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Weakly Prime and Weakly Semiprime Fuzzy Ideals

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Abstract

In this paper we introduce the notion of weakly prime (weakly semiprime) fuzzy ideals of fuzzy ring as a generalization of weakly prime (weakly semiprime) ideals. We investiagate several characterizations and properties of these concepts.

Introduction

The notion of fuzzy subset of a set $S \neq \phi$ is a function from a set S into [0,1], was first developed by Zadeh [1]. Liu in [2] introduced the concept of a fuzzy ring, Martines [3] introduced the notion of a fuzzy ideal of fuzzy ring.

D.D.Anderson and Eric Smith [4] introduced and studied the concept of weakly prime ideals, where a proper ideal P of a ring R is called weakly prime if $0 \neq ab \in P$ implies $a \in P$ or $b \in P$ [4] and P is called weakly semiprime ideal if for each $0 \neq x^2 \in R$ implies $x \in P$.

In this paper we introduce the notion of weakly prime (weakly semiprime) fuzzy ideals of fuzzy ring. We investigate some basic results about weakly prime and weakly semiprime fuzzy ideal of a fuzzy ring.

Throughout this paper we assume R to be commutative ring with identity 1 and for any fuzzy ring X over R, X(0)= 1.

S.1 Preliminaries

This section contains some definitions and properties of fuzzy subset, fuzzy ideals and fuzzy rings, which will be used in the next sections.

Let R be a commutative ring with identity. A fuzzy subset of R is a function from R into [0,1]. Let A and B be fuzzy subsets of R we write $A \subseteq B$ if $A(x) \leq B(x)$, for all $x \in R$, [1]. The set $\{x \in R, A(x) \geq t\}$ is called a level subset of R, [5]and denoted by A_t. If A and B are fuzzy subsets of R, then $\forall\, t \in [0,1]$
$A = B$ iff $A_t = B_t$, [1]
A_* denoted the set $\{x \in R; A(x) = A(0)\}$, [1], let $x_t: R \longrightarrow [0,1]$ be a fuzzy subset of R where $x \in R$, $t \in [0,1]$ defined by:
$x_t(y) = t$ if $x = y$ and $x_t(y) = 0$ if $x \neq y$, for each $y \in R$. x_t is called the fuzzy singleton. Let f be a mapping from a set M into a set N, let A and B be fuzzy subsets of M and N respectively. The image of A denotes by f (A) is the fuzzy subset of N defined by:

$$f(A)(y) = \begin{cases} \sup\{A(z) \mid z \in f^{-1}(y) \neq f, \text{if } f^{-1}(y) \neq f, \text{for all } y \in N\}, \\ 0 \qquad \text{otherwise} \end{cases}$$

and the inverse image of B, denoted by $f^{-1}(B)$ is the fuzzy subset of M, defined by:
$f^{-1}(B)(x) = B(f(x))$, for all $x \in M$, [1].

Let f be a function from a set M into a set N. A fuzzy subset of M is called f –invariant if $A(x) = A(y)$, whenever $f(x) = f(y)$, where $x, y \in M$, [6].

If A is a fuzzy subset of M and B is a fuzzy subset of N, then
1. $f^{-1}(f(A) = A$, whenever A is f –invariant, [6]
2. $f(f^{-1}(B)) = {}^{-}B$, [6].

Moreover the following definitions and properties are needed later

Definition 1.1 [2] :

Let X be a fuzzy subset of a ring R, X is called fuzzy ring of R if $X \neq \phi$ and for each $x, y \in R$:
1. X(0) = 1,
2. $X(x - y) \geq \min\{X(x), X(y)\}$,
3. $X(x\,y) \geq \max\{X(x), X(y)\}$.

Definition 1.2 [3] :

Let X be a fuzzy ring of a ring R, a fuzzy subset A of R is called a fuzzy ideal of X if
$A \subseteq X$
1. $A(x - y) \geq \min\{A(x), A(y)\}, \forall\, x, y \in R$,
2. $A(x\,y) \geq \min\{\max\{A(x), A(y)\}\ \forall x, y \in R$.

Proposition 1.3 [3]:

Let A and B be fuzzy ideals of a ring R, then $(AB)_t = A_t B_t$

Proposition 1.4 [3]:

A fuzzy subset X:R⟶[0,1] is a fuzzy ring iff X_t is a subring of R, for each $t \in [0,1]$.

