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On Weak Soft N-Open Sets and Weak Soft $\tilde{D}_N$- Sets in Soft Topological Spaces

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Abstract

The main purpose of our work is to introduce a new general type of soft open sets in soft topological spaces, namely, soft N-open sets and we prove that the set of each soft N-open sets in a soft topological space $(X,\tilde{\tau},E)$ forms a soft topology $\tilde{\tau}_N$ on X which is soft finer than $\tilde{\tau}$. Furthermore we use soft N-open sets to define and study new classes of soft sets, namely, weak soft N-open sets and weak soft $\tilde{D}_N$- sets in soft topological spaces. Moreover we investigate the relation between the soft N-open sets and each of weak soft N-open sets and weak soft $\tilde{D}_N$- sets.

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Keywords: soft N-open set, soft α-N-open set, soft pre-N-open set, soft b-N-open set, soft β-N-open set, soft $\tilde{D}$-set, soft $\tilde{D}_N$-set, soft $\tilde{D}_{\alpha-N}$-set, soft $\tilde{D}_{pre-N}$-set, soft $\tilde{D}_{b-N}$-set, and soft $\tilde{D}_{\beta-N}$-set.

Introduction

The concept of soft set theory was first introduced by Molodtsov D. [7] in 1999. He successfully applied the soft set theory into many directions such as Theory of measurement, Riemann integration, Smoothness of functions, Game theory, Theory of Probability and Operations research, etc., Shabir M. and Naz M. [10] in 2011 introduced the notion of soft topological spaces. Akdag M. and Ozkan A. [1,2] in 2014, Chen B. [5] in 2013, Arockiarani I. and Arokia Lancy A. [3] in 2013 introduced and investigated soft b-open sets, soft α-open sets, soft semi-open sets, soft β-open sets and soft pre-open sets respectively. The purpose of this paper is to introduce a new class of soft open sets in soft topological spaces, namely, soft N-open sets and we prove that the collection of each soft N-open sets in a soft topological space $(X,\tilde{\tau},E)$ forms a soft topology $\tilde{\tau}_N$ on X which is soft finer than $\tilde{\tau}$. Also, we use these soft open sets to define and study new classes of soft sets, namely, weak soft N-open sets and weak soft $\tilde{D}_N$- sets in soft topological spaces. Moreover, we discuss the relation between soft N-open sets and each of weak soft N-open sets and weak soft $\tilde{D}_N$- sets.

1. Preliminaries:

If $P(X)$ is the power set of X and E is the set of parameters for X. Then:

Definition (1.1)[7]:

A soft set over X is a pair (G,B), where G is a function given by $G: B \to P(X)$ and B is a non-empty subset of E.

Definition (1.2)[9]:

If (G,B) is a soft set over X, then $\tilde{b}=(e,\{b\})$ is said to be a soft point of (G,B) if $e\in B$ and $b\in G(e)$, and is denoted by $\tilde{b}\tilde{\in}(G,B)$.

Definition (1.3):

A soft set (G,B) is said to be a finite soft set if the set $G(e)$ is finite $\forall e\in B$.

Definition (1.4)[10]:

If $\tilde{\tau}$ is a collection of soft sets over X. Then $\tilde{\tau}$ is said to be a soft topology on X if $\tilde{\tau}$ satisfies the following:

i) $\tilde{X}$, $\tilde{\phi}$ belong to $\tilde{\tau}$.

ii) If $(G_1,E),(G_2,E)\tilde{\in}\tilde{\tau}$, then $(G_1,E)\tilde{\cap}(G_2,E)\tilde{\in}\tilde{\tau}$.

iii) If $(G_\alpha,E)\tilde{\in}\tilde{\tau}, \forall\alpha\in\Lambda$, then $\bigcup_{\alpha\in\Lambda}(G_\alpha,E)\tilde{\in}\tilde{\tau}$.

The triple $(X,\tilde{\tau},E)$ is said to be a soft topological space over X. The members of $\tilde{\tau}$ are said to be soft open sets in $\tilde{X}$. The complement of a soft open set is said to be soft closed.

Definition (1.5)[4]:

If (G,E) is a soft subset of a soft topological space $(X,\tilde{\tau},E)$. Then:

i) $cl(G,E)=\tilde{\bigcap}\{(F,E):(F,E)$ is soft closed in $\tilde{X}$ and $(G,E)\tilde{\subseteq}(F,E)\}$ is called the soft closure of (G,E).

ii) $int(G,E)=\tilde{\bigcup}\{(O,E):(O,E)$ is soft open in $\tilde{X}$ and $(O,E)\tilde{\subseteq}(G,E)\}$ is called the soft interior of (G,E).

Theorem (1.6)[6]:

Let (G,E) be a soft subset of a soft topological space $(X,\tilde{\tau},E)$. Then $\tilde{x}\tilde{\in}cl(G,E)$ if and only if for each soft open set (O,E) containing $\tilde{x}$, $(G,E)\tilde{\bigcap}(O,E)\neq\tilde{\phi}$.

Definitions (1.7):

A soft subset (G,E) of a soft topological space $(X,\tilde{\tau},E)$ is called:

i) Soft α-open set [2] if $(G,E)\tilde{\subseteq}$ $int(cl(int(G,E)))$.

ii) Soft pre-open set [3] if $(G,E)\tilde{\subseteq}$ $int(cl(G,E))$.

iii) Soft semi-open (soft s-open) set [5] if $(G,E)\tilde{\subseteq}cl(int(G,E))$.

iv) Soft b-open set [1] if $(G,E)\tilde{\subseteq}$ $int(cl(G,E))\tilde{\bigcup}cl(int(G,E))$.

v) Soft β-open set [3] if $(G,E)\tilde{\subseteq}$ $cl(int(cl(G,E)))$.

2. Soft N-Open Sets

Definition (2.1):

A soft subset (G,E) of a soft topological space $(X,\tilde{\tau},E)$ is said to be soft N-open if for each $\tilde{x}\tilde{\in}(G,E)$, there exists $(U,E)\tilde{\in}\tilde{\tau}$ such that $\tilde{x}\tilde{\in}(U,E)$ and $(U,E)-(G,E)$ is a finite soft set. The complement of a soft N-open set is said to be soft N-closed. The collection of each soft N-open subsets of $(X,\tilde{\tau},E)$ is denoted by $\tilde{\tau}_N$.

Every soft open set is soft N-open, but the converse is not true in general as shown by the following example:

Example (2.2):

Let $X=\{a,b,c\}$, $E=\{e_1,e_2\}$, and $\tilde{\tau}=\{\tilde{X},\tilde{\phi}\}$ be a soft topology over X. Then $(G,E)=\{(e_1,G(e_1)),(e_2,G(e_2))\}=\{(e_1,\{a,b\}),(e_2,\{a,b\})\}$ is a soft N-open set, but is not soft open.

Theorem (2.3):

Let $(X,\tilde{\tau},E)$ be a soft topological space. Then the collection of each soft N-open subsets of $\tilde{X}$ forms a soft topology on X.

Proof:

i) Since $\tilde{\phi},\tilde{X}\tilde{\in}\tilde{\tau}\Rightarrow\tilde{\phi},\tilde{X}\in\tilde{\tau}_N$.

ii) Let $(G_1,E),(G_2,E)\tilde{\in}\tilde{\tau}_N$.

