

## MTF Estimation for IR Optical Systems

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### Abstract

This paper deals with difficulty of estimating the modulation transfer function (MTF) of thermal IR (infrared) images, which almost content some of distortion. The distortion makes the features of the image to be embedded and cannot observe. This necessitates adopting a parameter work out through the frequency domain in order to appear the features needed to describe the image well. MTF is the most descriptive parameter used to evaluate both the contrast and resolution of the image, but it is characterized by its difficult estimation. The present work adopted two methods; spatial based method and frequency based method. The results of the two adopted methods were very similar. In order to ensure the correct results, the results of the two methods are compared with that extracted from Zemax software. The comparison shows great similarity in between, which indicates the correct path of the computation and accurate description of the proposed methods.

Keywords: IR optics, optical system design, OTF, and MTF.

### 1-Introduction

The resolution and performance of an optical system design can be characterized by MTF, which is a measurement of the system design ability to transfer contrast from the spatial domain into the frequency domain at a specific resolution. Computation of MTF is a mechanism that is often utilized by optical manufactures to incorporate resolution and contrast data into a single specification [1]. A plot of MTF against frequency  $f_s$  is shown in Fig. (1), when the optical system is free of aberration then the diffraction effect make to deviate the ideal image to be perfect. The existence of defocusing will damage the resultant image and then its MTF as Fig.(1) shows. MTF almost universally applicable measure of the performance of an image forming system, and can be applied not only to lenses but film, image tube, and even complete system such as camera carrying aircraft [2]. The adopted optical system is used usually in the tracing purpose [3], since it work out through the IR radiation of range (3-5)  $\mu m$ . The image consist of IR radiation has some distortion so it need to high accurate method to format the spot shape. The adopted optical system design (OSD) is designed to be a homing head optical system. In the present work, only spherical aberration and defocusing have been taken into account whereas other

aberrations have been neglected due to they were processed well in the designing stage, this make their effect is little on the image quality. The MTF behavior due to defocusing and aberration effects is presented in Fig.(1). The considered OSD is best as well as the MTF behavior approaches the perfect case. The representation of the adopted OSD and the mathematical treatments are carried out by the Zemax software, which can model, analysis, and assist in the design of the optical systems.

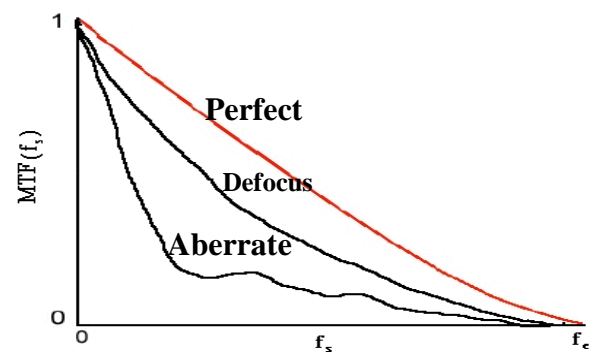


Fig.(1) MTFs of perfect, defocus, and aberrated systems.

### 2-Proposed Methods

MTF is one of the most important parameters determines the efficiency of the optical system. MTF is very sensitive to many different types of aberrations. It may be described by a set of points as a function of

frequency. A plot of this sort has a maximum value at zero frequency, which means that the OSD receives all the information of zero frequency. The relations refer that the maximum value of this curve is unity; therefore, it is a normalized curve. In other words, MTF values corresponding to a specific value of frequency in the MTF curve represents the percentage of the OSD to receive the information at that frequency. Therefore, the MTF value corresponding to a specific frequency indicates the amount of image contrast relative to the object contrast [4].

In order to determine the MTF, one must pass through OTF determination, because the MTF is the modulus of the resulting OTF. The OTF depends on the frequency of each point in the image. Because of the MTF represent the resolution of the image, it is important to compute this resolution accurately since the MTF is very sensitive for small amount of aberration in the optical system. Practically, MTF was computed by two different methods; the first depends on the mathematical relationship previously mentioned for the two cases of the perfect (no defocusing or aberration) and usual (with defocusing and aberration), while the second based on the fact that state the MTF is the modulus of the power spectrum of the PSF. In the following, a detailed explanation about each of them:

**2.1. Spatial Based Method**

The perfect OTF is real for all spatial frequencies, which means that there is no phase shift, in which the OTF and MTF are identical. This is the basic concept by which equation (1) is driven, so it can be used to find the MTF for the perfect system. With existence of aberration, the usual OTF is a complex function consisting of the product of a real modulus (MTF) and a factor having an imaginary exponential term; phase transfer function (PTF) [5].

