Variable Order Linear Multi-Step Methods for Solving Stochastic Ordinary Differential Equations

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Abstract

In this paper, we will study and introduce the higher-order weak variable order methods for approximation the solution of functionals diffusion of Itô kind. Under appropriate regularity conditions, it is shown that variable order method allows a considerable increase in the weak order of convergence of a discrete time one step approximation method. Numerical method experiments indicate the efficiency of variable order based on higher-order weak scheme for stochastic ordinary differential equations with additive noise.

Keywords**:** Stochastic ordinary differential equations, Linear multi-step methods, Variable order methods.

Introduction

The development of numerical methods for stochastic ordinary differential equations (SODE's for short) has intensified over the past two decades. As indicated in Paradoux and Talay [1], Kloeden and Platen [2] and Newton [3], their theoretical and practical investigation become of increasing importance. A challenging task is the construction of efficient higher-order approximations for the simulation of functionals of Itô diffusions, for example their moments and Lyapunov exponents, [4].

SODE's are differential equations in which one or more of its terms are stochastic processes, and therefore will give solutions which are itself stochastic processes, [5]. Also, they are used in a wide range of applications, such as environmental modeling, engineering and biological modeling, [6], [7].

Now days, there is a great many of researchers, which deals with numerical methods for solving SODE's. Yet the gap between the well developed theory of SODE's and its application still wide in range and a crucial task in bridging this gap is the development ofan efficient numerical methods for solving SODE's, therefore in this connection one of the numerical methods is the stochasticlinear multi-step methods (which is abbreviated by SLMM's), which is one of the most important of development numerical methods used to give the optimal accuracy to the approximate solution, [2].

In addition, variable order methods provide a class of higher order weak approximation methods which are efficient in many cases; there are also important practical situations in variable order methods providing general and efficient class of algorithms for the higher order weak approximation of SODE's. However, further investigations are still required to develop variable order methods for SODE's that have some performances comparable to those already known methods for solving ordinary differential equations.

With this aim, in this paper we will establish global error expansions for higher-order weak Taylor scheme, and we shall then use these expansions to construct variable order methods based on these higher-order schemes. This will allow range of SODE's to be handled numerically. Also, in this paper, we will study the numerical solution of SODE's by using variable order method of higher order weak approximation.

Preliminaries

In this section, some fundamental and basic necessary concepts related to the theory of SODE's are given.

Definition , [8]:

A stochastic process W_t, $t \in [0, \infty)$ is said to be a Brownian motion or Wiener process if:

1. $P(W_0 = 0) = 1$, where P refers to the probability.

2. For $0 < t_0 < t_1 < \ldots < t_n$; the increments $W_{t_1} - W_{t_2}$, $W_{t_2} - W_{t_1}$, …, $W_{t_n} - W_{t_{n-1}}$ are independent.
3. For an arbitrary t and $h > 0$, $W_{t+h} - W_t$ has a Gaussian distribution with mean 0 and variance h.

Stochastic Ordinary Differential Equations, [8], [9]

Among one of the general forms of SODE's is the following:

$$dy_t = f(t, y_t)\, dt + g(t, y_t)\, dW_t,\quad y_{t_0} = y_0 \quad \text{...... (1)}$$

where $f : I\times R \to R$, $g : I\times R \to R$ be a Borel measurable functions, we call f the drift function and g the diffusion function, y_t is a stochastic process and W_t is the Wiener process.

A solution y_t of the SODE's (1) must also satisfy another form of equation when it is written s a stochastic integral equation of the form:

$$y_t = y_{t_0} + \int_{t_0}^{t} f(s,y_s)\, ds + \int_{t_0}^{t} g(s,y_s)\, dW_s \quad \text{........ (2)}$$

Remark , [9]:

The second integral given in the right hand of equation (2) cannot be defined in the direct meaning, where W_s is the Weiner process. The variance of the Weiner process satisfies $\text{var}(W_t) = t$, and so this is increased as time increased even thought on the mean stays at 0.

