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# Calculation of Radiative Loss in Index Contrast of $Al_xGa_{1-x}As$ Waveguides

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## Abstracts

In this paper we used a theoretical and numerical investigation model to calculate the radiation losses, penetration depth and effective indices for $Al_xGa_{1-x}As$ planar optical waveguides at a wavelength of 1550 nm. Newton-Raphson method was used to find the radiation modes and its wavenumber. It was found that the change in the refractive index of $Al_xGa_{1-x}As$ optical waveguide is responsible of scattering effects and radiation towards the substrate.

Keywords: Radiation, Loss, Optical planar waveguides, refractive index.

## Introduction

Due to the limited control in waveguide fabrication the radiation losses naturally occur in most materials, and the energy does not remain in the core [1]. Lots of energy can flow either through the substrate or cladding region. One of reasons is due to physical discontinuities of the dielectric waveguide cause guided energy loss by radiation [2, 3].

Losses usually arise due to radiative scattering into the surrounding material and into backward-propagating modes due to the change in the refractive index of the core [4, 5].

There are several methods to calculate radiation modes such as the Fourier decomposition method (FDM), Perturbation calculus (PC) and the spectral decomposition method (SDM) [6, 7].

In this paper, we present a numerical model that gives insight into the physical processes involved in waveguide losses and which permits us to derive design guidelines for low-loss optical waveguides $Al_{0.20}Ga_{0.80}As$ / $Al_{0.61}Ga_{0.39}As$ [8-10].

The difficulty in the use of AlGaAs substrate is the propagation loss. While propagation loss due to scattering from imperfections as grown in the material and roughness introduced during fabrication process (such as sidewall roughness) have been minimized by superior growth and fabrication techniques [11, 12].

The different Al concentrations in the layers of $Al_xGa_{1-x}As$ cause grown little strain and this leads to change in the refractive index.

## Theory

The structure of a planar waveguide is displayed in Fig.(1). It consists of an $Al_{0.20}Ga_{0.80}As$ of thickness 15 μm and refractive index $n_{core}$ = 3.4516 surrounded by substrate an $Al_{0.61}Ga_{0.39}As$ and cladding with refractive indices $n_{sub.}$ =3.072 and $n_{cladd.}$ = 1 respectivelyand the operating wavelength is λ =1550 nm.

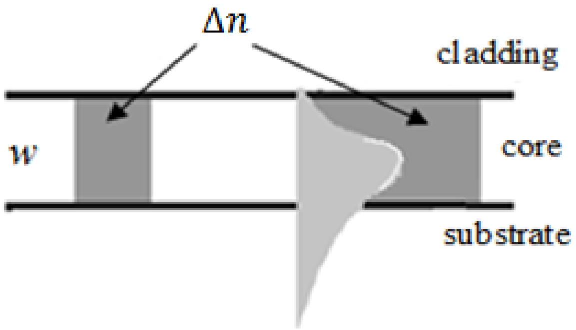

*Fig.(1) Basic geometry of the planar waveguide structure.*

Starting with the time harmonic ($e^{-i\omega t}$), from Maxwell's equations we obtain the Helmholtz's equation involving only the component of the electric field:

$$\frac{d^2E(x)}{dx^2} + [k_0^2 n^2(x) - \beta^2]E(x) = 0 \quad ..............(1)$$

Where $k_0$ is the wave number in the vacuum, $n$ is the refractive index of the medium and $\beta$ is the propagation constant in the *z*-direction. The electric field only has a *y*-component, as in the case of a TE mode. Equation 1 will have sinusoidal solutions in all layers of the waveguide when $0 \leq \beta \leq k_{sub}$ therefore,

$$E_y(x) = \begin{cases} A\,sin[\alpha\left(y+\frac{w}{2}\right)] + Bcos[\alpha\left(y+\frac{w}{2}\right)] & if\ y \leq w/2 \\ C\,sin(\gamma y) +\ D\,cos(\gamma y) & if\ -w/2 \leq y \leq w/2 \\ E\,sin[\alpha\left(y-\frac{w}{2}\right)] + Fcos[\alpha\left(y-\frac{w}{2}\right)] & if\ \ y \geq w/2 \end{cases} \qquad \text{....................................(2)}$$

