed.

***Proof*:**

Let f: (X,τ)→ (Y,σ) by a contra-pre gp^*- closed function and let A be a closed set in (X,τ), and by remark (2-3) we get A is a gp- closed set in (X,τ). Since f is contra- pre gp^*- closed function. Thus, f(A) is a preopen set in (Y,σ). Therefore,f: (X,τ)→ (Y,σ) is a preclosed function.

Remark (3-25):

The converse of proposition (3-24) need not be true. In example (3-2), a function f is contra-preclosed function, but it is not contra- pre gp^*- closed function. Since for a gp- closed set A={b} in (X,τ), we see f(A)=f({b})= {b} is not preopen set in (Y,σ).

Corollary (3-26):

Every contra- pre gp^*- closed function is contra- gp- closed.

***Proof*:**

It is follows from proposition (3-24) and proposition (3-2).

Remark (3-27):

The converse of corollary (3-26) need not be true .In example (3-6), it is observe that f is contra-gp- closed function, but is not contra-gp^*- closed. Since for gp- closed set A={c} in (X,τ), we see f(A)= f({c})= {b} is not preopen set in (X,τ).

Proposition (3-28):

Every contra- pre gp^*- closed function is contra- pre gp- closed.

***Proof*:**

Let f: (X,τ)→ (Y,σ) by a contra-pre gp- closed function and let A be a preclosed set in X. By using remark (2-3) we get A is a gp- closed set in (X,τ). Since f is contra- pre gp^*- closed function. Thus, f(A) is a preopen set in (Y,σ), and by using remark (2-3) we have f(A) is a gp- open set in (Y,σ). Therefore, a function f: (X,τ)→ (Y,σ) is contra-pre gp-closed.

Remark (3-29):

The converse of the proposition(3-28) is not true, It is observe that in example (3-7), f is contra-pre gp- closed function but it is not contra- pre gp^*- closed.

Proposition (3-30):

Every contra- pre gp^{**}- closed function is contra- gp^{**}- closed.

Proof:

It is clear.

Remark (3-31):

The converse of proposition (3-30) need not be true. In example (3-4), a function f is contra- pre gp^{**}- closed function, but is not contra- gp^*- closed function. The following proposition show the relation between contra- gp^*- closed function and contra-closed function:

Proposition (3-32):

Every contra- gp^*- closed function is contra- pre gp^*- closed.

***Proof*:**

Let f: (X,τ)→ (Y,σ) by a contra-gp^*-closed function and let A be a gp- closed set in (X,τ). Thus, f(A) is an open set in (Y,σ). By using remark (2-3), we get f(A) is a preopen set in (Y,σ) . Therefore, a function

f: (X,τ)→ Y,σ) is contra- pre gp^*- closed. The following example shows the converse of proposition (3-32) is not true in general.

Example (3-8):

Let X={a,b,c},τ={X,ϕ,{a},{a,c}}. Define a function f: (X,τ)→ (X,τ) by f(a)=b and f(b)= f(c)=a. We observe that f is contra-pre gp^*- closed function, but is not contra-gp^*-closed function. Since for a gp- closed set A={a,b} in(X,τ), we see that f(A)= f({a,b)}= {a,b} is not open set in (X,τ).

The following proposition addition the condition in order to the converse of proposition (3-32) is true.

Proposition (3-33):

If f: (X,τ)→ (Y,σ) by a contra-pre gp^*-closed function and (Y,σ) is a submaximal space. Then f is contra- gp^*- closed.

***Proof*:**

Let A be a gp- closed set in (X,τ). Thus f(A) is a preopen set in (Y,σ) . Since (Y,σ) is a submaximal space and by using theorem (2-7) we get f(A) is an open set in (Y,σ) . Hence, f: (X,τ)→ (Y,σ) is contra-pre gp^*- closed function.