Convergence Criteria, [2]:

There are four commonly used concepts for studying the convergence of random sequences, which are:

i. Convergence with probability one

A sequence of random variables $\{x_n(\omega)\}$ is said to be converges with probability one (written as p-w.P.1 or w.P.1) to $x(\omega)$, if:

$$P(\{\omega \in \Omega : \lim_{n\to\infty} x_n(\omega) = x(\omega)\}) = 1$$

This is also called almost sure convergence.

ii. Mean-square convergence

A sequence of random variables $\{x_n(\omega)\}$, such that $E(x_n^2) < \infty$, for all n is said to be mean square converge to $x(\omega)$, if:

$$\lim_{n\to\infty} E(|x_n - x|^2) = 0$$

where E refers to the mathematical expectation.

iii. Convergence in probability

A sequence of random variables $\{x_n(\omega)\}$ is said to be converge in probability to $x(\omega)$, if:

$$P(\{\omega \in \Omega : |x_n(\omega) - x(\omega)| \geq \varepsilon\}) = 0,\ \forall\ \varepsilon > 0$$

iv. Convergence in distribution

A sequence of random variables $\{x_n(\omega)\}$ is said to be converge in distribution to a random variable $x(\omega)$, if:

$$\lim_{n\to\infty} F(x_n) = F(x)$$

where *F(x)* is known the continuous distribution function.

It is remarkable that, in the above four definitions of convergence, the random variables are defined on a common probability space (Ω, F, P), where Ω is the sample space F is the class of all subset of Ω , and *P is aprobability* function with domine Ω and counter domine in the interval [0,1].

Now two main types of convergence criterion may be considered, which are the strong and the weak convergence depending on the problem under consideration.

Strong convergence criterion,[10]

Consider the sample path of the Wiener process, i.e., W_t is given (and hence known), therefore, there is no randomness in the SDE and hence no randomness in X_T, The increments in the given Wiener process are then used to obtain the numerical approximation $Y(T)$. The expectation of the absolute error is defined as:

$$\varepsilon = E(|X_T - Y(T)|)$$

here, the Euclidean norm is used, X_T is the Itô process at time T, while $Y(T)$ is the approximation obtained by approximately integrating the DE in a sequence of time steps.

The numerical scheme is consistent if the approximation $Y(T)$ converge to T as Δt tends to zero. Therefore, a discrete time approximation $Y(T)$ with maximum time step size δ converges strongly to Y at time T if [4]:

$$\lim_{\delta\to 0} E(|X_T - Y(T)|) = 0 \quad \text{.............................. (3)}$$

A discrete time approximation Y^p converge strongly with order $p > 0$ at time t if there exists a positive constant C, which does not depend on δ, and $\delta_0 > 0$, such that:

$\varepsilon(\delta) = E(|X_T - Y(T)|) \leq C\Delta^p$
for each $\delta \in (0, \delta_0)$, [10].

Weak convergence criterion, [10]:

The numerical approximation $Y(T)$ is also a random variable, because $Y(T)$ is obtained using samples of Wiener process increments. The convergence in distribution is analyzed in terms of means g($X(T)$) of certain test functions $g(x)$.

The test function g is bounded, infinitely differentiable and the mean exist as $|X|$ tends to infinity. The numerical scheme is weak p^{th} order accurate if the error:

$\varepsilon = |E(g(X(T))) - E(g(Y(T)))|$
is of order Δt^{p}. Thus:

$|E(g(X(T))) - E(g(Y(T)))| \leq C\Delta t^p$

A discrete time approximation Y with maximum step size δ converges weakly to x at time T as $\delta \longrightarrow 0$ with respect to a class C of the test function $g : R^d \longrightarrow$ R, if we have:

$$\lim_{\delta\to 0}|E(g(X_T)) - E(g(Y(T)))| = 0, \text{ for } g \in C \quad \ldots\ldots\ldots\ldots (4)$$

A discrete approximation Y converges weakly with order $\beta > 0$ to X at time T as $\delta \longrightarrow 0$, if for each polynomial g, there exists a positive constant C, which does not depend on δ, and a finite number δ_0, such that:

$|E(g(X_T)) - E(g(Y(T)))| \leq C\Delta t^p$
for each $\delta \in (0, \delta_0)$, [10].