Where *w* is the thickness of the waveguide's core, A, B, C, D, E, and F are constants describing field amplitude,$\alpha = \sqrt{\beta^2 - k_0^2 n_{sub}^2}$ and $\gamma = \sqrt{k_0^2 n_{core}^2 - \beta^2}$. The radiated energy in a waveguide due to the imperfection of the dielectric waveguide is calculated from a solution of the Helmholtz equation. It was represented by a change in the refractive index of the structure, we restrict to the change in refractive index, as shown in Fig.(1), assumed only the radiation modes that are able to carry energy away from the waveguide. In the presence of a transition due to an abrupt small refractive index change, with

$$\Delta n(x) = \Delta n_{max} h(x) \qquad \text{....................................(3)}$$

Where $\Delta n_{max}$ is the maximum change of the basic refractive index of the core, and $h(x)$indicates the shape of this change. The effective indices for $i^{\mathrm{th}}$ mode is$N_l$. Assuming that back-reflection can be neglected, the relative modal loss$\eta$ follows from the continuity of the fields at the transition [13]:

$$\eta \approx \frac{(\Delta n_{max})^2 \Delta N_o^2 - (\Delta N_o^2)^2}{(\Delta N_{o,rad}^2)^2} \qquad \text{....................................(4)}$$

With $\Delta N_o^2 \equiv N_{pert}^2 - N_0^2$

and $\Delta N_{o,rad}^2 \equiv N_0^2 - N_{rad}^2$

Where $N_{rad}$ is the effective index of the radiation modes and $N_{pert}$ is the effective index of the perturbed fundamental modal fields and $N_0$is the modal index. As shown in Fig.(2), the relative modal loss will increase by increasing the $\Delta n$ for a certain value.The total field is written as:

$$E_y = E_0 + E_r \qquad \text{................................................(5)}$$

Where $E_r$ is the excited field by the perturbation, and $E_0$ is the incoming field isgiven by:

$$E_0 = e_0(x)\exp(-i\beta z)$$

$$\frac{\partial^2 E_r}{\partial x^2} + \frac{\partial^2 E_r}{\partial z^2} + k_0^2 n^2(x) E_r + g(z)h(x)k_0^2 n^2 E_0 = 0 \qquad \text{....................................(6)}$$

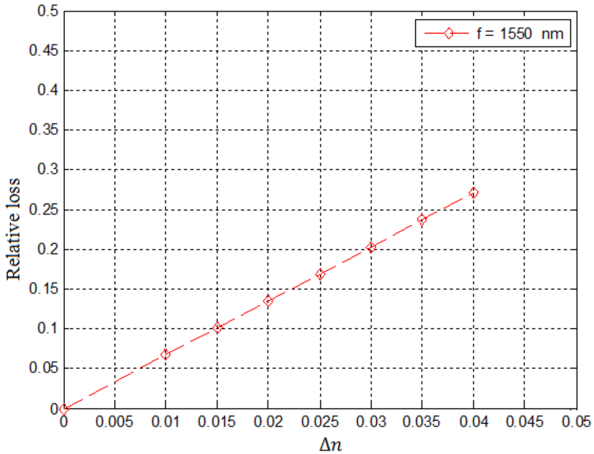

***Fig.(2) Relative loss as a function of Δn.***

For calculating the propagation constants of each mode, requires the use of numerical technique such as the Newton-Raphson method [1]. In Fig.(3) shows the radiation mode amplitude versus wave number in x-direction in the substrate.

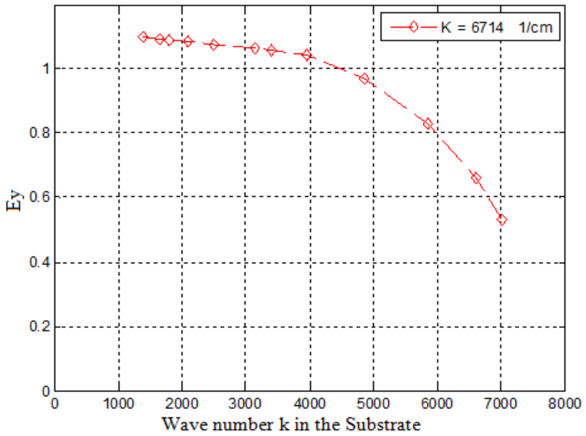

***Fig.(3) Mode amplitude in the substrate versus its wave number k.***

The effective width accounts for the amount to which the fields of the guided modes penetrate into the substrate and/or cladding [6]. The penetration depth, *d*, of a radiative mode into the substrateof the waveguide as shown in figure 4 and its value can be expressed:

$$d=\frac{\lambda}{2\pi}\frac{1}{\sqrt{{N_0}^2-n_{sub}^2}}\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(7)$$

Where $N_0$ is the effective index of the waveguide mode, it can be found by:$N_0=\frac{\beta}{k}$.