To prove that $(G_1,E)\tilde{\bigcap}(G_2,E)\tilde{\in}\tilde{\tau}_N$. Let $\tilde{x}\tilde{\in}(G_1,E)\tilde{\bigcap}(G_2,E)\Rightarrow\tilde{x}\tilde{\in}(G_1,E)$ and $\tilde{x}\tilde{\in}(G_2,E)$. Since (G_1,E) is soft N-open $\Rightarrow\exists(U_1,E)\tilde{\in}\tilde{\tau}$ such that $\tilde{x}\tilde{\in}(U_1,E)$ and $(U_1,E)-(G_1,E)$ is finite. Since (G_2,E) is soft N-open $\Rightarrow\exists(U_2,E)\tilde{\in}\tilde{\tau}$ such that $\tilde{x}\tilde{\in}(U_2,E)$ and $(U_2,E)-(G_2,E)$ is finite.

Since $\tilde{x}\tilde{\in}(U_1,E)$ and $\tilde{x}\tilde{\in}(U_2,E)\Rightarrow\tilde{x}\tilde{\in}(U_1,E)\tilde{\bigcap}(U_2,E)$. To prove that $((U_1,E)\tilde{\bigcap}(U_2,E))-((G_1,E)\tilde{\bigcap}(G_2,E))$ is finite.

$((U_1,E)\tilde{\bigcap}(U_2,E))-((G_1,E)\tilde{\bigcap}(G_2,E))$

$=((U_1,E)\tilde{\bigcap}(U_2,E))\tilde{\bigcap}((G_1,E)\tilde{\bigcap}(G_2,E))^c$

$=((U_1,E)\tilde{\bigcap}(U_2,E))\tilde{\bigcap}((G_1,E)^c\tilde{\bigcup}(G_2,E)^c)$

$=[((U_1,E)\tilde{\bigcap}(U_2,E))\tilde{\bigcap}(G_1,E)^c]\tilde{\bigcup}[((U_1,E)\tilde{\bigcap}(U_2,E))\tilde{\bigcap}(G_2,E)^c]$

$=[((U_1,E)\tilde{\bigcap}(U_2,E))-(G_1,E)]\tilde{\bigcup}[((U_1,E)\tilde{\bigcap}(U_2,E))-(G_2,E)]$

Since $((U_1,E)\tilde{\bigcap}(U_2,E))-(G_1,E)$ and $((U_1,E)\tilde{\bigcap}(U_2,E))-(G_2,E)$ are finite soft sets, then so is $[((U_1,E)\tilde{\bigcap}(U_2,E))-(G_1,E)]\tilde{\bigcup}[((U_1,E)\tilde{\bigcap}(U_2,E))-(G_2,E)]$. Hence $((U_1,E)\tilde{\bigcap}(U_2,E))-((G_1,E)\tilde{\bigcap}(G_2,E))$ is a finite soft set. Therefore $(G_1,E)\tilde{\bigcap}(G_2,E)\tilde{\in}\tilde{\tau}_N$.

(iii) Let $(G_\alpha, E) \tilde{\in} \tilde{\tau}_N, \forall \alpha \in \Lambda$. To prove that $\bigcup_{\alpha\in\Lambda}(G_\alpha, E) \tilde{\in} \tilde{\tau}_N$. Let $\tilde{x} \tilde{\in} \bigcup_{\alpha\in\Lambda}(G_\alpha, E) \Rightarrow \tilde{x} \tilde{\in} (G_{\alpha_0}, E)$ for some $\alpha_0 \in \Lambda$. Since $(G_{\alpha_0}, E) \tilde{\in} \tilde{\tau}_N \Rightarrow \exists\, (U,E) \tilde{\in} \tilde{\tau}$ such that $\tilde{x} \tilde{\in} (U,E)$ and $(U,E)-(G_{\alpha_0},E)$ is finite. Since $(G_{\alpha_0}, E) \tilde{\subseteq} \bigcup_{\alpha\in\Lambda}(G_\alpha, E) \Rightarrow (\bigcup_{\alpha\in\Lambda}(G_\alpha, E))^c \tilde{\subseteq} (G_{\alpha_0}, E)^c \Rightarrow (U,E) \tilde{\bigcap} (\bigcup_{\alpha\in\Lambda}(G_\alpha, E))^c \tilde{\subseteq} (U,E) \tilde{\bigcap} (G_{\alpha_0}, E)^c \Rightarrow (U,E) - (\bigcup_{\alpha\in\Lambda}(G_\alpha, E)) \tilde{\subseteq} (U,E)-(G_{\alpha_0},E)$. Since $(U,E)-(G_{\alpha_0},E)$ is a finite soft set, then so is $(U,E) - (\bigcup_{\alpha\in\Lambda}(G_\alpha, E)) \Rightarrow \bigcup_{\alpha\in\Lambda}(G_\alpha, E) \tilde{\in} \tilde{\tau}_N$ $\Rightarrow (X, \tilde{\tau}_N, E)$ is a soft topological space.

Definition (2.4):

If $(X, \tilde{\tau}, E)$ is a soft topological space and $(G,E) \tilde{\subseteq} \tilde{X}$. Then:

i) The soft N-closure of (G,E), denoted by $cl_N(G,E)$ is the intersection of each soft N- closed sets in $(X, \tilde{\tau}, E)$ which contains (G,E).

ii) The soft N-interior of (G,E), denoted by $int_N(G,E)$ is the union of each soft N-open sets in $(X, \tilde{\tau}, E)$ which are contained in (G,E).

Theorem (2.5):

If $(X, \tilde{\tau}, E)$ is a soft topological space and $(G,E),(H,E) \tilde{\subseteq} \tilde{X}$. Then:

i) $int(G,E) \tilde{\subseteq} int_N(G,E) \tilde{\subseteq} (G,E)$ and $(G,E) \tilde{\subseteq} cl_N(G,E) \tilde{\subseteq} cl(G,E)$.

ii) An arbitrary union of soft N-open sets is soft N-open.

iii) An arbitrary intersection of soft N-closed sets is soft N-closed.

iv) $int_N(G,E)$ is a soft N-open set in $\tilde{X}$ and $cl_N(G,E)$ is a soft N-closed set in $\tilde{X}$.

v) (G,E) is soft N-open iff $int_N(G,E) = (G,E)$ and (G,E) is soft N-closed iff $cl_N(G,E) = (G,E)$.

vi) $int_N(int_N(G,E)) = int_N(G,E)$ and $cl_N(cl_N(G,E)) = cl_N(G,E)$.

vii) $\tilde{X} - cl_N(G,E) = int_N(\tilde{X} - (G,E))$ and $\tilde{X} - int_N(G,E) = cl_N(\tilde{X} - (G,E))$.

viii) If $(G,E) \tilde{\subseteq} (H,E)$, then $int_N(G,E) \tilde{\subseteq} int_N(H,E)$ and $cl_N(G,E) \tilde{\subseteq} cl_N(H,E)$.

ix) $int_N((G,E) \tilde{\bigcap} (H,E)) = int_N(G,E) \tilde{\bigcap} int_N(H,E)$ and $cl_N((G,E) \tilde{\bigcup} (H,E)) = cl_N(G,E) \tilde{\bigcup} cl_N(H,E)$

x) $\tilde{x} \tilde{\in} int_N(G,E)$ iff there is a soft N-open set (V,E) in $\tilde{X}$ s.t $\tilde{x} \tilde{\in} (V,E) \tilde{\subseteq} (G,E)$.

xi) $\tilde{x} \tilde{\in} cl_N(G,E)$ iff for every soft N-open set (V,E) containing $\tilde{x}$, $(V,E) \tilde{\bigcap} (G,E) \neq \tilde{\phi}$.

xii) $\bigcup_{\alpha\in\Lambda} cl_N(G_\alpha,E) \tilde{\subseteq} cl_N(\bigcup_{\alpha\in\Lambda}(G_\alpha,E))$ and $\bigcup_{\alpha\in\Lambda} int_N(G_\alpha,E) \tilde{\subseteq} int_N(\bigcup_{\alpha\in\Lambda}(G_\alpha,E))$.

