CAD OF A THREE ELELCTRODE ELECTROSTATIC IMERRSION LENSES USING THE SPLINE MODEL

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Abstract

The focal properties and aberrations for three electrode electrostatic lenses are investigated in detail using the two-interval spline lens model. The synthesis procedure of optimization has been taken into account with the aid of the spline function. The potential ratio is investigated as an effective parameter in lens design.

The optimization procedure has shown the possibility of obtaining a practical design of electrostatic lenses operated under pre-assigned conditions to yield minimum aberrations. Non-relativistic velocities and neglecting the space-charge effects are the two main assumptions that have been taken into account throughout the work. The actual electrodes profile which is determined from equipotential surfaces has been constructed in three-dimensional diagrams.

Keywords: Electrostatic Lenses, Charged Particle Beams, Spherical and Chromatic Aberrations, Synthesis.

Introduction

The increasing complexity of electron optics instrumentation demands a continuous development of modeling techniques. In parallel, the theory of electron optical focusing and aberrations has reached a high degree of sophistication. The accurate analysis of optical parameters by perturbation calculus leads to a complex formalism that needs in any case the aid of computer algebra programs. There is however a rather simple and general method to characterize the aberrations of focusing system by the use of direct ray tracing. The price to be paid is that the process requires extremely accurate determination of trajectories of charged particles through the lens. As very high precision is required, each step of numerical simulation should be optimized to get accurate results with minimum computational efforts [1-3].

Optimization of electrostatic lenses is the search for their electron and ion optical element that would provide the required optical properties with minimum aberrations. Two optimization approaches are exist, namely analysis and synthesis [2]**.** The potential ratio is an effective parameter in all types of electrostatic lenses especially when their design is investigated by the synthesis procedure of optimization. The present work is aimed at performing the following:-

(a) Putting forward designs for electrostatic lenses operated under different modes of operation using the synthesis procedure of the optimization technique.
(b) Investigating the effect of the potential ratio on the optical properties such as the focal length and aberration coefficients for the various types of electrostatic lenses approximated by the spline lens model.

Electrostatic Spline Function

A cubic spline function is a third order polynomial used for interpolation and curve fitting with continuous first and second derivative. The spline assumes that shape, which minimizes its potential energy, and beam theory states that this energy is proportional to the integral with respect to the arc length of the square of the curvature of the spline. If the spline is a function of coordinate z and if the slope is small, the second derivative approximates the curvature [5, 6].

The cubic spline function, its first and second derivative can be written as [3]:-

$$\begin{aligned} U_k(z) &= A_k + B_k(z-z_{k-1}) + C_k(z-z_{k-1})^2 + D_k(z-z_{k-1})^3 \\ U'_k(z) &= B_k + 2C_k(z-z_{k-1}) + 3D_k(z-z_{k-1})^2 \\ U''_k(z) &= 2C_k + 6D_k(z-z_{k-1}) \end{aligned} \quad \text{.........} (1)$$

where $U_k(z)$ is the potential distribution for rotationally symmetric electrostatic lenses. z_{k-1} is the coordinate of the k-th interval's left end point, k=1,2,3,….n is the number of intervals *A*,*B*,*C* and *D* are coefficients of the spline function which are different for each region. The continuity conditions for the spline function, its first and second derivatives can be written as: -

$$U_{k+1}(z_k) = U_k(z_k)$$
$$U'_{k+1}(z_k) = U'_k(z_k) \quad \text{............................ (2)}$$
$$U''_{k+1}(z_k) = U''_k(z_k)$$