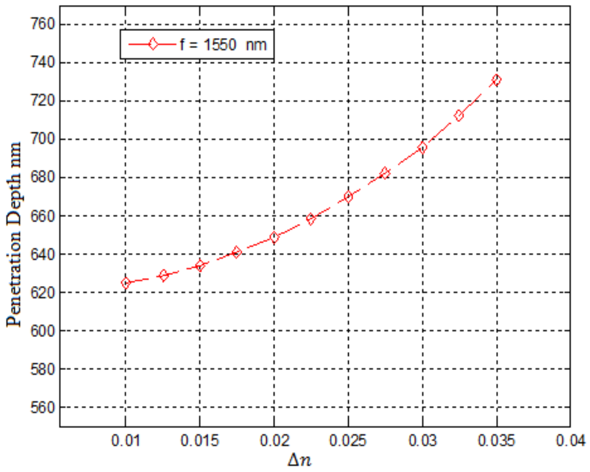

*Fig.(4) The Penetration depth nm of the TE$_0$ mode as a function of Δn.*

For the waveguide shown in Fig.(1), the mode spectra are illustrated in the form of classical mode curves where squares of modal effective indices $N_m^2$ are plotted versus the $(m+1)^2$ related to the mode order *m*. The values of the effective indices corresponding to the radative modes described by [14]:

$$N_m^2=n_{core}^2-\left(\frac{\lambda}{2w}\right)^2(m+1)^2\ ,\quad m=0,1,2,\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(8)$$

The effective refractive index, and thereforealso the penetration depth, is dependent on both the waveguide structure, and thewavelength and polarization of the propagating light as shown in Fig.(5), the value of the refractive index of the core is decreases as the composition ratio of Al increases.

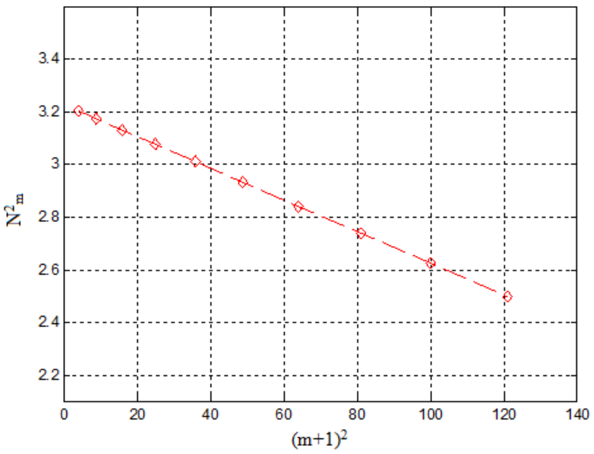

***Fig.(5) Effective indices mode curve of an Al$_{0.20}$Ga$_{0.80}$As core of the planar waveguide with width w=15μm, n$_{core}$ = 3.4516 and refractive index of the substrate n$_{sub}$ = 3.072 and the wavelength 1550 nm.***

## Conclusion

A numerical study has been performed to evaluate radiative loss arising from the change in refractive index of $Al_{0.20}Ga_{0.80}As$ towards the substrate. It has been shown that radiative loss is very sensitive to Δn. The amplitude of the radiation modes are calculated for different values of wave number in the substrate. The penetration depth, d, of a radiative mode into the substrate of the waveguide is increased by increasing Δn .Effective indices mode curve of an $Al_{0.20}Ga_{0.80}As$ core is inversely proportional to $(m+1)^2$.

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## الخلاصة

تم في هذا البحث تطبيق بعض الطرق النظرية و التحليل العددي لحساب فقدان الاشعاع، عمق الخرق و معاملات الانكسار لوجه موجة بصري $Al_xGa_{1-x}As$ لطول موجة 1550 nm لطول موجة لحساب انماط الاشعاع و العدد الموجي باستخدام طريقة نيوتن – رافسن. و وجد بان التغيير في معامل الانكسار لموجه الموجة البصري $Al_xGa_{1-x}As$ يكون مسؤولا عن تأثير التشتت والاشعاع باتجاه القاعدة.