Proof:(ix)

Since $(G,E) \tilde{\bigcap} (H,E) \tilde{\subseteq} (G,E)$ and $(G,E) \tilde{\bigcap} (H,E) \tilde{\subseteq} (H,E)$, then by (viii), $int_N((G,E) \tilde{\bigcap} (H,E)) \tilde{\subseteq} int_N(G,E)$ and $int_N((G,E) \tilde{\bigcap} (H,E)) \tilde{\subseteq} int_N(H,E) \Rightarrow int_N((G,E) \tilde{\bigcap} (H,E)) \tilde{\subseteq} int_N(G,E) \tilde{\bigcap} int_N(H,E)$ To prove that $int_N(G,E) \tilde{\bigcap} int_N(H,E) \tilde{\subseteq} int_N((G,E) \tilde{\bigcap} (H,E))$. Since $int_N(G,E) \tilde{\subseteq} (G,E)$ and $int_N(H,E) \tilde{\subseteq} (H,E) \Rightarrow int_N(G,E) \tilde{\bigcap} int_N(H,E) \tilde{\subseteq} (G,E) \tilde{\bigcap} (H,E)$.

Since $int_N(G,E)$ and $int_N(H,E)$ are soft N-open sets in $\tilde{X}$, then so is $int_N(G,E) \tilde{\bigcap} int_N(H,E)$.

But $int_N((G,E) \tilde{\bigcap} (H,E))$ is the largest soft N-open set which contained in $(G,E) \tilde{\bigcap} (H,E)$, therefore $int_N(G,E) \tilde{\bigcap} int_N(H,E) \tilde{\subseteq} int_N((G,E) \tilde{\bigcap} (H,E))$.

Hence $int_N((G,E)\tilde{\cap}(H,E)) = int_N(G,E)\tilde{\cap}$ $int_N(H,E)$. From theorem (2.3) and definition (2.4), it is easy to prove other cases.

3. Weak Soft N-Open Sets

In this section we introduce and study new kinds of soft N-open sets called soft α-N-open sets, soft pre-N-open sets, soft b-N-open sets and soft β-N-open sets which are weaker than soft N-open sets.

Definitions (3.1):

A soft subset (G,E) of a soft topological space $(X,\tilde{\tau},E)$ is called:

i) soft α-N-open set if $(G,E)\tilde{\subseteq}$ $int_N(cl(int_N(G,E)))$

ii) soft pre-N-open set if $(G,E)\tilde{\subseteq}$ $int_N(cl(G,E))$.

iii) soft b-N-open set if $(G,E)\tilde{\subseteq}$ $int_N(cl(G,E))\tilde{\cup}cl(int_N(G,E))$.

iv) soft β-N-open set if $(G,E)\tilde{\subseteq}$ $cl(int_N(cl(G,E)))$.

Proposition (3.2):

If $(X,\tilde{\tau},E)$ is a soft topological space, then the following hold:

i) Each soft open (resp. soft α-open, soft pre-open, soft b-open, soft β-open) set is soft N-open (resp. soft α-N-open, soft pre-N-open, soft b-N-open, soft β-N-open) set.

ii) Each soft N-open set is soft α-N-open.

iii) Each soft α-N-open set is soft pre-N-open.

iv) Each soft pre-N-open set is soft b-N-open.

v) Each soft b-N-open set is soft β-N-open.

Proof:

(i) It is obvious.

(ii) If (G,E) is a soft N-open set, then $(G,E)=int_N(G,E)$. Since $(G,E)\tilde{\subseteq}cl(G,E)$, then $(G,E)\tilde{\subseteq}$ $cl(int_N(G,E))$ and $(G,E)\tilde{\subseteq}$ $int_N(cl(int_N(G,E)))$. Therefore (G,E) is soft α-N-open.

(iii) If (G,E) is a soft α-N-open set, then $(G,E)\tilde{\subseteq}int_N(cl(int_N(G,E)))\tilde{\subseteq}$ $int_N(cl(G,E))$. Therefore (G,E) is soft pre-N-open.

(iv) If (G,E) is soft pre-N-open set, then $(G,E)\tilde{\subseteq}int_N(cl(G,E))\tilde{\subseteq}int_N(cl(G,E))$ $\tilde{\cup}cl(int_N(G,E))$. Therefore (G,E) is soft b-N-open.

(v) If (G,E) is soft b-N-open set, then $(G,E)\tilde{\subseteq}int_N(cl(G,E))\tilde{\cup}cl(int_N(G,E))\tilde{\subseteq}$ $cl(int_N(cl(G,E)))\tilde{\cup}cl(int_N(cl(G,E)))$ $=cl(int_N(cl(G,E)))$. Therefore (G,E) is soft β-N-open.

The converse of proposition (3.2) may not be true in general as shown by the following examples:

Example (3.3):

Let $X=\{a,b,c\}$, $E=\{e_1,e_2\}$ and $\tilde{\tau}=\{\tilde{X},\tilde{\phi}\}$ be a soft topology over X. Then $(G,E)=\{(e_1,G(e_1)),(e_2,G(e_2))\}=\{(e_1,\{a,b\}),\ (e_2,\{a,b\})\}$ is a soft N-open (resp. soft α-N-open) set, but is not soft open (resp. is not soft α-open) set.

Example (3.4):

Let $X=N$, $E=\{e\}$ and $\tilde{\tau}=\{\tilde{X},\tilde{\phi},(U_1,E),(U_2,E),(U_3,E)\}$ be a soft topology over X, where $(U_1,E)=\{(e,\{1\})\}$, $(U_2,E)=\{(e,\{2\})\}$ and $(U_3,E)=\{(e,\{1,2\})\}$. Then $(G,E)=\{(e,N-\{1\})\}$ is a soft pre-N-open set, but is not soft pre-open.

Example (3.5):

Let $X=N$, $E=\{e\}$ and $\tilde{\tau}=\{\tilde{X},\tilde{\phi},(U_1,E),(U_2,E)\}$ be a soft topology over X, where $(U_1,E)=\{(e,\{1\})\}$ and $(U_2,E)=\{(e,\{1,2\})\}$. Then $(G,E)=\{(e,N-\{1\})\}$ is a soft b-N-open (resp. soft β-N-open) set, but is not soft b-open (resp. is not soft β-open).

Example (3.6):

Let $X=N$, $E=\{e_1,e_2\}$ and $\tilde{\tau}=\{\tilde{X},\tilde{\phi},(U,E)\}$ be a soft topology over X, where $(U,E)=\{(e_1,\{1\}),(e_2,\{1\})\}$. Then $(G,E)=\{(e_1,\{1,2\}),(e_2,\{1,2\})\}$ is a soft α-N-open set, but is not soft N-open.

Example (3.7):

Let $X = N$, $E = \{e_1, e_2\}$ and $\tilde{\tau} = \{\tilde{X}, \tilde{\phi}\}$ be a soft topology over X. Then $(G,E) = \{(e_1, \{1\}), (e_2, \{1,2\})\}$ is a soft pre-N-open set (since (G,E) is soft pre-open), but is not soft α-N-open.