$$OTF(f_s) = \left[ \frac{2}{\pi} \left( \arccos\left(\frac{f_s}{f_c}\right) - \frac{f_s}{f_c} \right) \sqrt{1 - \left(\frac{f_s}{f_c}\right)^2} \right] \dots\dots\dots(1)$$

where  $f_s$  is the spatial frequency and  $f_c = 1/\lambda(f/no)$  which is the value of the cutoff frequency. The OTF can be calculated

along the spatial frequency that expanded through the range from 0 into the cutoff frequency ( $f_c$ ). Therefore, it should be determined the cutoff value which equal to  $(1/\lambda(f/no))$  and then MTF can be taken as the modulus of equation (2).

$$OTF(f_s) = \iint \exp[-inkf_s \Delta W(x, y, f_s)] dx dy \dots\dots\dots(2)$$

Equation (2) can be written in the discrete form by the following [6]

$$OTF(f_s) = \sum_{y=0}^1 \sum_{x=0}^1 \exp(-inkf_s \Delta W(x, y, f_s)) \dots\dots(3)$$

Equation (3) can be used to estimate the MTF by determining the aberrated wavefront  $\Delta W(x, y; f_s)$ , which depends in turn on the defocusing coefficient  $C_{020}$  and spherical aberration coefficient  $C_{040}$  by the following relation.

$$\Delta W = C_{040}\rho^4 + C_{020}\rho^2 \dots\dots\dots(4)$$

where  $\rho = \sqrt{x_1^2 + y_1^2}$  which is the radial distance measured from the optical axis in the image plane. The computation of  $C_{020}$  and  $C_{040}$  is carried out by the ray tracing using the following relationship

$$C_{040} = \frac{1}{8} \sum_{i=1}^N -(n_i h_i c_i + n_i \alpha_i)^2 h_i \left( \frac{n_i \alpha_{i+1} + n_{i+1} \alpha_i}{n_i n_{i+1}} \right) \dots\dots\dots(5)$$

Where N is the number of surfaces,  $n_i$  is the  $i^{th}$  refractive index,  $h_i$  is the  $i^{th}$  height of the ray from the optical axis,  $\alpha_i$  is the  $i^{th}$  angle of incident,  $c_i$  is the  $i^{th}$  radius of curvature of the  $i^{th}$  surface. Also, the defocusing coefficient  $C_{020}$  associated with the spherical aberration can be concluded from the spherical aberration coefficient by the following relationship [6].

$$C_{020} = \frac{3}{2} C_{040} \dots\dots\dots(6)$$

**2.2. Frequency Based Method**

In this method, OTF can be derived from PSF which has been described as the image of an axial point object formed by diffraction limited system. The relation by which can determine the perfect OTF is given by equation (7).

$$OTF = |\text{FT}\{\text{PSF}\}|^2 \dots\dots\dots(7)$$

Therefore, one can employ equation (7) to determine the behavior of MTF. That means the determined PSF enable to determine MTF using the common relationship that relates them given in equation (7). The perfect PSF give perfect MTF, while the usual PSF belong to the adopted design give usual MTF which describe the resolution of the image consisting in the suggested design [7].

The blurred spot image of a single point is called the point spread function PSF of the optical system [8]. For a lens with a circular aperture of diameter  $D_o$ , the PSF is the zero-order Bessel function ( $J_o$ ) [9]

$$PSF = \frac{P}{\lambda^2 (f/ no)^2} \left| \int_0^1 P J_o \left[ \left( \frac{\pi r}{\lambda (f/ no)} \right) \cdot \rho \right] \rho \, d\rho \right|^2 \dots\dots\dots(8)$$

where

$$J_o(x) = \sum_{m=0}^{\infty} \frac{(-1)^m (x/2)^{2m}}{(m!)^2} \dots\dots\dots(9)$$

In perfect optical system, the PSF is the *Airy disc*, which is the Fraunhofer diffraction pattern for a circular pupil [10]. The mathematical expression that describes an actual wavefront as it passes through the exit pupil is called a pupil function. When this function is expressed in normalized coordinates of a usual optical system (both diffraction and aberration are exist), it is understood to have zero value outside the unit-radius circle.) [11].