Given a fuzzy ring X and a fuzzy set A: R⟶[0,1] with A ≠ O_1, then A is a fuzzy ideal of X iff A_t is an ideal of X_t, $\forall\, t \in [0,1]$, [3].

Definition 1.5 [7]:

A fuzzy ideal A of a fuzzy ring X is called a prime if A ≠ λ_R (where λ_R denotes the characteristic function of R such that $\lambda_R(x)=1$, $\forall x\in R$) and it satisfies:min{A(xy), X(x), X(y)}≤max{A(x),A(y)}, for all $x,y\in$ R.

Proposition 1.6 [7] :

Let A be a fuzzy ideal of fuzzy ring X, then A is a prime fuzzy ideal of X iff A_t is a prime ideal of X_t, $\forall\, t \in [0,1]$.

Remark 1.7 [8]:

Let a_t and b_k be two fuzzy singletons of a set S. If $a_t = b_k$, then $a = b$ and $t = k$, where $t, k \in [0,1]$.

Definition 1.8 [9]:

Let A be a fuzzy ideal of a ring R is called maximal ideal if:

1- A is not constant
2- For any fuzzy ideal B of R if A ⊆ B then either $A_* = B_*$ or B = λ_R.

S.2 Weakly Prime Fuzzy Ideals

In this section, we introduce the notion of weakly prime fuzzy ideal of fuzzy ring as a generalization of (ordinary) notion weakly prime ideal of a ring, where an ideal I of a ring R is called weakly prime if for each a, $b \in$ R such that $0 \neq a\cdot b\in I$, then $a\in$ I or $b\in$ I [4]. We shall give some properties of this concept.

Definition 2.1:

Let X be a fuzzy ring of a ring R. Let A be a fuzzy ideal of X, A is called a weakly prime fuzzy ideal of X if for each a_t, $b_s \subseteq$ X, $t,s \in (0,1]$, such that $O_1 \neq a_t \cdot b_s \subseteq X$ then $a_t \subseteq A$ or $b_s \subseteq A$, where

$$O_1(x) = \begin{cases} 1 & x = 1 \\ 0 & x \neq 1 \end{cases}$$

Proposition 2.2:

Let A be a fuzzy ideal of a fuzzy ring X of a ring R. If A is weakly prime fuzzy ideal of X, then A_t is a weakly prime ideal of X_t, $\forall\, t \in (0,1]$.

Proof:

Let $a, b \in X_t$ such that $0 \neq a \cdot b \in A_t$, hence $0_t \neq (a\ b)_t \subseteq A$, so $0_t \neq a_t \cdot b_t \subseteq A$ $\forall\, t \in (0,1]$. But A is weakly prime, $a_t \subseteq A$ or $b_t \subseteq A$. Hence $a \in A_t$ or $b \in A_t$. Thus A_t is weakly prime ideal of X_t, $\forall\, t \in (0, 1]$.

Remark 2.3:

The converse of the proposition 2.2 is not true in general as the following example shows:

Example:

Let X : Z_4 ⟶ [0,1] defined by: X(a) = 1, $\forall\, a \in Z_4$,

Let A : Z_4 ⟶ [0,1] defined by:

$$A(x) = \begin{cases} \frac{1}{2} & x \in \{\bar{0},\bar{2}\}, \\ 0 & \text{otherwise} \end{cases}$$

It is clear that A is a fuzzy ideal of fuzzy ring X and $A_t = \{\bar{0},\bar{2}\}$, $\forall\, t \in (0,1]$ is a prime ideal and hence is weakly prime. But A is not weakly prime fuzzy ideal since $\bar{2}_{\frac{1}{2}} \in A$ since $A(\bar{2}) = \frac{1}{2} \geq \frac{1}{2}$ also $\bar{2}_{\frac{1}{2}} \neq \bar{0}_{\frac{1}{2}}$ since $\bar{2} \neq \bar{0}$.

On the other hand,

$$2_{0.6} \cdot_4 \bar{3}_{\frac{1}{2}} = (\bar{2} \cdot_4 \bar{3})_l\,, \qquad \lambda = \min\{\tfrac{1}{2}, 0.6\}$$
$$= (\bar{2})_{\frac{1}{2}}$$

But $\bar{2}_{0.6} \notin A$ since $A(\bar{2}) = \frac{1}{2} \not\geq 0.6$,

$\bar{3}_{\frac{1}{2}} \notin A$ since $A(\bar{3}) = \bar{0} \not\geq \frac{1}{2}$

Proposition 2.4:

Let A be a weakly prime fuzzy ideal of a fuzzy ring X such that A is not a prime fuzzy ideal, then $A^2 = O_1$.