Example (3.8):

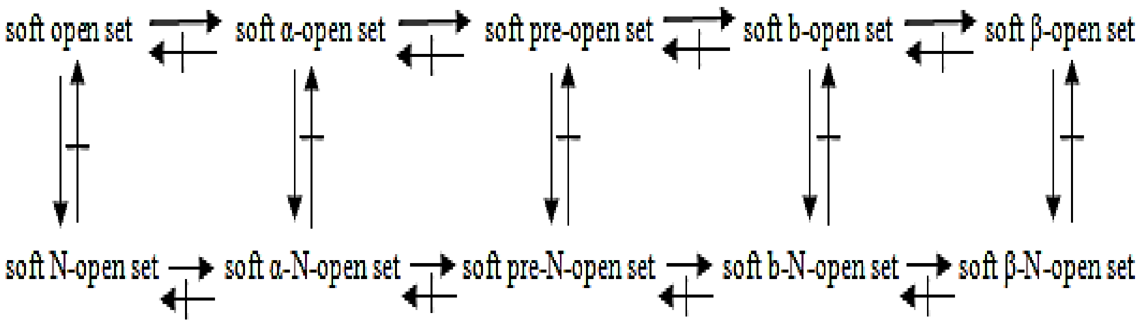
Let $(\Re, \tilde{\mu}, E)$ be the soft usual topological space, where $E = \{e\}$.

Then $(G,E) = \{(e, (0,1])\}$ is a soft b-N-open set (since (G,E) is soft b-open), but is not soft pre-N-open.

Example (3.9):

Let $(\Re, \tilde{\mu}, E)$ be the soft usual topological space, where $E = \{e\}$.Then (G,E) $= \{(e, Q \cap [0,1])\}$ is a soft β-N-open set (since (G,E) is soft β-open), but is not soft b-N- open.

The following diagram show the relation between the different types of soft open sets and types of weak soft N-open sets.



<u>Theorem (3.10) :</u>

If (G,E) is a soft pre-N-open subset of a soft topological space $(X, \tilde{\tau}, E)$ such that $(V,E) \tilde{\subseteq} (G,E) \tilde{\subseteq} cl(V,E)$ for each soft subset (V,E) of $\tilde{X}$, then (V,E) is also a soft pre-N-open set in $\tilde{X}$.

Proof:

Since $(G,E) \tilde{\subseteq} cl(V,E) \Rightarrow$
$cl(G,E) \tilde{\subseteq} cl(cl(V,E)) = cl(V,E) \Rightarrow$
$int_N(cl(G,E)) \tilde{\subseteq} int_N(cl(V,E))$. Since
$(G,E) \tilde{\subseteq} int_N(cl(G,E))$ and
$(V,E) \tilde{\subseteq} (G,E)$
$\Rightarrow$ $(V,E) \tilde{\subseteq} int_N(cl(V,E))$. Thus (V,E) is a soft pre-N-open set.

<u>Theorem (3.11):</u>

A soft subset (G,E) of a soft topological space $(X, \tilde{\tau}, E)$ is soft semi-open if and only if (G,E) is a soft β-N-open set and $int_N(cl(G,E)) \tilde{\subseteq} cl(int(G,E))$.

Proof:

Let (G,E) be soft semi-open, then $(G,E) \tilde{\subseteq} cl(int(G,E)) \tilde{\subseteq} cl(int_N(cl(G,E)))$ and hence (G,E) is a soft β-N-open set. Also, since $(G,E) \tilde{\subseteq} cl(int(G,E)) \Rightarrow cl(G,E) \tilde{\subseteq}$ $cl(int(G,E)) \Rightarrow int_N(cl(G,E)) \tilde{\subseteq} cl(int(G,E))$

Conversely, let (G,E) be a soft β-N-open set and $int_N(cl(G,E)) \tilde{\subseteq} cl(int(G,E))$. Then $(G,E) \tilde{\subseteq} cl(int_N(cl(G,E))) \tilde{\subseteq} cl(cl(int(G,E)))$ $= cl(int(G,E))$ and hence (G,E) is soft semi-open.

<u>Proposition (3.12) :</u>

Let $(X, \tilde{\tau}, E)$ be a soft topological space. If (V,E) is a soft open set in $\tilde{X}$, then $(V,E) \tilde{\cap} cl(G,E) \tilde{\subseteq} cl((V,E) \tilde{\cap} (G,E))$ for any soft subset (G,E) of $\tilde{X}$.

Proof:

Let $\tilde{x} \tilde{\in} (V,E) \tilde{\cap} cl(G,E)$ and (U,E) be any soft open set in $\tilde{X}$ such that $\tilde{x} \tilde{\in} (U,E)$. Since $\tilde{x} \tilde{\in} cl(G,E)$, then by theorem (1.6), $(U,E) \tilde{\cap} (G,E) \neq \tilde{\phi}$. Since $(U,E) \tilde{\cap} (V,E)$ is a soft open set in $\tilde{X}$ and $\tilde{x} \tilde{\in} (U,E) \tilde{\cap} (V,E)$,

then $((U,E)\tilde{\cap}(V,E))\tilde{\cap}(G,E)=$
$(U,E)\tilde{\cap}((V,E)\tilde{\cap}(G,E))\neq\tilde{\phi}$.
Therefore $\tilde{x}\tilde{\in}cl((V,E)\tilde{\cap}(G,E))$.
Thus $(V,E)\tilde{\cap}cl(G,E)\tilde{\subseteq}cl((V,E)\tilde{\cap}(G,E))$ for any soft subset (G,E) of $\tilde{X}$.

Propositions (3.13):
Let $(X,\tilde{\tau},E)$ be a soft topological space, then:

i) The intersection of a soft α-N-open set and a soft open set is soft α-N-open.
ii) The intersection of a soft pre-N-open set and a soft open set is soft pre-N-open.
iii) The intersection of a soft b-N-open set and a soft open set is soft b-N-open.
iv) The intersection of a soft β-N-open set and a soft open set is soft β-N-open.

Proof:
i) Let (G,E) be a soft α-N-open set and (V,E) be a soft open set in $\tilde{X}$. Since every soft open set is soft N-open, then $(G,E)\tilde{\subseteq}$ $int_N(cl(int_N(G,E)))$ and $(V,E)=int_N(V,E))$.
By proposition (3.12), we have $(V,E)\tilde{\cap}(G,E)\tilde{\subseteq}int_N(V,E)\tilde{\cap}$ $int_N(cl(int_N(G,E)))$.
$=int_N((V,E)\tilde{\cap}cl(int_N(G,E)))$.
$\tilde{\subseteq}int_N(cl((V,E)\tilde{\cap}int_N(G,E)))$.
$=int_N(cl(int_N((V,E)\tilde{\cap}(G,E))))$.
Therefore $(G,E)\tilde{\cap}(V,E)$ is a soft α-N-open set. Similarly, we can prove the other cases.

Remark (3.14):
The intersection of two soft pre-N-open (resp. soft α-N-open, soft b-N-open, soft β-N-open) sets need not be soft pre-N-open (resp. soft α-N-open, soft b-N-open, soft β-N-open) as shown by the following examples:

Example (3.15):
Let $(\Re,\tilde{\mu},E)$ be the soft usual topological space, where $E=\{e\}$. Then $(G_1,E)=\{(e,Q)\}$ and $(G_2,E)=\{(e,Q^c\cup\{1\})\}$ are soft pre-N-open sets, but $(G_1,E)\tilde{\cap}(G_2,E)=(G,E)=\{(e,\{1\})\}$ is not soft β-N-open, since $(G,E)\tilde{\not\subset}$ $cl(int_N(cl(G,E)))=cl(int_N(G,E))=cl(\{\tilde{\phi}\})$ $=\tilde{\phi}$.