The following example show that contra-closed function and contra-pre gp^*-closed function are independent**.**

Example (3-9):

Let X=Y={a ,b,c}},τ={X,ϕ,{a},{a ,c}} and σ={Y,ϕ}.Define a function f:(X,τ)→(Y,σ) by f(a)=band f(b)=f(c)=a**.** Then f is contra-pre gp^*- closed. But f is not contra- closed. Since for a closed set A=X in (X,τ), we see that f(A)= f(X)= {a ,b} is not open set in (Y,σ). Also, In example (3-2) f is contra- closed, but is not contra- pre gp^*- closed. Since for gp- closed set A={c} in (X,τ), we see f(A)=f({c})={c} is not preopen.

The following proposition addition the condition to make every contra-pre gp*- closed function is contra- closed function.

Proposition (3-34):

If f: (X,τ)→ (Y,σ) is contra-pre gp*- closed function and Y is a sub maximal space. Then f is contra- closed.

***Proof*:** Let A be a gp- closed set in (X,τ) and by remark (2-3) we get A is gp- closed set in (X,τ).Thus, f(A) is a preopen set in (Y,σ). Since (Y,σ) is a submaximal space and by theorem (2-9) we get f(A)is an open set in (Y,σ). Hence, f is contra- closed function.

Remark (3-35):

The following diagram shows the relation between contra-gp- closed function types.