Stochastic Liner Multi-Step Methods, [11], [12], [13]:

By the following notations and definitions, we denote by | . | the Eucledian norm in R^n and || . || the corresponding matrix norm. The mean-square norm of a vector valued square integrable variable $Z \in L_2(\Omega, R^n)$ will be defined by:

$\|Z\|_{L_2} = (E|Z|^2)^{1/2}$

Let us denote by $C^{s-1,s}$ the class of all functions $V(t, y(t)) : J \times R^n \longrightarrow R^n$ having continuous partial derivatives up to order $s - 1$ with respect to the first variable and continuous partial derivatives of order s with respect to the second variable. Moreover, let C^k be the class of functions V satisfying a linear growth condition of the form:

$$|V(t, y)| \leq k(1 + |y|^2)^{1/2}, \ \forall\ t \in J, y \in R^n \quad \ldots\ldots (5)$$

Furthermore, we introduce the notation:

$$I_r^{t,t+h}(V) = \int_t^{t+h} V(s_j, y(s_j))dW_r(s_j) \quad \ldots (6)$$

where dW_0(s) = ds.

If $V = 1$, then the integral $I_1^{t,t+h}$ is the one Wiener process and the increment $\Delta W = W(t + h) - W(t)$ of the scalar Wiener process W.

The next lemma presents the order of the one stochastic integral.

Lemma , [12]:

If $V \in C^k$ is any function and for $t \in J$, $h > 0$, such that $t + h \in J$, then:

$$E(I_r^{t,t+h}(V)|F_t) = 0, \text{ if } r \neq 0 \quad \ldots\ldots\ldots\ldots (7)$$

$$\|E(I_r^{t,t+h}(V)|F_t)\|_{L_2} \leq \|I_r^{t,t+h}(V)\|_{L_2} = O(h^{i_1 + \frac{i_2}{2}}) \quad \ldots\ldots\ldots\ldots (8)$$

where i_1 is the number of zero indices r_{i_1} and i_2 is the number of non-zero indices r_{i_2}.

Now, we consider a stochastic linear k-step method for the approximation of the solution of the SODE given by equation (1), for $n = k, k + 1, \ldots, N, N \in$N; which takes the form:

$$\sum_{j=0}^{k}\alpha_j y_{n-j} = h\sum_{j=0}^{k}\beta_j f(t_{n-j}, y_{n-j}) + \sum_{j=0}^{k} G_j(t_{n-j}, y_{n-j}) I^{t_{n-j}, t_{n-j+1}} \quad \ldots\ldots\ldots (9)$$

and setting without loss of generality $\alpha_0 = 1$ and require the given initial and starting values $y_0, y_1, \ldots, y_{k-1} \in L_2(\Omega, R^n)$, is the space of all integrable functions defined from Ω to R,such that y_n is
F $_{t_n}$-measurable for $n = 0, 1, \ldots, k - 1$, [6].

As in the deterministic case, usually only $y_0 = y(t_0)$ is given by the stochastic initial value problem and the values $y_1, y_2, \ldots, y_{k-1}$ need to be computed numerically and this can be made by any suitable one-step method, where one has to be careful to achieve the desired accuracy. If $\beta_0 = 0$, then the Stochastic linear multi-Step method (SLMM for short) given by equation (9) is said to be explicit, otherwise it is implicit.

Variable Order Methods

Using the SLMM's in connection with variable order methods used for solving ODE's to derived a new approach for solving SODE's with more accurate results , in which this method will be reffered to as the variable method for solving SODE's:

Consider the SODE:

$$dy_t = f(t)\,dt + g(t)\,dW_t;\quad y_{t_0} = y_0 \quad \text{.............. (10)}$$

In this investigation, approximation are studied for expectations of functions of the solution, i.e., $E(g(y(T)))$, where g is a real-valued smooth function, that is, weak approximation. The weak error is defined as:

$$E(g(y(T)) - g(y(h))) \quad \text{.................................. (11)}$$

The primary goal of this investigation is to prove that the variable order method has a weak error expansion of the form:

$$E(g(y(T)) - g(y(h))) = a_1 h + a_2 h^2 + \ldots \quad \text{...... (12)}$$

where a_1, a_2, … are constants independent of h and by using several approximations $E(g(y(h_0)))$, $E(g(y(h_1)))$, $E(g(y(h_2)))$, …; with $h_0 > h_1 > h_2 > \ldots$; where h_0, h_1, h_2, … are the step sizes.