Example (3.16):
Let $X=N$, $E=\{e\}$ and $\tilde{\tau}=\{\tilde{X},\tilde{\phi},(U_1,E),(U_2,E),(U_3,E)\}$ be a soft topology over X, where $(U_1,E)=\{(e,\{1\})\}$, $(U_2,E)=\{(e,\{2\})\}$ and $(U_3,E)=\{(e,\{1,2\})\}$.
Then $(G_1,E)=\{(e,N-\{1\})\}$ and $(G_2,E)=\{(e,\{1,3\})\}$ are soft α-N-open sets, but $(G_1,E)\tilde{\cap}(G_2,E)=(G,E)=\{(e,\{3\})\}$ is not soft α-N-open, since $(G,E)\tilde{\not\subset}$ $int_N(cl(int_N(G,E)))=int_N(cl(\tilde{\phi}))=\tilde{\phi}$.

Theorem (3.17):
If $\{(G_\alpha,E):\alpha\in\Lambda\}$ is a family of soft b-N-open (resp. soft α-N-open, soft pre-N-open, soft β-N-open) sets of a soft topological space $(X,\tilde{\tau},E)$, then $\bigcup_{\alpha\in\Lambda}(G_\alpha,E)$ is soft b-N-open (resp. soft α-N-open, soft pre-N-open, soft β-N-open).

Proof :
Since
$(G_\alpha,E)\tilde{\subseteq}int_N(cl(G_\alpha,E))\tilde{\cup}cl(int_N(G_\alpha,E))$ for every $\alpha\in\Lambda$, we have: $\bigcup_{\alpha\in\Lambda}(G_\alpha,E)\tilde{\subseteq}$
$\bigcup_{\alpha\in\Lambda}[int_N(cl(G_\alpha,E))\tilde{\cup}cl(int_N(G_\alpha,E))]$
$=[\bigcup_{\alpha\in\Lambda}int_N(cl(G_\alpha,E))]\tilde{\cup}[\bigcup_{\alpha\in\Lambda}cl(int_N(G_\alpha,E))]$
$\tilde{\subseteq}[int_N(\bigcup_{\alpha\in\Lambda}cl(G_\alpha,E))]\tilde{\cup}[cl(\bigcup_{\alpha\in\Lambda}int_N(G_\alpha,E))]$
(By theorem (2.5),(xii)).
$\tilde{\subseteq}[int_N(cl(\bigcup_{\alpha\in\Lambda}(G_\alpha,E)))]\tilde{\cup}[cl(int_N(\bigcup_{\alpha\in\Lambda}(G_\alpha,E)))]$
(By theorem (2.5),(xii)).
Therefore $\bigcup_{\alpha\in\Lambda}(G_\alpha,E)$ is soft b-N-open. Similarly, we can prove the other cases.

Proposition (3.18):
If (G,E) is a soft b-N-open set in $(X,\tilde{\tau},E)$ such that $int_N(G,E)=\tilde{\phi}$, then (G,E) is a soft pre-N-open set.

Proof:

Since (G,E) is soft b-N-open, then $(G,E)\tilde{\subseteq}\mathrm{int}_N(\mathrm{cl}(G,E))\tilde{\bigcup}\mathrm{cl}(\mathrm{int}_N(G,E))$. But $\mathrm{int}_N(G,E)=\tilde{\phi}$, hence $\mathrm{cl}(\mathrm{int}_N(G,E))=\tilde{\phi}$, thus $(G,E)\tilde{\subseteq}\mathrm{int}_N(\mathrm{cl}(G,E))$. Therefore (G,E) is a soft pre-N-open set.

Definition (3.19):

A soft topological space $(X,\tilde{\tau},E)$ is called a soft door space if each soft subset of $\tilde{X}$ is either soft open or soft closed.

Propositions (3.20):

If $(X,\tilde{\tau},E)$ is a soft door space, then:

i) Each soft pre-N-open set is soft N-open.

ii) Each soft β-N-open set is soft b-N-open.

Proof:

i) Let (G,E) be a soft pre-N-open set. If (G,E) is soft open, then (G,E) is soft N-open. Otherwise, (G,E) is soft closed. Hence $(G,E)\tilde{\subseteq}\mathrm{int}_N(\mathrm{cl}(G,E))$ $=\mathrm{int}_N(G,E)$, thus $(G,E)=\mathrm{int}_N(G,E)$. Therefore (G,E) is a soft N-open set.

ii) Let (G,E) be a soft β-N-open set. If (G,E) is soft open, then (G,E) is soft b-N-open. Otherwise, (G,E) is soft closed. Hence $(G,E)\tilde{\subseteq}\mathrm{cl}(\mathrm{int}_N(\mathrm{cl}(G,E)))$ $=\mathrm{cl}(\mathrm{int}_N(G,E))\tilde{\subseteq}$ $\mathrm{int}_N(\mathrm{cl}(G,E))\tilde{\bigcup}\mathrm{cl}(\mathrm{int}_N(G,E))$. Thus (G,E) is a soft b-N-open set.

Definitions (3.21):

A soft subset (G,E) of a soft topological space $(X,\tilde{\tau},E)$ is said to be:

i) Soft N-$\tilde{t}$-set if $\mathrm{int}(G,E)=\mathrm{int}_N(\mathrm{cl}(G,E))$.

ii) Soft N-$\tilde{B}$-set if $(G,E)=(U,E)\tilde{\bigcap}(V,E)$, where $(U,E)\tilde{\in}\tilde{\tau}$ and (V,E) is a soft N-$\tilde{t}$-set.

Proposition (3.22):

Let (G_1,E) and (G_2,E) be two soft subsets of a soft topological space $(X,\tilde{\tau},E)$. If (G_1,E) and (G_2,E) are soft N-$\tilde{t}$-sets, then $(G_1,E)\tilde{\bigcap}(G_2,E)$ is also a soft N-$\tilde{t}$-set.

Proof:

If (G_1,E) and (G_2,E) are soft N-$\tilde{t}$-sets. Then: $\mathrm{int}_N(\mathrm{cl}((G_1,E)\tilde{\bigcap}(G_2,E)))$

$\tilde{\subseteq}\mathrm{int}_N(\mathrm{cl}(G_1,E)\tilde{\bigcap}\mathrm{cl}(G_2,E))$

$=\mathrm{int}_N(\mathrm{cl}(G_1,E))\tilde{\bigcap}\mathrm{int}_N(\mathrm{cl}(G_2,E))$

$=\mathrm{int}(G_1,E)\tilde{\bigcap}\mathrm{int}(G_2,E)$

$=\mathrm{int}((G_1,E)\tilde{\bigcap}(G_2,E))$.

Since $\mathrm{int}((G_1,E)\tilde{\bigcap}(G_2,E))\tilde{\subseteq}$ $\mathrm{int}_N(\mathrm{cl}((G_1,E)\tilde{\bigcap}(G_2,E)))$, then $\mathrm{int}((G_1,E)$ $\tilde{\bigcap}(G_2,E))=\mathrm{int}_N(\mathrm{cl}((G_1,E)\tilde{\bigcap}(G_2,E)))$.

Thus $(G_1,E)\tilde{\bigcap}(G_2,E)$ is a soft N-$\tilde{t}$-set.

The following example shows that soft pre-N-open sets and soft N-$\tilde{B}$-sets are independent.

