Scheduling Critical Activities on Multi-objectives Stochastic Projects

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Abstract

In this paper, we consider the problem of stochastic multi-objectives project when some or all activities are interrupted and are on different allocations (i.e. are independent on each other). Some theories concerning multi-objectives problems are presented, and a new objective function is constructed). An approach has been built to schedule the critical activities, by constructing some expressions based on the project lateness costs due to the interruption activities, to obtain the corresponding critical cost index. A simple tested problem is presented.

Keywords: Operations Researchs, Project Network, Multi- objectives problems, Project scheduling, Stochastic activity duration, Mathematical programming.

Introduction

Recently, Projects planning and optimal timing, under uncertainty are extremely critical for many organizations, see [1]. Having an effective mathematical model will give project managers a significant tool for replanning projects in response events and outcomes, see [2]. As a result, the uncertainty associated with such risky projects should be reduced. No one factor can account for, or prevent, failure in a project. It is common practice for a project team to develop a comprehensive risk assessment and risk management plan. Identifying the most critical activities with regard to schedule risk is a problem faced by all project managers. The problem of identifying critical activities (CA) in a deterministic problems is well understood. Since a project could be delayed if these activities were not completed in the scheduled time, Standard Critical Path Method (CPM) analyses can be used to identify the longest path(s), known as the critical path(s), in an activity network. Multiple critical paths may exist, but all will be of equal length. A project’s critical tasks are those that lie along a critical path. These methods are described in many sources. For more details see [3].

Numerous papers have been written in identifying (CA) with stochastic activity durations. In [4], a recursive algorithm is developed for determining the CDF of the project distribution. In [5] & [6] bounds are obtained for the PDF, and developed the distribution for project network with exponentially distributed activity times. In [7] a Branch and-Bound algorithm is presented for solving a discrete version of continuous density functions, when activity times must be either normal or crashed (i.e., a binary state). Also, see [8].

Identifying critical activities in a stochastic project is difficult problems. Several methods had been proposed contain series draw backs which lead to identifying critical activities incorrectly, leaving project mangers without means to identify and rank the most probable sources of project delays, and with activities represent the best opportunities for successfully addressing schedule risk. The activities times (durations) are random variables. Activity starting and ending times, as well as activity slack times, are therefore random variables. A new direction for identifying critical path activities (CPA) in stochastic project (SP) is based on different philosophy, than in deterministic project (DP), where each critical activity must correspond to zero time slack activity, while such condition need not to be necessary in (SP). We immediately encounter difficulties developing concepts analogous to total slack and "critical" activities for stochastic project. Such concept is the critically index, defined as the probability that an activity will lie on a critical path. However, an activity may lie on a critical path without introducing risk of project delay.

Many real lives–problems, one is usually confronted with several objectives, which are in mutual conflict in which we cannot just formulate a model and leave it to an optimization expert to calculate an optimal solution. Many algorithms appeared in the literature have been designed to obtain solutions to decision problems which must accomplish with multiple objectives, where each algorithm has its own claim of power. Empirical tests have been reported in the literature various models and methodologies are frequently developed in the theoretical sense without addressing the practically of applying them in a real–world setting. Applications which use only illustrative data may also mislead the practitioner to believe that the model may be practical in a wider setting. From the managerial point of view, there is a need to investigate which method would be better in what situations.

Many of the recent works deals with the determination of efficient solutions set, and with their utilization in solving problems. An enormous researches effort in an area known as "Efficient Solution" is constructed "local efficiency" sets. A motivated some works are discussed in the context of "proper efficiency”, see [3], [9], & [10].