Proof:

Since A is weakly prime fuzzy ideal, then A_t is weakly prime ideal, $\forall\, t \in (0,1]$. Hence $(A_t)^2 = (0)$ by [4,Th.1]. But $(A_t)^2 = (A^2)_t$, so $(A)_t^2 = (0)$ for all $t \in (0,1]$. Thus $A^2 = O_1$.

Remark 2.5:

The converse of proposition 2.4 is not true in general as the following example illustrate:

Example:

Consider the same example of remark 2.3 $A_t = \{\bar{0}, \bar{2}\}$ and $(A_t)^2 = 0$, hence $A^2 = O_1$. But A is not weakly prime fuzzy ideal.

Recall that for a fuzzy ring X, then $\sqrt{O} = \cap\{P : P$ is a prime fuzzy ideal of X$\}$ $\left(\sqrt{O_1}\right)_t = \cap\{P_t : P$ is a prime fuzzy ideal of X$\}$, [4].

Corollary 2.6:

If A is weakly prime fuzzy ideal, then $A \subseteq \sqrt{O_1}$ or $\sqrt{O_1} \subseteq A$.

Definition 2.7 [10]:

Let A, B be two fuzzy submodule of fuzzy module X of an R-module M. The residual quotient of A and B denoted by (A:B) it is the fuzzy subset of R defined by:

$(A:B) = \{r_t : r_t$ fuzzy singleton of R s.t. $r_t B \subseteq A\}$.

$(A:B)(r) = \sup\{t \in (0,1] : r_t B \subseteq A\}$, if $B = (b_k)$, then $(A:b_k) = \{r_t : r_t\, b_k \subseteq A$, r_t is fuzzy singlton of R$\}$ and $< b_k > = \{r_s\, b_k ; r_s \in X\}$.

We next give characterizations of weakly prime fuzzy ideals.

Theorem 2.8:

Let A be a fuzzy ideal of a fuzzy ring X of R. Then the following are equivalent:

1- A is weakly prime fuzzy ideal of X.
2- For each $a_t \subseteq X - A$; $(A:x_t) = A \cup (0:x_t)$
3- For $x_t \subseteq X - A$; $(A:x_t) = A$ or $(A:x_t) = (0_t:x_t)$
4- For fuzzy ideals B and C of X with $O_1 \neq BC \subseteq A$, either $B \subseteq A$ or $C \subseteq A$.

Proof:

$(1 \Rightarrow 2)$ Let $y_s \in (A:x_t)$, where $x_t \in X - A$. Then $y_s \cdot x_t \subseteq A$. If $y_s \cdot x_t = O_1$, then $y_s \in (O_1 : x_t)$. If $O_1 \neq y_s\, x_t \subseteq A$, then $y_s \subseteq A$, since A is weakly prime fuzzy ideal (i.e.) $y_s \subseteq A$, so $(A : x_t) \subseteq A \cup (O_1 : x_t)$. On the other hand, $A \subseteq (A:x_t)$. Hence $A \cup (O_1:x_t) \subseteq (A:x_t)$.

$(2 \Rightarrow 3)$ it is clear, so it omitted

$(3 \Rightarrow 1)$ clear

$(1 \Rightarrow 4)$ If $O_1 \neq BC \subseteq A$, then for all $t \in (0,1]$ $(O_1)_t \neq (BC)_t \subseteq A_t$. Hence $O_t \neq B_t C_t \subseteq A_t$, for all $t \in (0,1]$.

But A is weakly prime fuzzy ideal, so A_t is weakly prime ideal (by prop. 2.1). Hence $B_t \in A_t$ or $C_t \in A_t$. Thus $B \subseteq A$ or $C \subseteq A$.

$(4 \Rightarrow 1)$ Suppose $x_t, y_s \subseteq X$ such that $O_1 \neq x_t \cdot y_s \subseteq A$, then $O_1 \neq < x_t >< y_s > \subseteq A$, so by (4) $< x_t > \subseteq A$ or $< y_s > \subseteq A$. Hence $x_t \subseteq A$ or $y_s \subseteq A$. Thus A is weakly prime fuzzy ideal.