Example (3.23):

Let $(\Re,\tilde{\mu},E)$ be the soft usual topological space, where $E=\{e\}$. Then $(G,E)=\{(e,Q^c)\}$ is a soft pre-N-open set, but is not a soft N-$\tilde{B}$-set (since $(G,E)=\tilde{\Re}\tilde{\bigcap}$ (G,E), where $\tilde{\Re}\tilde{\in}\tilde{\tau}$, but (G,E) is not a soft N-$\tilde{t}$-set) and $(H,E)=\{(e,(0,1])\}$ is a soft N-$\tilde{B}$-set (since $(H,E)=\tilde{\Re}\tilde{\bigcap}(H,E)$, where $\tilde{\Re}\tilde{\in}\tilde{\tau}$ and (H,E) is a soft N-$\tilde{t}$-set), but is not soft pre-N-open (since $(H,E)\tilde{\not\subset}\mathrm{int}_N(\mathrm{cl}(H,E))=\{(e,(0,1))\}$.

Proposition (3.24):

If $(X,\tilde{\tau},E)$ is a soft topological space and $(G,E)\tilde{\subseteq}\tilde{X}$. Then the following statements are equivalent:

i) (G,E) is a soft open set.

ii) (G,E) is a soft pre-N-open and a soft N-$\tilde{B}$-set.

Proof:

$(i)\Rightarrow(ii)$. If (G,E) is a soft open set, then $(G,E)=\mathrm{int}_N(G,E)\tilde{\subseteq}\mathrm{int}_N(\mathrm{cl}(G,E))$, hence (G,E) is soft pre-N-open. Also, $(G,E)=(G,E)\tilde{\bigcap}\tilde{X}$, where $(G,E)\tilde{\in}\tilde{\tau}$ and $\tilde{X}$ is a soft N-$\tilde{t}$-set and hence (G,E) is a soft N-$\tilde{B}$-set.

$(ii) \Rightarrow (i)$. Since (G,E) is a soft N-$\tilde{B}$-set, then $(G,E)=(U,E)\tilde{\cap}(V,E)$, where $(U,E)\tilde{\in}\tilde{\tau}$ and (V,E) is a soft N-$\tilde{t}$-set. By hypothesis, (G,E) is a soft pre-N-open set, then:

$(G,E)\tilde{\subseteq}\mathrm{int}_N(\mathrm{cl}(G,E))$
$=\mathrm{int}_N(\mathrm{cl}((U,E)\tilde{\cap}(V,E)))$
$\tilde{\subseteq}\mathrm{int}_N(\mathrm{cl}(U,E)\tilde{\cap}\mathrm{cl}(V,E))$
$=\mathrm{int}_N(\mathrm{cl}(U,E))\cap\mathrm{int}_N(\mathrm{cl}(V,E))$
$=\mathrm{int}_N(\mathrm{cl}(U,E))\tilde{\cap}\mathrm{int}(V,E)$. Hence $(G,E)=(U,E)\tilde{\cap}(V,E)=((U,E\tilde{\cap}(V,E))\tilde{\cap}(U,E)$
$\tilde{\subseteq}(\mathrm{int}_N(\mathrm{cl}(U,E))\tilde{\cap}\mathrm{int}(V,E))\tilde{\cap}(U,E)$
$=(\mathrm{int}_N(\mathrm{cl}(U,E))\tilde{\cap}(U,E))\tilde{\cap}\mathrm{int}(V,E)$
$=(U,E)\tilde{\cap}\mathrm{int}(V,E)=\mathrm{int}(U,E)\cap\mathrm{int}(V,E)$
$=\mathrm{int}((U,E)\tilde{\cap}(V,E))=\mathrm{int}(G,E)$. Thus $(G,E)=\mathrm{int}(G,E))$ and hence (G,E) is soft open.

Definitions (3.25):

A soft subset (G,E) of a soft topological space $(X,\tilde{\tau},E)$ is said to be:

i) Soft N-$\tilde{t}_\alpha$-set if $\mathrm{int}(G,E)=\mathrm{int}_N(\mathrm{cl}(\mathrm{int}_N(G,E)))$.

ii) Soft N-$\tilde{B}_\alpha$-set if $(G,E)=(U,E)\tilde{\cap}(V,E)$, where $(U,E)\tilde{\in}\tilde{\tau}$ and (V,E) is a soft N-$\tilde{t}_\alpha$-set.

Proposition (3.26):

Let (G_1,E) and (G_2,E) be two soft subsets of a soft topological space $(X,\tilde{\tau},E)$. If (G_1,E) and (G_2,E) are soft N-$\tilde{t}_\alpha$-sets, then $(G_1,E)\tilde{\cap}(G_2,E)$ is also a soft N-$\tilde{t}_\alpha$-set.

Proof:

If (G_1,E) and (G_2,E) are soft N-$\tilde{t}_\alpha$-sets. Then:

$\mathrm{int}_N(\mathrm{cl}(\mathrm{int}_N((G_1,E)\tilde{\cap}(G_2,E))))$
$=\mathrm{int}_N(\mathrm{cl}(\mathrm{int}_N(G_1,E)\tilde{\cap}\mathrm{int}_N(G_2,E)))$
$\tilde{\subseteq}\mathrm{int}_N(\mathrm{cl}(\mathrm{int}_N(G_1,E))\tilde{\cap}\mathrm{cl}(\mathrm{int}_N(G_2,E)))$
$=\mathrm{int}_N(\mathrm{cl}(\mathrm{int}_N(G_1,E)))\tilde{\cap}\mathrm{int}_N(\mathrm{cl}(\mathrm{int}_N(G_2,E)))$
$=\mathrm{int}(G_1,E)\tilde{\cap}\mathrm{int}(G_2,E)$
$=\mathrm{int}((G_1,E)\tilde{\cap}(G_2,E))$.

Since $\mathrm{int}((G_1,E)\tilde{\cap}(G_2,E))\tilde{\subseteq}\mathrm{int}_N(\mathrm{cl}(\mathrm{int}_N((G_1,E)\tilde{\cap}(G_2,E))))$, then $\mathrm{int}((G_1,E)\tilde{\cap}(G_2,E))=\mathrm{int}_N(\mathrm{cl}(\mathrm{int}_N((G_1,E)\tilde{\cap}(G_2,E))))$. Thus $(G_1,E)\tilde{\cap}(G_2,E)$ is a soft N-$\tilde{t}_\alpha$-set.

The following example shows that soft α-N-open sets and soft N-$\tilde{B}_\alpha$-sets are independent.

Example (3.27):

Let $(\Re,\tilde{\mu},E)$ be the soft usual topological space, where $E=\{e\}$. Then $(H,E)=\{(e,(0,1])\}$ is a soft N-$\tilde{B}_\alpha$-set which is not soft α-N-open. Also, in example (2.2), $(G,E)=\{(e_1,\{a,b\}),(e_2,\{a,b\})\}$ is a soft α-N-open set, but is not a soft N-$\tilde{B}_\alpha$-set.

Proposition (3.28):

If $(X,\tilde{\tau},E)$ is a soft topological space and $(G,E)\tilde{\subseteq}\tilde{X}$. Then the following statements are equivalent:

i) (G,E) is a soft open set.

ii) (G,E) is a soft α-N-open and a soft N-$\tilde{B}_\alpha$-set.