Now, to successively eliminate the terms in the error expansion, thereby producing approximations using methods of higher and higher order. The sequence of step sizes used was $h_j = h/2^j$; j = 0, 1, 2, …; where h is some starting step size. If a_1 in (12) is not zero, then the approximation scheme $E(g(Y(T)))$ is only of order h. To obtain approximations of order h^2, and we proceed as follows:

Find the weak error expansion using two different step sizes h_0 and h_1, such that $h_1 < h_0$, as follows:

$$\left.\begin{aligned} E(g(y(T)) - g(y(h_0))) &= a_1 h_0 + a_2 h_0^2 + a_3 h_0^3 + \ldots \\ E(g(y(T)) - g(y(h_1))) &= a_1 h_1 + a_2 h_1^2 + a_3 h_1^3 + \ldots \end{aligned}\right\} \quad \text{............................... (13)}$$

and upon subtracting h_0 times the second equation from h_1 times the first equation and solving for $E(g(y(T)))$, one may get:

$$E(g(y(T))) = \frac{h_1 E(g(y(h_0))) - h_0 E(g(y(h_1)))}{h_1 - h_0} - a_2 h_0 h_1 - a_3 h_0 h_1 (h_0 + h_1) - a_4 (h_0^2 + h_0 h_1 + h_1^2) - \ldots$$

$$= E(g(y(h_1))) + \frac{E(g(y(h_1))) - E(g(y(h_0)))}{\frac{h_0}{h_1} - 1} - a_2 h_0 h_1 - a_3 h_0 h_1 (h_0 + h_1) - a_4 (h_0^2 + h_1 h_2 + h_1^2) - \ldots$$

Thus , letting:

$$E_1(g(y(h_0))) = E(g(y(h_1))) + \frac{E(g(y(h_1))) - E(g(y(h_0)))}{\frac{h_0}{h_1} - 1}$$

which is an O(h_0^2) approximation to $E(g(Y(T)))$. Since $h_1 < h_0$ and any two pair h_j and h_{j+1} may be used in the above elimination process, one may see that in general:

$$E_1(g(y(h_j))) = E(g(y(h_{j+1}))) + \frac{E(g(Y(h_{j+1}))) - E(g(Y(h_j)))}{\frac{h_j}{h_{j+1}} - 1} \quad \text{................. (14)}$$

which is also an O(h_j^2) approximation to $E(g(Y(T)))$. Now, we have:

$$E(g(y(T))) = E_1(g(y(h_0))) - a_2 h_0 h_1 - a_3 h_0 h_1 (h_0 + h_1) - a_4 h_0 h_1 (h_0^2 + h_0 h_1 + h_1^2) - \ldots \quad \text{........ (15)}$$

and

$$E(g(y(T))) = E_1(g(y(h_1))) - a_2 h_1 h_2 - a_3 h_1 h_2 (h_1 + h_2) - a_4 h_1 h_1 (h_1^2 + h_1 h_2 + h_2^2) - \ldots \quad \text{........ (16)}$$

and upon eliminating the terms involving a_2, we obtain:

$$E(g(y(T))) = E_2(g(y(h_0))) + a_3 h_0 h_1 h_2 + a_4 h_0 h_1 h_2 (h_0 + h_1 + h_2) + \ldots$$