Proposition 2.9:

Let A be a weakly prime fuzzy ideal of X such that is not prime, then $A\sqrt{O_1} = O_1$.

Proof:

Since A is weakly prime fuzzy ideal, then A_t is weakly prime ideal of X_t, for all $t \in (0,1]$. Hence $A_t \sqrt{O} = O$ by [4,Th.4], which implies $A_t \ (\sqrt{O_1})_t = (A\sqrt{O_1})_t = (O_1)_t = O$. Thus $A\sqrt{O_1} = O_1$.

Corollary 2.10:

If A and B are weakly prime fuzzy ideal of X such that are not prime, then $AB = O_1$.

Proof:

Since A and B are weakly prime, then A_t and B_t are weakly, for all $t \in (0,1]$. Hence $A_t \cdot B_t = O$ by [4,Th.4], so $(A\ B)_t = O$. Thus $AB = O_1$.

For the following result, we need the following lemma.

Lemma 2.11:

Let X be a fuzzy ring of R such that $X(a) = 1$, for all $a \in R$. If X is a fuzzy local ring (with unique fuzzy maximal ideal) then X_* is a local ring.

Proof:

Let A_1 and A_2 be two maximal ideals of X_*, now define $B_1 : R \longrightarrow [0,1]$ and $B_2 : R \longrightarrow [0,1]$ by:

$$B_1(x) = \begin{cases} 1 & \text{if } x \in A_1 \\ 0 & \text{otherwise} \end{cases} \quad \text{and}$$

$$B_2(x) = \begin{cases} 1 & \text{if } x \in A_2 \\ 0 & \text{otherwise} \end{cases}$$

It is clear that B_1 and B_2 are fuzzy ideals of X and $(B_1)_* = A_1$ and $(B_2)_* = A_2$. To prove B_1 and B_2 are fuzzy maximal ideals of X. If $B_1 \subset C \subset X$. Hence $(B_1)_* \subseteq C_* \subseteq X_*$, where C is a fuzzy ideal of X, which implies $(B_1)_* = C_*$

or $C_* \subseteq X_*$. But $C_* \subseteq X_*$ impossible, hence $(B_1)_* = C_*$. Thus $B_1 = C$. Thus B_1 is a fuzzy maximal ideal of X.

Similarly, B_2 is a fuzzy maximal ideal of X. But X is fuzzy local ring, so $B_1 = B_2$. Thus X_* is a local ring.

Theorem 2.12:

Let X be a fuzzy local ring (with unique maximal fuzzy ideal B) such that $B^2 = O_1$, then every proper ideal of X_* is weakly prime.

Proof:

Since X is local fuzzy ring so X has a unique maximal ideal say B, then X_* is local ring with unique maximal ideal say I. Hence $I = B_*$. Since $B^2 = O_1$, then $(B^2)_* = (O_1)_* = O$. Hence $(B_*)^2 = O$. Thus every proper ideal of X_* is weakly prime ideal by [4, Th.8].

Recall the following definition.

Definition 2.13 [11]:

Let X be a fuzzy ring of a ring R, let $a_t \in X$ a_t is called an irreducible of X, if $a_t = b_k\ c_s$ for some $b_k, c_s \in X$ implies either $< a_t > = < b_k >$ or $< a_t > = < c_s >$.

Lemma 2.14:

Let X be a fuzzy ring, let $a_t \subseteq X$, if a_t is an irreducible, then a is an irreducible in X_t, for all $t \in (0,1]$.

Proof:

Suppose $a = bc$ such that $b, c \in X_t$, so $a_t = (bc)_t$, for all $t \in [0,1]$, let $k, s \in [0,1]$ such that $t = \min\{k,s\}$ and $b_k \subseteq X$, $c_s \subseteq X$. Hence $a_t = b_k\ c_s$, but a_t is an irreducible, then $< a_t > = < b_k >$ or $< a_t > = < c_s >$, suppose $< a_t > = < b_k >$, since $a = bc$, then $<a> \subseteq < b >$, let $y \in < b >$. Thus $y = rb$, $r \in R$. $y_k = (rb)_k = r_k b_k \subseteq < b_k > = < a_t >$. Hence $y_k = a_t\ d_s = (ad)_\lambda$, where $\lambda = \min\{t,s\}$, $d_s \subseteq X$. Hence $k = \lambda$ and $y = ad$. This implies $y \in < a >$. Thus $<a>=<b>$. Similarly, if $< a_t > = < c_s >$.