$$P(x, y) = P_o e^{-ink\Delta W} \dots\dots\dots(10)$$

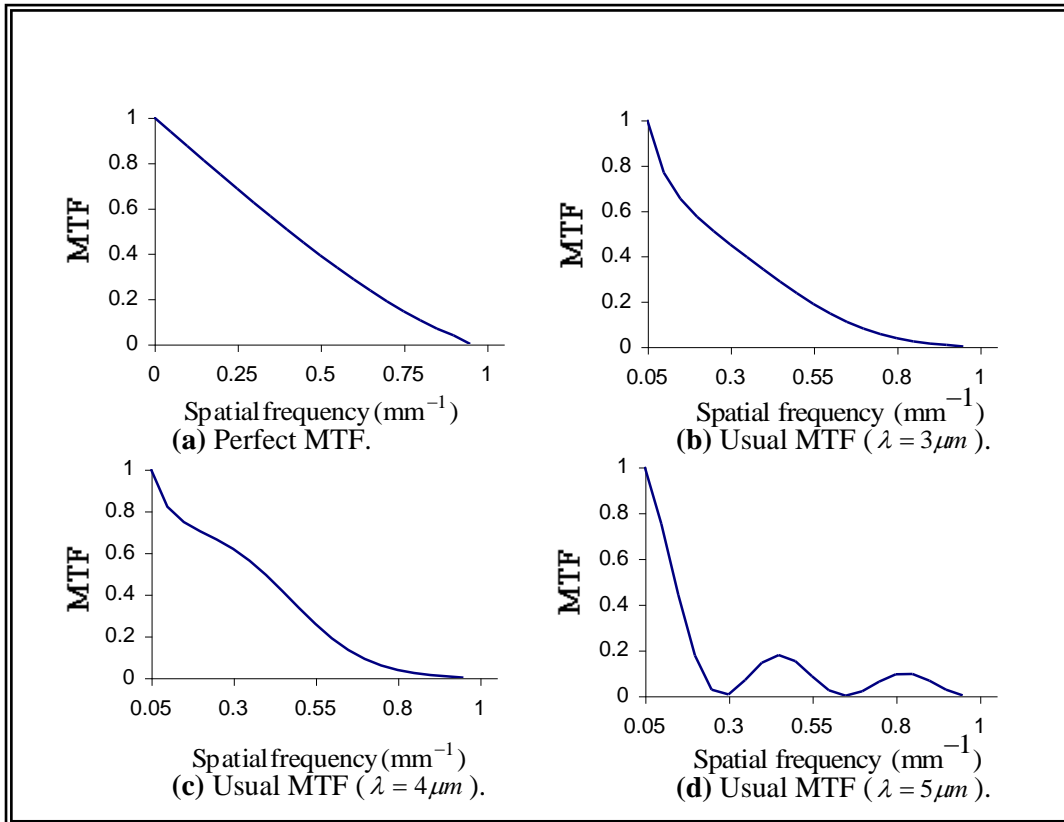
When there is no aberration  $C_{040}\rho^4 + C_{020}\rho^2 \approx 0$  then