In this paper, a new approach is developed, by constructing different critical activities indices, depending on the objectives problem, consists of independent stochastic activities, in which some of these activities are interrupted for an uncertain amount of time. Also the initial processing time is considered to be known, however, the length of the interrupted and the final processing time are uncertain. Based on [11], who developed general expressions for determining activities late starting and ending time distributions, we built an approach for identifying and scheduling the critical activities, according to the type of the weighted cost per unit time, constructed by evaluating the total expected interruption, and slackness times, respectively, for an activity, and the total expected lateness, given that the total latest finishing time is greater than or equal to the given due date. Some theories and experimental results are presented to point out the simplicity of our approach. In case of multiojectives, a new philosophy for scheduling critical activities are considered by maximizing (or minimizing) the critical costs indices on each objective.

The Problem

We assume that activities $(i \in N)$ are not lexicographically ordered such that duration (t_i) can be simply denoted by:

$$t_i = t_i^n \equiv uninterrupted = estimated\ processing\ time$$

and if activity is interrupted, then it is simply given by:

$$t_i = t_i^p + t_i^r \quad \text{.......... (1)}$$

Where (t_i^p) is the processing time before the interruption and (t_i^r) is the processing time during the interruption.

Notations

If we define the following notations, see[11]

$\boldsymbol{p_i(t)} = the\ activity\ time\ probability\ density\ function\ PDF\ activity\ i$

$\boldsymbol{P_i(t)} = The\ activity\ time\ cumulative\ distribution\ function\ CDF$ activity i

$\boldsymbol{p_{E,i}(t)} = The\ earliest\ start\ time\ PDF\ for$ activity i

$\boldsymbol{p_{L,i}(t)} = The\ latest\ start\ time\ PDF\ for$ activity i

$\boldsymbol{P_{E,i}(t)} = The\ earliest\ start\ time\ CDF\ for$ activity i

$\boldsymbol{P_{L,i}(t)} = The\ latest\ start\ time\ CDF\ for$ activity i

$\boldsymbol{f_{E,i}(t)} = The\ earliest\ finish\ time\ PDF\ for$ activity i

$\boldsymbol{f_{L,i}(t)} = The\ latest\ finish\ time\ PDF\ for$ activity i

$\boldsymbol{F_{E,i}(t)} = The\ earliest\ finish\ time\ CDF\ for$ activity i

$\boldsymbol{F_{L,i}(t)} = The\ latest\ finish\ time\ CDF\ for$ activity i

By assuming the early start schedule distributions to be continuous distributions, we can define the following:

$$P_{E,i}(t) = \prod_j F_{E,j}(t) \quad \text{.......... (2)}$$

$$p_{E,i}(t) = \frac{dP_{E,i}(t)}{dt} \quad \text{.......... (3)}$$

$$F_{E,i}(t) = \int_0^t P_i(t - t_1) p_{E,i}(t_1) dt_1 \quad \text{.......... (4)}$$

$$f_{E,i}(t) = \frac{dF_{E,i}(t)}{dt} \quad \text{.......... (5)}$$

$$P_{L,i}(t) = \int_{t_1=0}^{t} \int_{t_2=t}^{\infty} p_i(t_2 - t_1) f_{L,i}(t_2) dt_2 dt_1 \quad \text{.......... (6)}$$

$$p_{L,i}(t) = \frac{dP_{L,i}(t)}{dt} \quad \text{.......... (7)}$$

$$F_{L,i}(t) = \prod_j \{1 - P_{L,i}(t)\} \quad \text{.......... (8)}$$

$$f_{L,i}(t) = \frac{dF_{L,i}(t)}{dt} \quad \text{.......... (9)}$$

The early starting and ending time distribution for every activity is determined by proceeding sequentially forward, using equations (2-5), while, by setting $F_{L,n} = F_{E,n}$ and processing sequentially backwards, beginning with activity n and terminating with activity 1, the late start schedule distributions can be calculated using equations (6-9).

In [11], the sources of schedule risk in a stochastic project network is identified, the general expression for determining an activity's late starting and ending time distributions are developed, to identify the critical activities using the activity critically index to those found using stochastic activity metrics.