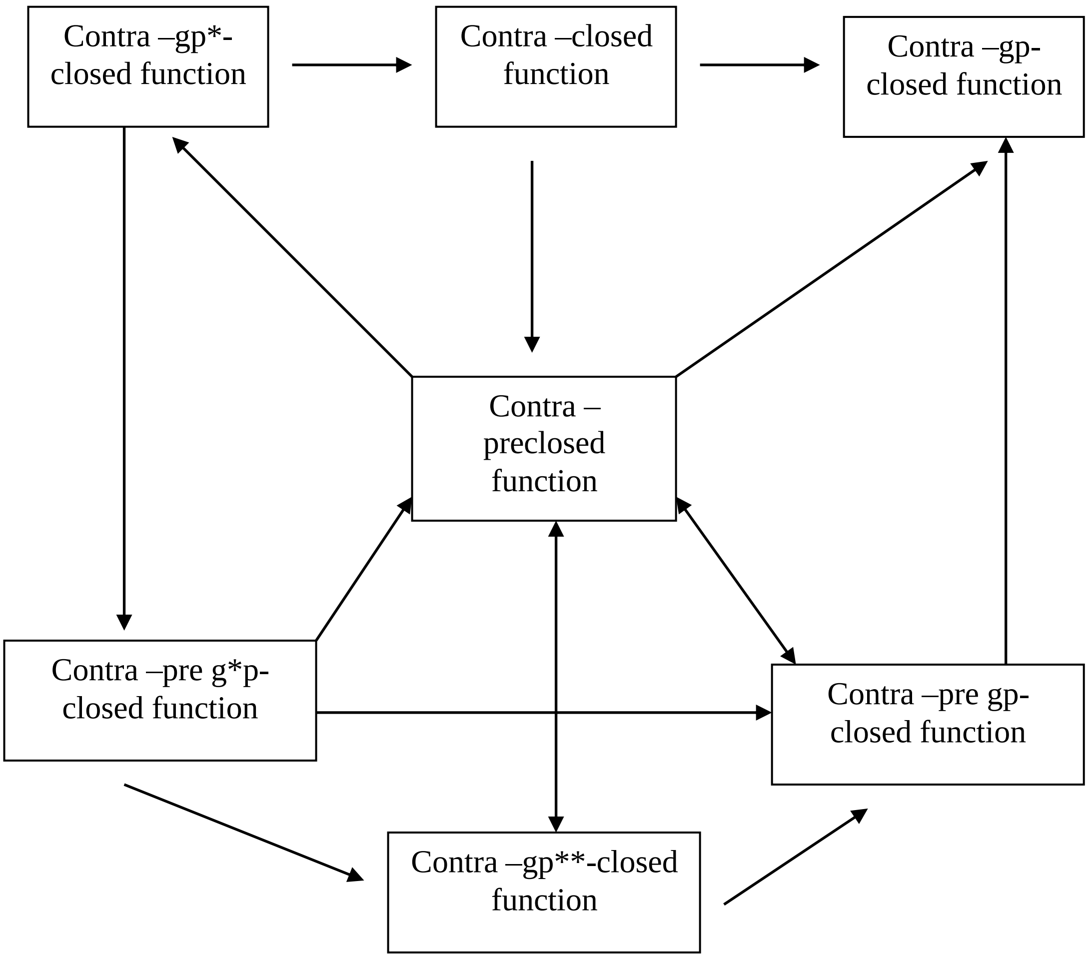

Diagram (1).

4-Composition of contra-gp-closed function

In this section, will be give the composition of these types of contra-gp-closed function with closed (preclosed, gp-closed and pre gp- closed) function respectively.

Proposition (4-1):

Let f: $(X,\tau)\rightarrow(Y,\sigma)$ and g: $(Y,\sigma)\rightarrow(Z,\mu)$ be any two functions. Then gof: $(X,\tau)\rightarrow(Z,\mu)$ is a contra-gp- closed function, if f is a closed function and g is

1-contra-gp- closed function.
2-contra-gp*- closed function.
3-contra-gp**- closed function.
4-contra-pre gp- closed function.
5-contra- pre gp*- closed function.

Proof:

(1) let A be a closed set in (X,τ). Thus, f(A) is a closed set in (Y,σ). By using remark (2-3) we get f(A) is a preclosed set in (Y,σ). Since g is contra- gp- closed function. Then g(f(A)) is a gp- open set in (Z,μ). But g(f(A)) = gof(A). Hence, gof: $(X,\tau)\rightarrow(Z,\mu)$ is a contra-gp- closed function.

(2) Since g is contra-gp*- closed function, and by using corollary (3-7) we get g is contra-gp- closed, and by step-1- we obtain gof a contra-gp- closed.

(3) Since g is contra-gp*- closed function and by using proposition (3-10) we get g is contra-gp- closed function and by step-1- we obtain the composition gof:$(X,\tau)\rightarrow(Z,\mu)$ is a contra-gp- closed function.

(4) Since g is contra-pre gp- closed function, and by proposition (3-14) we have g is contra-gp- closed function and by step-1- we get
gof: $(X,\tau)\rightarrow(Z,\mu)$ is a contra-gp- closed function.

(5) Since g is g is contra-pre gp*- closed function and by using corollary (3-26) we obtain g is contra-gp- closed function and by step-1- we have gof:$(X,\tau)\rightarrow(Z,\mu)$ is a contra-gp- closed function.

Similarly, we proof the following corollaries.

Corollary (4-2):

Let f: $(X,\tau)\rightarrow(Y,\sigma)$ and g: $(Y,\sigma)\rightarrow(Z,\mu)$ be any two functions. Then gof: $(X,\tau)\rightarrow(Z,\mu)$ is a contra-gp- closed function if f is a preclosed function and g is

1. contra-gp*- closed function.
2. contra-gp**- closed function.
3. contra-pre gp- closed function
4. contra–pre gp*- closed function.

Corollary (4-3):

Let f: $(X,\tau)\rightarrow(Y,\sigma)$ and g: $(Y,\sigma)\rightarrow(Z,\mu)$ be any two functions. Then gof: $(X,\tau)\rightarrow(Z,\mu)$ is a contra-gp- closed function if f is gp- closed function and g is

1-contra gp*-closed function.
2-contra gp**-closed function.