$$P(x,y)=P_o \dots\dots\dots(11)$$

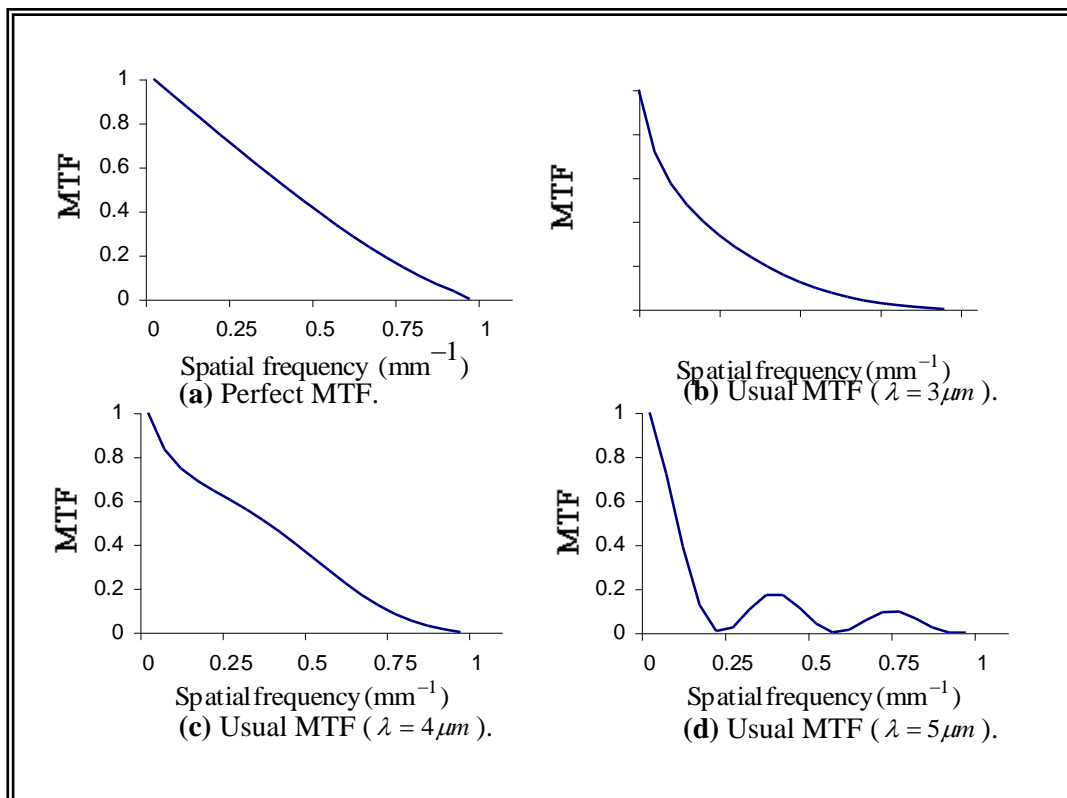
**3- Results and Discussion**

Practically both proposed methods are applied to estimate the MTF for two cases of perfect and usual. In order to measure the accuracy of the work, one can compare the behavior of the usual MTF with the perfect one. In fact, the adopted optical system was setting to show best image of less aberration and less associated defocusing at wavelength range of 3-5  $\mu m$ . The MTF of individual wavelength shows different behavior, so it expected that the amount of aberration and defocusing were also different. Fig.(2) shows the resulted MTF behaviors for the perfect and

usual (3, 4, and 5  $\mu m$  wavelength) cases that estimated by the spatial based method. It seen that the usual MTF includes greater decay due to the residual aberration in the optical system. The smooth behavior of the MTF curve refers to the uniformity of the energy distribution along the two dimensions of the image. In other words, it refers to less noise affect the image along the allowed frequency (i.e. 3-5  $\mu m$ ). It should be noticed that at  $\lambda = 4\mu m$  there is a middle protrusion in the MTF curve pushes it into outward. It thought that this protrusion (corresponding to the spread points of frequency  $0.45 \text{ mm}^{-1}$ ) is resulting from the residual aberration exist in the optical system, while the fast decay of the MTF curve is due to the defocusing constituted in the optical system. Fig.(3) show the MTF curves for both cases of the perfect and usual system. It is noticeable that there is a great similarity between the behavior of MTF in Fig.(2) and its corresponding MTF of Fig.(3), this ensure the correct estimation of MTF in both methods. Also, the MTF of 3  $\mu m$  wavelength shows fast decay in comparison with other wavelengths, which refers to appear high amount of defocusing and less aberration. Conversely, the MTF of 5  $\mu m$  wavelength shows some oscillation especially at the high frequency region, which refers to high aberration and less defocusing appears in such wavelength. Whereas the MTF of 5  $\mu m$  wavelength shows an equalized amounts of both defocusing and aberration. The comparison between Figs. (2) and (3) with their correspondent extracted by Zemax shown in Fig.(4) was carried out. One can find that both of the MTF curves belong to the adopted methods have similar behaviors comparing with their correspondent curves extracted by Zemax. The similarity of MTF behaviors ensures the correct path of the computation.