Where:

$$E_2(g(y(h_0))) = E_1(g(y(h_1))) + \frac{E_1(g(y(h_1))) - E_1(g(y(h_0)))}{\frac{h_0}{h_2} - 1}$$

which is an O(h_0^3) approximation to $E(g(Y(T)))$. More generally:

$$E_2(g(y(h_j))) = E_1(g(y(h_{j+1}))) + \frac{E_1(g(y(h_1))) - E_1(g(y(h_0)))}{\frac{h_0}{h_2} - 1} \quad \text{.................... (17)}$$

which is also an O(h_j^3) approximation to $E(g(Y(T)))$. Similarly, continuing in this manner, the following recursively sequence may be derived:

$$E_0(g(y(h_j))) = E(g(y(h_j))) \quad \text{........................... (18)}$$

$$E_n(g(y(h_j))) = E_{n-1}(g(y(h_{j+1}))) + \frac{E_{n-1}(g(y(h_{j+1}))) - E_{n-1}(g(y(h_j)))}{\frac{h_j}{h_{j+n}} - 1} \quad \text{......... (19)}$$

On the basis of the results for $E(g(y(h_j)))$ and $E_2(g(y(h_j)))$, it seems that $E_n(g(y(h_j)))$ provides an O(h_j^{n+1}) approximation to $E(g(Y(T)))$. This may be verified directly by following the evolution of the general term $a_n h^n$ in the error expansion, but is perhaps obtained more easily by the following alternative approach obtained from equations (18) and (19), which is given in the following table:

Level	**O(h_j)**	**O(h_j^2)**	**O(h_j^3)**	**O(h_j^4)**	
0	$E_0(g(y(h_0)))$				
1	$E_0(g(y(h_1)))$ →	$E_1(g(y(h_0)))$			
2	$E_0(g(y(h_2)))$ →	$E_1(g(y(h_1)))$ →	$E_2(g(y(h_0)))$		
3	$E_0(g(y(h_3)))$ →	$E_1(g(y(h_2)))$ →	$E_2(g(y(h_2)))$ →	$E_3(g(y(h_0)))$	…
⋮	⋮	⋮	⋮	⋮	⋱

Algorthim illustrate the procedure for solving SODE's using variable order method:

Algorthim

1. Input x_0, y_0 (initialcondition), $h_0, h_1 \ldots$;
   $h_j := \frac{h_j}{2^j}$,j:=0,1,2,…,(Step size).
1. find the numerical solution y_j with
   $h_j := \frac{h_j}{2^j}$, using euler method
   $$y_{t_{n+1}} = y_{t_n} + hf(y_{t_n}) + g(y_{t_n})(w_{n+1} - w_n).$$
2. Evatute $E(g(y(h_j)))$, j=0,1,…where g is any Polynomial.
3. Find
   $$E_0(g(y(h_j))) = E(g(y(h_j)))$$
   $$E_n(g(y(h_j))) = E_{n-1}(g(y(h_{j+1}))) + \frac{E_{n-1}(g(y(h_{j+1}))) - E_{n-1}(g(y(h_j)))}{\frac{h_j}{h_{j+n}} - 1}$$
   For n:=1,2,…

Now, two examples are given to illustrate the above scheme of variable order weak approximation:

Example, [14]:

Consider the SODE:

$$dy_t = y_t\,dt + \frac{1}{2} y_t\,dW_t$$

with the initial condition $y_0 = 1$, and for comparison purpose, the exact solution is given by:

$$y_t = y_0 \exp(0.875t + \frac{1}{2} W_t)$$

and using Euler's method (which is a one step explicit method) with step sizes $h_0 = 0.1$, $h_1 = 0.05$ and $h_2 = 0.025$,and is defind for all n=0,1,…,N;by:

$$y_{t_{n+1}} = y_{t_n} + hf(y_{t_n}) + g(y_{t_n})(w_{n+1} - w_n)$$

Therefore, using equations (18) and (19), we get tables (1)-(3) the approximate variable order method, exact results and the absolute error, respectively for the weak solution:

Table (1)
The approximate result for the weak solution using variable order method.

Level	O(h_j)	O(h_j^2)	O(h_j^3)	O(h_j^4)	
0	1.462				
1	1.472	1.483			
2	1.481	1.49	1.492		
3	1.51	1.54	1.558	1.791	…
⋮	⋮	⋮	⋮	⋮	⋱

Table (2)
The exact result for the weak solution using variable order method.