Corollary (4-4):

Let f: $(X,\tau)\rightarrow(Y,\sigma)$ and g: $(Y,\sigma)\rightarrow(Z,\mu)$ be any two functions. Then their composition gof: $(X,\tau)\rightarrow(Z,\mu)$ is a contra-pre gp- closed function if f is pre gp- closed function and g is

1- contra gp*-closed function.
2- contra gp**-closed function.
3- contra – pre gp*-closed function.

Proposition (4-5):

Let f:$(X,\tau)\rightarrow(Y,\sigma)$ is contra-gp*- closed function and g:$(Y,\sigma)\rightarrow(Z,\mu)$ is a contra- closed function . if (Y,σ) is a locally indiscrete, then gof:$(X,\tau)\rightarrow(Z,\mu)$ is contra- gp*- closed (and contra- pre gp*- closed) function.

***Proof*:**

Let A be a gp- closed set in (X,τ). Thus, f(A) is an open set in (Y,σ). Since (Y,σ) is a locally indiscrete. Then f(A) is a closed set in (Y,σ), and since g is a contra- closed function. Then g(f(A)) is an open set in (Z,μ). But g(f(A))= gof(A).

Therefore, gof:$(X,\tau)\rightarrow(Z,\mu)$ is contra- gp*- closed function, and by using proposition (3-32) we get, gof:$(X,\tau)\rightarrow(Z,\mu)$ is contra- pre gp*- closed function.

Proposition (4-6):

If f: $(X,\tau)\rightarrow(Y,\sigma)$ is contra-gp*- closed function, g:$(Y,\sigma)\rightarrow(Z,\mu)$ is a contra-preclosed function and (Y,σ) is locally indiscrete, then gof:$(X,\tau)\rightarrow(Z,\mu)$ is contra-pre gp*- closed (and contra- gp**- closed) function.

***Proof*:**

Let A be a gp- closed set in(X,τ). Thus, f(A) is an open set in (Y,σ). Since (Y,σ) is a locally indiscrete. Then f(A) is a closed set in (Y,σ), and since g is a contra-preclosed function. Then g(f(A)) is a preopen set in Z. Since g(f(A))= gof(A).

Hence, gof:$(X,\tau)\rightarrow(Z,\mu)$ is contra-pre gp*- closed function. And by using proposition (3-30) we get, gof:$(X,\tau)\rightarrow(Z,\mu)$ is contra gp**- closed function.

Similarly, we prove the following propositions:

Preposition (4-7):

If f: $(X,\tau)\to(Y,\sigma)$ is contra-gp*- closed function , g:$(Y,\sigma)\to(Z,\mu)$ is a contra-gp- closed function and (Y,σ) is locally indiscrete. Then gof:$(X,\tau)\to(Z,\mu)$ is contra- gp**- closed function.

Proposition (4-8):

If f: $(X,\tau)\to(Y,\sigma)$, g:$(Y,\sigma)\to(Z,\mu)$ are contra-gp*- closed functions and let (Y,σ) is a locally indiscrete, Then gof:$(X,\tau)\to(Z,\mu)$ is (contra- gp*- closed, contra- pre gp*- closed and contra-gp**- closed) function respectively.

Remark (4-9):

If f: $(X,\tau)\to(Y,\sigma)$ is a preclosed function and

1-g:$(Y,\sigma)\to(Z,\mu)$ is a contra-gp- closed function. Then gof:$(X,\tau)\to(Z,\mu)$ is not contra-gp- closed function.

2- g:$(Y,\sigma)\to(Z,\mu)$ is a contra-pre gp*- closed function. Then gof:$(X,\tau)\to(Z,\mu)$ is not (contra- closed, contra- preclosed and contra-gp- closed) function respectively.

The following proposition and results give the condition to make Remark (4-9) it is true**:**

Proposition (4-10):

If f: $(X,\tau)\to(Y,\sigma)$ is a preclosed function , g:$(Y,\sigma)\to(Z,\mu)$ is a contra-gp- closed function and (Y,σ) is submaximal space. Then gof:$(X,\tau)\to(Z,\mu)$ is contra- gp- closed function.