The CDF for the total late (TL_i^k) distribution for activity i due to k-th type of the interruption, is calculated, using the early and late of the lateness time distributions as follows:

$$\boldsymbol{TL_i^k} = \int_0^{\infty}\int_{-\infty}^{z_1+t} p_{E,i}^{K}(z_1)p_{L,i}^{k}(z_2)dz_2dz_1 \quad \text{..... (10)}$$

The early and late of the starting time distributions are used to calculate expected total late for activity i as follows:

$$E[TL_i^k] = \int_0^{\infty}\int_{-\infty}^{z_1+t}(z_2 - z_1)p_{E,i}^{K}(z_1)p_{L,i}^{k}(z_2)dz_2dz_1$$
$$\cong \sum_{z_1}\sum_{z_1}(z_2 - z_1)p_{E,i}^{K}(z_1)p_{L,i}^{k}(z_2) \quad \text{.......... (11)}$$

In this paper, we are constructing an expression for determining the total expected lateness, given that the total latest finishing time is greater than or equal to the known due date D, by considering the total interruption time, as follow, respectively:

$$E(L|F_{L,n} \geq D, TL_i^k) = \int_D^{\infty}(t - D)p_{L,n}^{k}(t|TL_i^k)dt$$
$$\cong \sum_{t_i \geq D}(t_i - D)p_{L,n}^{k}(t|TL_i^r) \quad \text{.................. (12)}$$

where $L = F_{L,n} - D$, represents project lateness.

Then, we can introduce the k-th type interrupted critical index of the activity j, which effect to the project result, as follow:

$$r_j^k = \frac{c_j^k}{E[TL_j^k]} * E(L|F_{L,n} \geq D, TL_j^k) \quad \text{............. (13)}$$

Where c_j^k is the k-th type interrupted costs of activity j per unit of time.

Multi-objectives decision making (MODM) procedures seek to obtain the "most preferred" of the feasible solutions across all the objectives which the decision maker wishes to optimize. Usually, no solution can be found which allows concurrent optimization of all objectives, because of the conflicting nature of the individual objective. Nearly, all the literatures {see [9], [12], [13] & [14] }, proposed the properties of different types of solution sets, by constructing a new objective function based on the linear combination of the original objective functions, whose coefficients are constants, denoted by (w_k) with

$(0 \leq w_k < 1)$ *and* $(\sum_{k=1}^{K} w_k = 1)$

In [1], a new approach for solving multi-objective problems is constructed, by interpolating multi-objectives functions with variable coefficients, in order to find more suitable values of these coefficients, by considering K optimum solutions points as the base points in constructing new weight coefficients as variables functions, denoted by $t_k(X)$ defined as;

$$w_k(t) = \prod_{k \neq 1}\left(\frac{t - t^{k*}}{t^{l*} - t^{k*}}\right) \quad \text{.......................... (14)}$$

and a new multi-objective functions problem can be formulated as following:

$$F(t) = \sum_{k=1}^{K} w_k(t)f_k(t) \quad \text{............................ (15)}$$

Subject to:

$g_i(t) \leq 0, \quad for\ i = 1,2,3,\dots m$

$t_k^l \leq w_k(t) \leq t_k^u$

Where $t = (t_1, t_2, \dots, t_n)^T$, ($t_k^l$ &t_k^u) are the given lower and upper bounds of the weight function { $f_k(t)$ }, and (t^{k*}) are the optimum points of { $f_k(t)$ }, which is unique vector and in practical problems, such vector is always unfeasible (otherwise there would be no conflicts), but it is conceivable that the nearest feasible solution could be an acceptable compromise for the decision maker.

In this paper, a new approach is developed for scheduling critical activities for multi-objective optimization problem, in which it can be stated as follows:

Find $X = (x_1, x_2, \dots, x_n)^T$, which;

$subject\ to$

$g_i(X) \leq 0, \quad for\ i = 1,2 \dots m$ .... (16)

Where X is the set of the project activities, and (14) can be formulated using (12) & (13).