**Fig.(2) Perfect and usual MTF estimated by the spatial based method.**

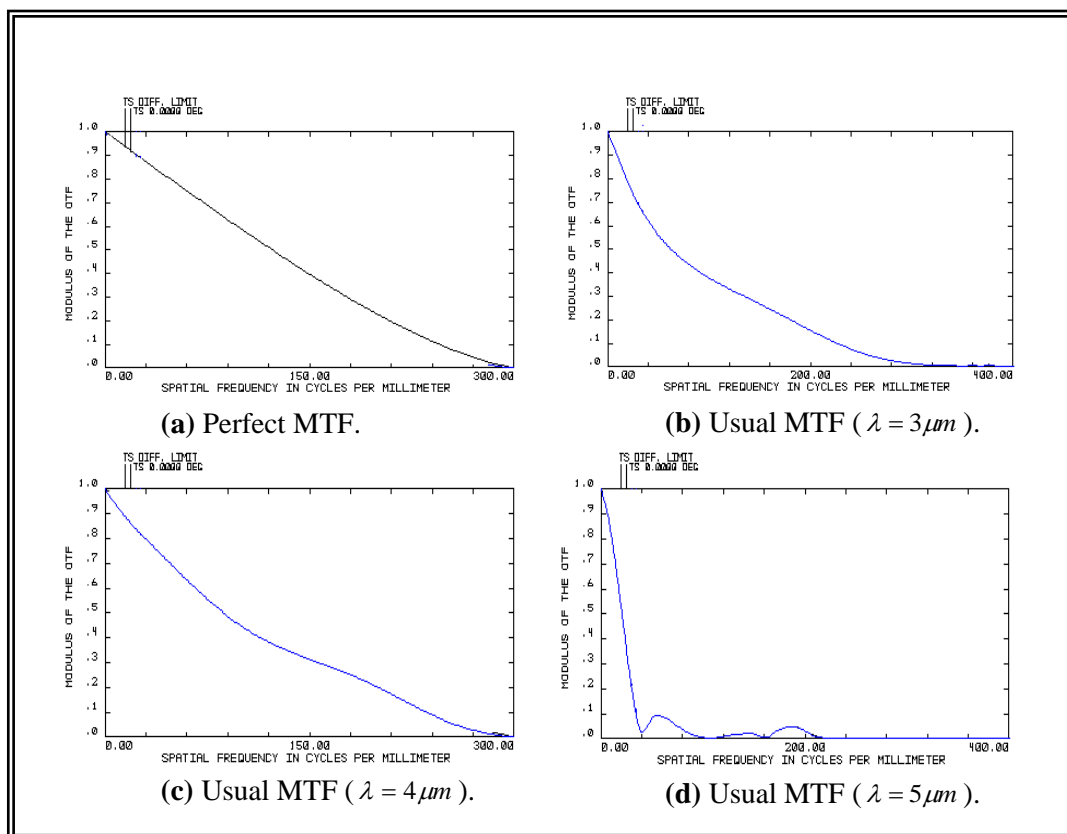


**Fig.(3) Perfect and usual MTF estimated by frequency based method.**

In addition, the MTF does not match zero value unless approaching a region is closing to cutoff frequency, which refers to include some information of high frequency band in the constituted image. The MTF plot show that the optical system operates as an exponential low-pass filter with a specific cutoff frequency. Therefore, one can say that the adopted system receive all the allowed frequencies in between zero and cutoff frequency with their natural rates and capabilities of the optical system. This points out to high resolution of the consisting image. High resulting resolution (i.e. good MTF) that characterize the consisting image ensure good quality of the image that appeared sharper features and higher contrast, and then refer to the good quality of the suggested OSD.

#### 4-Conclusions

The MTF is sensitive measure concerning the image quality in the frequency domain. MTF, it is very effective to the amount of image aberrations; it decays as the aberration increases. The amount of the decay is directly proportional to the contrast of the image. The cutoff frequency in the MTF curve varies with the resolution of the image; as the cutoff frequency increases the resolution increases. Within the vicinity of the cutoff frequency the image improves since the MTF curve approaches the ideal case.



*Fig.(4) Perfect and usual MTF extracted by Zemax.*

## 5-References

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## الخلاصة

يعالج هذا البحث الصعوبة في حساب دالة التضمين الانتقالي (MTF) لصور الأشعة تحت الحمراء والتي غالباً ما تكون مصحوبة بالتشويه الذي يجعل الصورة غائبة ولا يمكن رؤيتها بوضوح ولذلك كان من الطبيعي أن نستخدم عامل يعمل ضمن مجال التردد لكي يظهر الصفات المغمورة في الصورة و يستطيع أن يصف معلومات الصورة بشكل جيد. أستخدمنا أهم عوامل المجال الترددي والتي تمتاز بصعوبة حسابها. أستخدمت في هذا البحث طريقتين، طريقة الاعتماد المكاني وطريقة الاعتماد الترددي. أن المقارنة بين نتائج الطريقتين كانت متطابقة وتشابه كثيراً النتائج المستحصلة من برنامج زيمكس. وهذا يؤكد صحة مسار الحسابات وواقعية الطرق المستخدمة.