Level	O(h_j)	O(h_j^2)	O(h_j^3)	O(h_j^4)	
0	1.424				
1	1.419	1.414			
2	1.419	1.419	1.42		
3	1.439	1.46	1.474	1.482	…
⋮	⋮	⋮	⋮	⋮	⋱

Table (3)
The absolute error between the approximate and exact result for the weak solution using variable order.

Level	O(h_j)	O(h_j^2)	O(h_j^3)	O(h_j^4)	
0	0.038				
1	0.053	0.069			
2	0.062	0.071	0.072		
3	0.071	0.08	0.082	0.309	…
⋮	⋮	⋮	⋮	⋮	⋱

Example, [14]:

Consider the SODE given by:

$$dy_t = -(1+0.01\,y_t^2)(1-y_t^2)dt + 0.1(1-y_t^2)dW_t$$

with the initial condition $y_0 = 0$, and for comparison purpose, the exact solution is given in [5] by:

$$y_t = \frac{e^{-2t+0.2W_t}-1}{e^{-2t+0.2W_t}+1}$$

and using Euler's method with step sizes $h_0 = 0.1$, $h_1 = 0.05$ and $h_2 = 0.025$. Therefore, using equations (18) and (19), we get tables (1)-(3) the approximate variable order method, exact results and the absolute error, respectively for the weak solution:

Table (1)
The approximate result for the weak solution using variable order method.

Level	$O(h_j)$	$O(h_j^2)$	$O(h_j^3)$	$O(h_j^4)$	
0	1.003×10^{-4}				
1	9.975×10^{-5}	9.915×10^{-5}			
2	9.924×10^{-4}	9.872×10^{-5}	9.857×10^{-5}		
3	1.157×10^{-4}	1.335×10^{-4}	1.458×10^{-4}	1.676×10^{-4}	…
⋮	⋮	⋮	⋮	⋮	⋱

Table (2)
The exact result for the weak solution using variable order method.

Level	$O(h_j)$	$O(h_j^2)$	$O(h_j^3)$	$O(h_j^4)$	
0	9.894×10^{-5}				
1	9.89×10^{-5}	9.887×10^{-5}			
2	9.877×10^{-5}	9.863×10^{-5}	9.855×10^{-5}		
3	1.154×10^{-4}	1.334×10^{-4}	1.456×10^{-4}	1.526×10^{-4}	…
⋮	⋮	⋮	⋮	⋮	⋱

Table (3)
The absolute error between the approximate and exact result for the weak solution using variable order.

Level	$O(h_j)$	$O(h_j^2)$	$O(h_j^3)$	$O(h_j^4)$	
0	1.367×10^{-6}				
1	8.455×10^{-7}	3.2×10^{-7}			
2	4.682×10^{-7}	9×10^{-8}	2×10^{-8}		
3	3.163×10^{-7}	10×10^{-10}	2×10^{-7}	1.5×10^{-5}	…
⋮	⋮	⋮	⋮	⋮	⋱

From the present study, we make

Conclusion

1. Variable order method give very high accurate result in comparsion between the approximate result and exact result.
2. Other numerical method for solving SODE's with higher order may be used insteadof Euler method using variable order method.
3. Solution of non-Linear SODE's using variable order method give more accurate result than the solution of linear SODE's.

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الخلاصة

في هذا البحث، سنقوم بدراسة واستحداث طرائق ذات الرتب المتغيرة الضعيفة (weak variable order methods) ومن الرتب العليا لتقريب حل المعادلات التفاضلية التصادفية من نوع Itô. حيث قمنا تحت شروط مناسبة باثبات ان طرية الرتبة المتغيرة تؤدي غلى زيادة ملحوظة في الرتبة الضعيفة لتقارب حلول طرائق الخطوة الواحدة. حيث وضحت الامثلة العددية قيد الدرراسة دقة طريقة الرتبة المتغيرة المستندة على طرائق ذات رتب عليا ضعيفة للمعادلات التفاضلية التصادفية والتي تمتلك ضوضاء (additive noise).