Proof:

$(i)\Rightarrow(ii)$. If (G,E) is a soft open set, then $(G,E)=\mathrm{int}_N(G,E)\tilde{\subseteq}\mathrm{cl}(\mathrm{int}_N(G,E))\tilde{\subseteq}\mathrm{int}_N(\mathrm{cl}(\mathrm{int}_N(G,E)))$, thus (G,E) is soft α-N-open. Also, $(G,E)=(G,E)\tilde{\cap}\tilde{X}$, where $(G,E)\tilde{\in}\tilde{\tau}$ and $\tilde{X}$ is a soft N-$\tilde{t}_\alpha$-set, therefore (G,E) is a soft N-$\tilde{B}_\alpha$-set.

$(ii)\Rightarrow(i)$. Since (G,E) is a soft N-$\tilde{B}_\alpha$-set, then $(G,E)=(U,E)\tilde{\cap}(V,E)$, where $(U,E)\tilde{\in}\tilde{\tau}$ and (V,E) is a soft N-$\tilde{t}_\alpha$-set. By hypothesis, (G,E) is a soft α-N-open set, then:

$(G,E)\tilde{\subseteq}\mathrm{int}_N(\mathrm{cl}(\mathrm{int}_N(G,E)))$
$=\mathrm{int}_N(\mathrm{cl}(\mathrm{int}_N((U,E)\tilde{\cap}(V,E))))$
$=\mathrm{int}_N(\mathrm{cl}(\mathrm{int}_N(U,E)\cap\mathrm{int}_N(V,E)))$
$\tilde{\subseteq}\mathrm{int}_N(\mathrm{cl}(\mathrm{int}_N(U,E))\tilde{\cap}\mathrm{cl}(\mathrm{int}_N(V,E)))$

$= \text{int}_N(\text{cl}(\text{int}_N(U,E))) \tilde{\cap} \text{int}_N(\text{cl}(\text{int}_N(V,E)))$ $\tilde{\subseteq} \text{int}_N(\text{cl}(U,E)) \tilde{\cap} \text{int}(V,E)$. Hence $(G,E) = (U,E) \tilde{\cap} (V,E) = ((U,E \tilde{\cap} (V,E)) \tilde{\cap} (U,E)$ $\tilde{\subseteq} (\text{int}_N(\text{cl}(U,E)) \tilde{\cap} \text{int}(V,E)) \tilde{\cap} (U,E)$ $= (\text{int}_N(\text{cl}(U,E)) \tilde{\cap} (U,E)) \tilde{\cap} \text{int}(V,E)$ $= (U,E) \tilde{\cap} \text{int}(V,E) = \text{int}(U,E) \cap \text{int}(V,E)$ $= \text{int}((U,E) \tilde{\cap} (V,E)) = \text{int}(G,E)$. Thus $(G,E) = \text{int}(G,E))$ and hence (G,E) is soft open.

Definition (3.29):

A soft subset (G,E) of a soft topological space $(X,\tilde{\tau},E)$ is said to be a soft N-set if $(G,E) = (U,E) \tilde{\cap} (V,E)$, where $(U,E) \tilde{\in} \tilde{\tau}$ and $\text{int}(V,E) = \text{int}_N(V,E)$.

The following example shows that soft N-open sets and soft N-sets are independent.

Example (3.30):

Let $(\Re,\tilde{\mu},E)$ be the soft usual topological space, where $E = \{e\}$. Then (H,E) $= \{(e,(0,1) \cap Q)\}$ is a soft N-set which is not soft N-open. Also, in example (2.2), $(G,E) = \{(e_1,\{a,b\}), (e_2,\{a,b\})\}$ is a soft N-open set, but is not a soft N-set.

Proposition (3.31):

If $(X,\tilde{\tau},E)$ is a soft topological space and $(G,E) \tilde{\subseteq} \tilde{X}$. Then the following statements are equivalent:

i) (G,E) is a soft open set.

ii) (G,E) is a soft N-open and a soft N-set.

Proof:

$(i) \Rightarrow (ii)$. It is obvious.

$(ii) \Rightarrow (i)$. Since (G,E) is a soft N-set, then $(G,E) = (U,E) \tilde{\cap} (V,E)$, where $(U,E) \tilde{\in} \tilde{\tau}$ and $\text{int}(V,E) = \text{int}_N(V,E)$. By hypothesis, (G,E) is a soft N-open set, then:

$(G,E) = \text{int}_N(G,E) = \text{int}_N((U,E) \tilde{\cap} (V,E))$ $= \text{int}_N(U,E) \cap \text{int}_N(V,E) = (U,E) \tilde{\cap} \text{int}(V,E)$ $= \text{int}(U,E) \tilde{\cap} \text{int}(V,E) = \text{int}((U,E) \tilde{\cap} (V,E))$ $= \text{int}(G,E)$. Thus (G,E) is soft open.

Definitions (3.32):

A soft topological space $(X,\tilde{\tau},E)$ is said to satisfy:

i) The soft N-condition if every soft N-open set is soft N-$\tilde{t}$-set.

ii) The soft N-$\tilde{B}_\alpha$-condition if every soft α-N-open set is soft N-$\tilde{B}_\alpha$-set.

iii) The soft N-$\tilde{B}$-condition if every soft pre-N-open set is soft N-$\tilde{B}$-set.

4. Weak Soft $\tilde{D}_N$-Sets

Now, we introduce and study new concepts called soft $\tilde{D}$-sets, soft $\tilde{D}_N$-sets, soft $\tilde{D}_{\alpha-N}$-sets, soft $\tilde{D}_{pre-N}$-sets, soft $\tilde{D}_{b-N}$-sets and soft $\tilde{D}_{\beta-N}$-sets.

Definition (4.1):

A soft subset (G,E) of a soft topological space $(X,\tilde{\tau},E)$ is said to be a soft $\tilde{D}$-set (resp. soft $\tilde{D}_N$-set, soft $\tilde{D}_{\alpha-N}$-set, soft $\tilde{D}_{pre-N}$-set, soft $\tilde{D}_{b-N}$-set, soft $\tilde{D}_{\beta-N}$-set) if there exists two soft open (resp. soft N-open, soft α-N-open, soft pre-N-open, soft b-N-open, soft β-N-open) sets (U_1,E) and (U_2,E) in $\tilde{X}$ such that $(U_1,E) \neq \tilde{X}$ and $(G,E) = (U_1,E) \setminus (U_2,E)$.

Remark (4.2):

In definition (4.1), if $(U_1,E) \neq \tilde{X}$ and $(U_2,E) = \tilde{\phi}$, then each proper soft open (resp. soft N-open, soft α-N-open, soft pre-N-open, soft b-N-open, soft β-N-open) subset of $\tilde{X}$ is a soft $\tilde{D}$-set (resp. soft $\tilde{D}_N$-set, soft $\tilde{D}_{\alpha-N}$-set, soft $\tilde{D}_{pre-N}$-set, soft $\tilde{D}_{b-N}$-set, soft $\tilde{D}_{\beta-N}$-set).

The converse of Remark (4.2) may not be true in general as shown by the following examples.

Example (4.3):

Let $X = N$, $E = \{e\}$ and $\tilde{\tau} = \{\tilde{X}, \tilde{\phi}, (U,E)\}$ be a soft topology over X, where $(U,E) = \{(e,\{1\})\}$. Then $(G,E) = \{(e,\{2\})\}$ is soft $\tilde{D}_{\alpha-N}$-set and soft $\tilde{D}_N$-set, but is not soft α-N-open set and is not soft N-open set.

Example (4.4):

Let $X=N$, $E=\{e\}$ and $\tilde{\tau}=\{\tilde{X},\tilde{\phi},(U,E)\}$ be a soft topology over X, where $(U,E)=\{(e,N-\{1\})\}$. Then $(G,E)=\{(e,\{1\})\}$ is a soft $\tilde{D}_{pre-N}$-set (resp. soft $\tilde{D}_{b-N}$-set, soft $\tilde{D}_{\beta-N}$-set), but is not soft β-N-open set.