Now, we need to state, the following basic definitions and theories:

Definition (1):

An activity ($X \in S$) is said to be local critical in S, if it is corresponding to the maximum k-th type interrupted critical cost index r_X^k..

Definition (2):

An activity ($X \in S'$) is said to be global critical (S'), if it is corresponding to the all types of the interrupted critical costs indices v(X).

Where

$v(j) = \max\{r_j^k\}$, for all types k. .... (17)

In order to have more demonstration about critical activities, we consider the feasible region Ω define as following

$\Omega=\{x \in R^n\colon\ g_i(x) \leq 0,\ i \in I\}$,

where $g_i(x) \leq 0$ is refer to the i-th activity constraint.

The region represented by all but the i-th constraint is given by

$\Omega_j= \{x \in R^n\colon\ g_i(x) \leq 0,\ \ i \in I\backslash\{j\}\}$,

where $I\backslash\{j\}$ is the set I with the element j removed.

The following definitions can be stated:

Definition (3):

The constraint $g_i(x) \leq 0$ is said to be not critical activity in the description of Ω if $\Omega= \Omega_j$, and otherwise is said to be critical.

Definition (4):

The constraint $g_i(x) \leq 0$ is weakly critical activity, if $0 \leq g_i(x) \leq \varepsilon$ for some +ve scalar.

The correct identification of critical activities constraints is important from both a theoretical and a practical point of view. Theoretically, the identification of the critical activities constraints is not difficult. However, as far as we are aware of, to date no technique can successfully identify all critical activities constraints. To overcome this, we are presenting the following definition.

Definition (5):

The projection $P_j(x_k)$ of the point x_k onto the hyperplane $H_j = \{x \in R^n\colon\ g_i(x) \leq 0\ \}$, is defined by

$P_j(x_k) = x_k + g_j(g_j^T(x_k)$.

Consequently, we have

$\| P_j(x_k) - x_k \|= (g_j(g_j^T(x_k)) = \mathrm{dis}(x_k, H_j)$.

In [16], a definition of local inactive nonlinear constraint is presented, which is of no use, in identifying whether the constraint is active or not, since local redundant constraint may be non-redundant in another local feasible region), Therefore, we prefer to define a weakly critical activity, which its existence is necessary to keep the hole feasible region of the problem unchanged. In doing so, we set δ>0, and define

$U_\delta(x) = \{y \in R^n\colon \| y - x \| \leq \delta\}$.

We are using the usual definition of the distance function dis(.,.) between the point and set of linear equations. Suppose that "ε" the infmum distance between the point x_k and the set of the constraints Ω at a local region, we can present the following definition.

Definition (6):

The activity $g_r(x) \leq 0$ is ***locally critical*** at x if there exist an open set $\delta(\varepsilon)$, *such that* $g_r(x) \in \Omega \cap \bar{\delta}(\varepsilon)$ & $\Omega_r \cap \bar{\delta}(\varepsilon) = \emptyset$, *where* $\bar{\delta}(\varepsilon)$ *is the closuer of* δ.

We can state the above definition into another way:

Definition (7):

The activity $g_r(x) \leq 0$ is weakly critical at x, if for some $(\varepsilon), \Omega \cap \bar{\delta}(\varepsilon) \neq \Omega_r \cap \bar{\delta}(\varepsilon)$. Otherwise, it is not critical.

The Approach

In an important related paper [3], literature review is presented on determining the criticality of activities in stochastic project networks, evaluating a number of approaches for assessing criticality and sensitivity. In several large construction projects ,it has been observed that firms have been earned significant returns by optimally managing the time- cost trade-of decisions needed to avoid penalties associated with delaying the successful completion of a project within due data. The overall importance of the stochastic project problem has been noted in much of the project management literatures [16].