***Proof*:**

Let A be a closed set in (X,τ). Thus, f(A) is a preclosed set in (Y,σ). Since (Y,σ) is a submaximal space and by using corollary (2-8) we get f(A) is a closed set in (Y,σ). Since g is contra-gp- closed function. Then g(f(A)) is a gp- open set in (Z,μ). Since g(f(A))= gof(A). Therefore, gof:$(X,\tau)\to(Z,\mu)$ is contra-gp- closed function.

Proposition (4-11):

If f:$(X,\tau)\to(Y,\sigma)$ is a preclosed function, g:$(Y,\sigma)\to(Z,\mu)$ is a contra- pre gp*- closed and (Z,μ) is submaximal space.

Then gof:$(X,\tau)\to(Z,\mu)$ is contra- closed function.

***Proof*:**

Let A be a closed set in (X,τ). Thus, f(A) is a preclosed set in (Y,σ) , and by remark (2-3) we get, f(A) is a gp- closed set in (Y,σ) .Since g is a contra-pre gp*- closed function. Then g(f(A)) is a preopen set in (Z,μ).

Since (Z,μ) is a is submaximal space and by theorem (2-7) we get, g(f(A)) is an open set in (Z,μ). Since, g(f(A))=gof(A). Hence, gof:$(X,\tau)\to(Z,\mu)$ is contra- closed function.

The following corollary has easy proof. Thus it is omitted.

Corollary (4-12):

If f: $(X,\tau)\to(Y,\sigma)$ is a preclosed function, g:$(Y,\sigma)\to(Z,\mu)$ is a contra- pre gp*-closed function, and (Z,μ) is submaximal space. Then gof:$(X,\tau)\to(Z,\mu)$ is

(i) contra-preclosed function.

(ii) contra-gp- closed function.

Remark (4-13):

If f: $(X ,\tau)\to(Y, \sigma)$ is contra-gp*- closed function and g:$(Y ,\sigma)\to(Z ,\mu)$ is pre gp- closed function. Then gof:$(X,\tau)\to(Z,\mu)$ is not necessarily gp- closed function.

The following proposition the condition to make remark (4-13) its true:

Proposition (4-14):

If f: $(X,\tau)\to(Y,\sigma)$ is contra-gp*-closed function, g:$(Y,\sigma)\to(Z,\mu)$ is pre gp-closed function, X is semi- pre $T_{1/2}$-space and (Y,σ) is a locally indiscrete gof:$(X,\tau)\to(Z,\mu)$ is gp- closed function.

***Proof*:**

Let A be a closed set in (X,τ). Since (X,τ) is semi - pre $T_{1/2}$- space and by remark (2-9) we get A is gp- closed wet in (X,τ). Thus f(A) is an open set in (Y,σ). Since (Y,σ) is locally indiscrete and by define (2-5) we obtain f(A) is closed set in (Y,σ), and by remark (2-3) we have f(A) is a preclosed set in (Y,σ). Since g is a pre gp- closed function. Then g(f(A)) is a gp- closed set in(Z,μ).But g(f(A))= gof(A).Hence, gof:$(X,\tau)\to(Z,\mu)$ is a gp- closed function.

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الخلاصة

في هذا البحث، سنقدم نوع جديد من الدوال– ضد المغلقة وتسمى دالة– ضد المغلقة– gp و سنستخدم هذه الدالة لتقديم ودراسة أنواع أخرى من الدوال ضد المغلقة– gp وتدعى (دالة– ضد المغلقة– gp^*، دالة– ضد المغلقة– gp^{**}، دالة– ضد الشبه المغلقة– gp و دالة– ضد الشبه المغلقة– gp^*) وإيجاد العلاقة فيما بينها، كذلك سوف نبرهن بعض خواص تلك الدوال.