Proposition (4.5):

In any soft topological space $(X,\tilde{\tau},E)$.

i) Each soft $\tilde{D}$-set is soft $\tilde{D}_N$-set.

ii) Each soft $\tilde{D}_N$-set is soft $\tilde{D}_{\alpha-N}$-set.

iii) Each soft $\tilde{D}_{\alpha-N}$-set is soft $\tilde{D}_{pre-N}$-set.

iv) Each soft $\tilde{D}_{pre-N}$-set is soft $\tilde{D}_{b-N}$-set.

v) Each soft $\tilde{D}_{b-N}$-set is soft $\tilde{D}_{\beta-N}$-set.

Proof:

Follows from proposition (3.2).

The converse of proposition (4.5) number (i) and (iii) may not be true in general it is shown in the following examples.

Example (4.6):

Let $X=N$, $E=\{e\}$ and $\tilde{\tau}=\{\tilde{X},\tilde{\phi},(U,E)\}$ be a soft topology over X, where $(U,E)=\{(e,\{1\})\}$. Then $(G,E)=\{(e,\{2\})\}$ is a soft $\tilde{D}_N$-set, since $\exists\ (G_1,E)=\{(e,N-\{1\})\}$ and $(G_2,E)=\{(e,N-\{2\})\}$ are soft N-open sets such that $(G_1,E)\neq\tilde{X}$ and $(G,E)=(G_1,E)\setminus(G_2,E)$, but (G,E) is not soft $\tilde{D}$-set.

Example (4.7):

Let $(\Re,\tilde{\mu},E)$ be the soft usual topological space, where $E=\{e\}$. Then $(G,E)=\{(e,Q)\}$ is a soft $\tilde{D}_{pre-N}$-set, since $\exists\ (G_1,E)=\{(e,Q)\}$ and $(G_2,E)=\tilde{\phi}$ are soft pre-N-open sets such that $(G_1,E)\neq\tilde{\Re}$ and $(G,E)=(G_1,E)\setminus(G_2,E)$, but (G,E) is not soft $\tilde{D}_{\alpha-N}$-set.

Proposition (4.8):

If $(X,\tilde{\tau},E)$ is a soft door space. Then:

i) Each soft $\tilde{D}_{pre-N}$-set is soft $\tilde{D}_N$-set.

ii) Each soft $\tilde{D}_{\beta-N}$-set is soft $\tilde{D}_{b-N}$-set.

Proof:

Follows from Proposition (3.20).

Proposition (4.9):

In any soft topological space satisfies soft N-condition soft $\tilde{D}_N$-set is soft $\tilde{D}$-set.

Proof:

Suppose that (G,E) is a soft $\tilde{D}_N$-set, then there are two soft N-open sets (U_1,E) and (U_2,E) in $\tilde{X}$ such that $(U_1,E)\neq\tilde{X}$ and $(G,E)=(U_1,E)\setminus(U_2,E)$. Hence $(U_1,E)=int_N(U_1,E))\tilde{\subseteq}int_N(cl(U_1,E))$ and $(U_2,E)=int_N(U_2,E))\tilde{\subseteq}int_N(cl(U_2,E))$. Since $(X,\tilde{\tau},E)$ is satisfy the N-condition, then (U_1,E) and (U_2,E) are soft N-$\tilde{t}$-sets. Therefore $(U_1,E)\tilde{\subseteq}int(U_1,E)$ and $(U_2,E)\tilde{\subseteq}int(U_2,E)$. Hence (U_1,E) and (U_2,E) are soft open sets. Thus (G,E) is a soft $\tilde{D}$-set.

Proposition (4.10):

In any soft topological space satisfies soft N-$\tilde{B}_\alpha$-condition soft $\tilde{D}_{\alpha-N}$-set is soft $\tilde{D}$-set.

Proof:

Follows from Proposition (3.28).

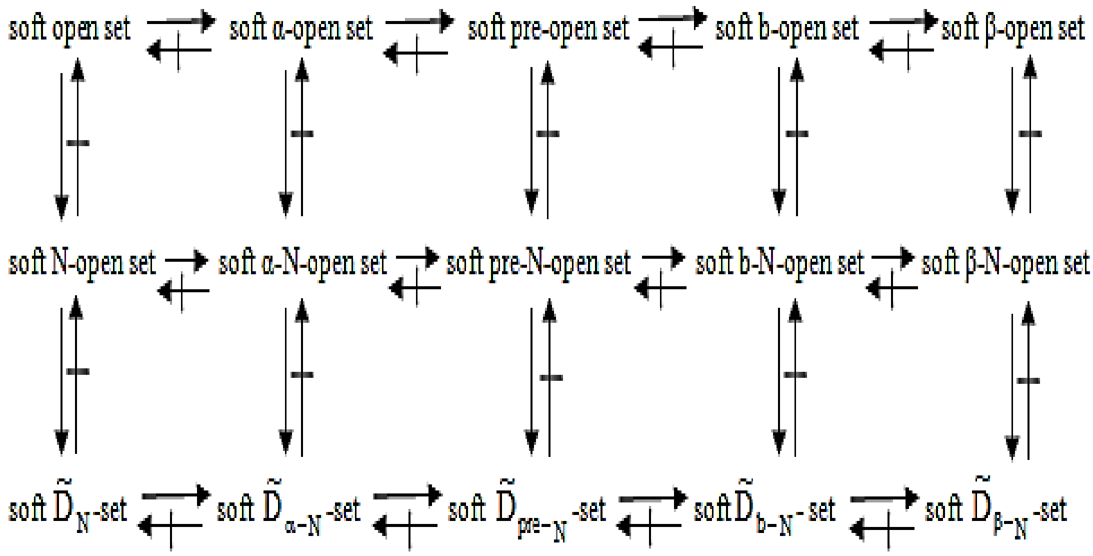
Proposition (4.11):

In any soft topological space satisfies soft N-$\tilde{B}$-condition soft $\tilde{D}_{pre-N}$-set is soft $\tilde{D}$-set.

Proof:

Follows from Proposition (3.24).

The following diagram shows the relation between the soft open sets and each of soft N-open sets, weak soft N-open sets and weak soft $\tilde{D}_N$-set.



<u>Definition (4.12):</u>

A soft function $f:(X,\tilde{\tau},E)\to(Y,\tilde{\sigma},E)$ is said to be soft N-continuous (resp. soft pre-N-continuous, soft N-$\tilde{B}$-continuous, soft α-N-continuous, soft N-$\tilde{B}_\alpha$-continuous, soft N*-continuous) if $f^{-1}((V,E))$ is soft N-open (resp. soft pre-N-open, soft N-$\tilde{B}$-set, soft α-N-open, soft N-$\tilde{B}_\alpha$-set, soft N-set) in $\tilde{X}$ for each soft open set (V,E) in $\tilde{Y}$.

By propositions (3.24), (3.28) and (3.31) we get the following results.

<u>Theorem (4.13):</u>

For a soft function $f:(X,\tilde{\tau},E)\to(Y,\tilde{\sigma},E)$, the following statements are equivalent:

i) f is soft continuous.

ii) f is soft pre-N-continuous and soft N-$\tilde{B}$-continuous.

iii) f is soft α-N-continuous and soft N-$\tilde{B}_\alpha$-continuous.

iv) f is soft N-continuous and soft N*-continuous.

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