In this paper, in order to obtain the more suitable efficient scheduling, $F_{L,n}$ is calculated, and from (13), the critical activities scheduling, corresponding to each k-th type of the interrupted critical indices $\boldsymbol{r_j^k}$ is identified. Then, new critical activities scheduling is constructed, corresponding to new global critical index defined in (17). In practical problems, it is conceivable that the nearest feasible scheduling could be an acceptable compromise for the decision maker, then a simple ranking of activities can be identified, whether are most likely to introduce a delay into a project.

Now the above approach can illustrated in the following steps:

Step 1: Calculate (2), (4), (6), (8), (10), (11), (12), (13) &(17).

Step 2: Set k=1,

Step 3: Set $\mathcal{E}(S, r_X^k) = \{1,2, \dots, n\}$,

Step 4: Choose the activity with maximum value of critical cost index r_X^k first, and dropped it from $\mathcal{E}(\, S, r_X^k)$,

Step 5: If $\mathcal{E}(\, S, r_X^k) \neq \emptyset$, then go to step 4,

Step 6: Set k=k+1. if k≤ K, go to step 3,

Step 7: Set $G(\, S', v) = \{\, 1,2, \dots, n\}$,

Step 8: Choose the activity j with maximum value of critical cost index $v(j)$ first, and dropped it from $G(\, S', v)$,

Step 9: If $G(\, S', v) \neq \emptyset$, then go to step 8,

Step 10: Stop.

Tested Problem

As we believe that, no theoretical difficulties are raised, we perform our approach, on a simple illustrative problem, taken from [11], and generated randomly, -to construct an extension one, by convention, tasks 1 & 9 have zero activity duration with probability 1.0. Therefore, any delay introduced into the project schedule will be introduced by one of the remaining tasks 2-8, independed on both their locations within the project network and their activity distributions extracted from the implementation this project several times, at different states, given in the table below:

Task	1	2	3	4	5	6	7	8	9
Distribution	Det	Beta	Beta	Beta	Beta	Beta	Beta	Beta	Det
Mean	-	35.0	43.0	52.0	19.0	20.0	32.0	40.0	-
SD	-	10.5	1-.3	18.4	8.0	12.0	15.4	8.9	-
r_j^1	-	0.44	0.37	0.59	0.63	0.61	0.65	0.73	-
r_j^2		0.33	0.31	0.65	0.66	0.58	0.62	0.71	-
r_j^3		0.61	0.58	0.68	0.75	0.64	0.77	0.81	
V(j)		0.61	0.58	0.68	0.75	0.64	0.77	0.81	

Given the project due date $D = 4\, months$, if we consider three objectives, the first one is to minimize the total stopping times TL_j^1, the second objective is to minimize the total slack times TL_j^2 and the third objective is to minimize the total lateness TL_j^3. Therefore, according to their expected total slacks times, the ranking of critical activities is 8, 5, 4, 7, 4, 2, 3. While, according to their expected total interruption times, the ranking of critical activities is 8, 7, 5, 4, 6, 2, 3.and according to their expected total lateness, the ranking of critical activities is 8, 7, 5, 4, 6, 2, 3. Now, performing our approach, to consider all these objectives, simultaneously, by using (15), the ranking of critical activities is 8, 7, 5, 4, 6, 2, 3.

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الخلاصة

تناول هذا البحث المشاريع التصادفية ذات الأهداف المتعددة عندما تكون هناك بعض أو كل الأنشطة ذات توقفات مفاجئه وفي أماكن مختلفة (بمعنى لاتعتمد على بعضها في التنفيذ) وقد تم تناولنا بعظاً من الجانب النظري لهذه المسائل. حيث تم بناء أسلوب لجدولة الأنشطة الحرجة من خلال تكوين بعض العلاقات المتبنية على كلف التأخير للمشروع بسبب العطلات المفاجئه للحصول على المؤشر الحرج لكل نشاط. وأخيراً تم عرض مثال بسيط لتوضيح المشكلة.