Recall that x is called weakly prime element of a ring R if $< x >$ is weakly prime ideal, [4].

We fuzziy this definition as let $a_t \in X$, a_t is weakly prime element of X if $< a_t >$ is weakly prime fuzzy ideal of X, where X is a fuzzy ring of R.

Definition 2.15 [11]:

Let R be a ring with unity 1 and A be a fuzzy subring of R. If $1_r \subseteq A$ with $r \in [0,1]$. Then $x_t \subseteq A$, x_t is a non-zero fuzzy singleton with $t \in [0,1]$ is said to be left (right) unit fuzzy singleton in A if there exists $y_s \subseteq A$ with $s \in [0,1]$ such that $y_s \cdot x_t = 1_r$ $(x_t \cdot y_s = 1_r)$ where $r = \min\{t,s\}$. If x_t is a left and right unit, then x_t is called a unit in A. In the commutative fuzzy subring $y_s \cdot x_t = x_t \cdot y_s = 1_r$, where $r = \min\{t,s\}$.

Proposition 2.16:

Let x_t be a nonzero nonunit of X, then x_t is prime $\Rightarrow$ x_t is weakly prime $\Rightarrow$ x_t is irreducible.

Proof:

x_t is prime if $< x_t >$ is a prime fuzzy ideal, hence $< x_t >$ is weakly prime fuzzy ideal, so x_t is weakly element. Now, if x_t is weakly prime, to prove x_t is irreducible element. Suppose $O_1 \neq x_t = b_k\ c_s \in < x_t >$ since $< x_t >$ is weakly prime fuzzy ideal, so $b_k \in < x_t >$ or $c_s \in < x_t >$, if $b_k \in < x_t >$, then $< b_k > \subseteq < x_t >$. But $< x_t > \subseteq < b_k >$, thus $< b_k > = < x_t >$. Similarly, if $c_s \in < x_t >$. Thus $< c_s > \subseteq < x_t >$. Thus x_t is an irreducible element.

Now, we can give the following result.

Theorem 2.17:

Let A be a proper fuzzy ideal of a fuzzy ring X. If every nonzero element of A is an irreducible, then A is weakly prime.

Proof:

To prove A is weakly prime, let $a_t, b_k \subseteq X$ such that $O_1 \neq a_t \cdot b_k \subseteq A$, so $a_t\ b_k$ is an irreducible. Hence $< a_t\ b_k > = < a_t >$ or $< a_t\ b_k > = < b_k >$, hence $a_t \in < a_t\ b_k > \subseteq A$ or $b_k \in < a_t\ b_k > \subseteq A$. Then $a_t \subseteq A$ or $b_k \subseteq A$. Thus A is weakly prime fuzzy ideal.

S.3 Weakly Semiprime Fuzzy Ideals

In this section, we introduce the notion of weakly semiprime ideal of fuzzy ring as a generalization of (ordinary) notion weakly semiprime ideal, where an ideal I is called weakly semiprime ideal of R if for any $x \in R$ such that $0 \neq x^2 \in I$ implies $x \in A$.

We shall give main properties of this concept.

Definition 3.1:

Let A be a non constant fuzzy ideal of fuzzy irng X of R, A is called a weakly semiprime fuzzy ideal of X if for any fuzzy singleton x_t of R, $O_1 \neq x_t^2 \subseteq A$ implies $x_t \in A$.

Remark 3.2:

Every weakly prime fuzzy ideal is weakly semiprime, but the converse is not true in general for the following example shows:

Example:

Let X : Z ⟶ [0,1] defined by: X(a) = 1 for all $a \in Z$, let A : Z ⟶ [0,1] defined by

$$A(x) = \begin{cases} 1 & x \in 6Z, \\ 0 & \text{otherwise} \end{cases}$$

It is easy to show A is a fuzzy ideal of fuzzy ring X, $A_t = 6Z$ is semiprime ideal since $\sqrt{(6)} = (6)$, hence A is weakly semiprime. But A is not weakly prime because if A is weakly prime, then A_t is weakly prime, however A_t=6Z is not weakly prime since $0 \neq 2 \cdot 3 = 6 \in A_t$ such that $2 \notin A_t$ and $3 \notin A_t$.

Proposition 3.3

Let X be a fuzzy ring of R and A is a fuzzy ideal of X, then A is weakly semiprime of X iff A_t is weakly semiprime ideal of X_t for all $t \in (0,1]$.

Proof:

Let $O \neq a^2 \in A_t$, $a \in X_t$, hence $O_t \neq a_t^2 \subseteq A$. But A is weakly semiprime of X, so $O_t \neq a_t \in A$. Hence $a \in A_t$. Thus A_t is weakly semiprime ideal.

Conversely, to prove A is a weakly semiprime fuzzy ideal of X, let $O_t \neq a_t^2 \subseteq A$, hence $O \neq a^2 \in A_t$. Thus $a_t \in A$ since A_t is weakly semiprime. So $a_t \subseteq A$. Thus A is weakly semiprime fuzzy ideal.

Proposition 3.4:

If A is weakly semiprime fuzzy ideal of fuzzy ring X, then $A = \sqrt{A}$ or $A \subseteq \sqrt{O_1}$

Proof: Since A is weakly semiprime, then A_t is weakly semiprime ideal of X_t, for all $t \in (0,1]$, by prop.3.3. Hence $A_t = \sqrt{A_t}$ or $A_t = \sqrt{O}$. It is clear that $A_t = \left(\sqrt{A}\right)_t$, which implies $A = \sqrt{A}$. If $A_t = \sqrt{(O)} = L(R)$. Thus $A = \sqrt{O_t}$.

Proposition 3.5:

Let A be a fuzzy ideal of fuzzy ring X. Then A is weakly semiprime fuzzy ideal iff $A(x^2) > 0$ implies $A(x) = A(x^2)$, $x \in R$.

Proof:

To prove $A(x) = A(x^2)$, let $A(x^2) = t$, $t > 0$, hence $x^2 \in A_t$. Since A is weakly semiprime fuzzy ideal, then $x \in A_t$. Thus $A(x) \geq t$, but $t = A(x^2) = A(x)$. Hence $A(x)=t$, so $A(x^2)=A(x)$.

Converserly, to prove A is weakly semiprime, let $O_t \neq x_t^2 \subseteq A$, $t > 0$, hence $A(x^2) \geq t$. But $A(x^2) = A(x) \geq t$, so $A(x) \geq t$, hence $O_t \neq x_t \in A$. Thus A is weaklt semiprime fuzzy ideal.

Now, we shall study the behaviour of weakly semiprime fuzzy ideal under homomorphisms.

Proposition 3.6:

Let f:R_1⟶ R_2 be a ring epimorphism, let X and Y be two fuzzy rings of R_1, R_2 respectively and let A, B be fuzzy ideals of X, Y respectively, then

1- If A is weakly semiprime of X, then f (A) is weakly semiprime of Y. A is invariant .
2- If B is weakly semiprime fuzzy ideal of Y, then f^{-1}(B) is weakly semiprime of X.

Proof (1):

If $(f(A))(y^2) > 0$, for each $y \in R_2$, $y = f(x)$, for some $x \in R$ since f is onto.

Moreover $y \neq 0$, so $x \neq 0$.

$$\begin{aligned} f(A)(f(x))^2 &= f(A)(f(x^2)) \\ &= f^{-1}(f(A)(x^2)) \\ &= A(x^2)\text{, since A is } f\text{-invariant} \\ &= A(x)\text{,since A weakly semiprime} \\ &= f^{-1}(f(A)) \\ &= f(A)\, f(x). \end{aligned}$$

Thus f (A) is weakly semiprime fuzzy ideal of Y.

Proof (2):

To prove $f^{-1}(B)(x^2) = f^{-1}(B(x))$, $x \in R$, if $f^{-1}(B)(x^2) > 0$, hence

$$\begin{aligned} f^{-1}(B)(x^2) &= B(f(x^2)) \\ &= B(f(x))^2\text{, since } f \text{ homo.} \\ &= B(f(x))\text{, since B is weakly semiprime} \\ &= f^{-1}(B(x)). \end{aligned}$$

Thus f^{-1} (B) is weakly semiprime fuzzy ideal of X.

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الخلاصة

في هذا البحث قدمنا مفهوم المثاليات الأولية الضعيفة (شبه الاولية الضعيفة) الضبابية في حلقة ضبابية كتعميم لمفهوم المثاليات الأولية الضعيفة (شبه الاولية الضعيفة). ثم اعطينا العديد من التشخيصات والخواص الاساسية لهذان المفهومان.