From a mathematical point of view, the cubic spline function with zero second derivative at both ends is called the natural spline, which is a unique function possessing the minimum possible curvature over all its length. In this sense the natural cubic spline is the smoothest function interpolates the data [5]. In dealing with the cubic spline function, there are eight coefficients that are interconnected by three continuity equations, two equations for zero slope at the ends of each interval, and three equations for fitting three given potential values. The following two equations are for satisfying the zero slope conditions:

$$B_1 = 0\,,\, B_n + 2C_n h + 3D_n h^2 = 0$$

where h=z_k-z_{k-1} is the length of each interval of the spline function. Since all the optical properties depend on the potential ratios, it is convenient to choose the potential U(0) at z = 0 to be unity. There are actually two free parameters, namely the potentials U(L/2) and U(L) at z=L/2 and z=L, respectively. The length of each interval will be the same to achieve the simplest non-trivial case; otherwise it becomes more complicated. The solution of a system of eight algebraic equations yields the following coefficients [5]:

$$A_1 = U(0)$$
$$B_1 = 0$$
$$C_1 = [12U(L/2) - 3U(L) - 9U(0)]/L^2$$
$$D_1 = [10U(0) - 16U(L/2) + 6U(L)]/L^3 \quad \text{.......... (3)}$$
$$A_2 = U(L/2)$$
$$B_2 = 3[U(L) - U(0)]/2L$$
$$C_2 = 6[U(0) - 2U(L/2) + U(L)]/L^2$$
$$D_2 = [16U(L/2) - 6U(0) - 10U(L)]/L^3$$

where A1 and A2 are given by the potential values at the beginning of each interval, B1 = 0 according to the condition of vanishing field, B2 is the value of the potential gradient at z = L/2 and it is independent of the value of the potential at that point [3].

Results and Discussion

The potential distribution of the electrostatic lenses was approximated by using the cubic polynomial of the spline function. However, the first and second derivatives of the potential were approximated by the first and second derivatives of the spline function respectively (equation 1). The axial extension was divided into two intervals and the potential and electric field distributions were defined in each interval by the spline model with continuous first and second derivatives (equation2).

The paraxial ray equation was solved along each interval numerically by using fourth-order Runge-Kutta method. The aberration integrals were solved by using the numerical integration method of Simpson rule. The optical properties and the spherical and chromatic aberrations have been investigated under different magnification conditions. The potential distribution, axial dimensions, optical properties and aberrations were normalized in order to make a meaningful comparison between different lenses. The actual electrodes configuration was determined from the potential distribution by using the approximated series of the potential equation, which includes its first two terms (ad-hoc assumption) in order to find the equipotential surfaces representing the final electrodes shape [7].

$$R = [4(U_k(z) - V)/U''(z)]^{1/2} \quad \text{.................. (4)}$$

The general condition of the zero-slope (i.e. vanishing field) at the terminals of each distribution (lens boundaries) was fulfilled. Immersion electrostatic lenses consisting of two or three electrodes and unipotential lenses operated at acceleration or deceleration mode with initial conditions of zero or finite magnification were taken into consideration. In the present work only the three-electrode immersion lens is shown.

Fig.(1) shows the axial distribution of the potential function U(z) for a three electrode immersion lens operated in the acceleration mode [i.e. U(zi)/U(zo) > 1.0]. The dotted line is the first derivative of the potential function; it is terminated at both ends of the corresponding distribution to satisfy the vanishing field condition. There are two inflection points in this distribution at the relative axial distance z/L = 0.2 and 0.75 which indicate that the number of electrodes is three.

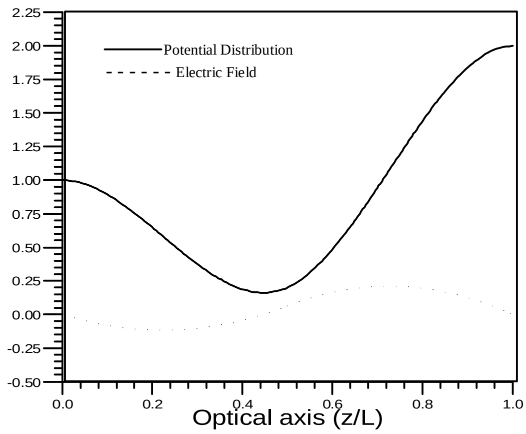

Fig.(1) : The axial potential distribution and its first derivative for a three-electrode immersion lens.

The ion beam trajectory is shown in Fig.(2) for the zero and finite magnification conditions. Under finite magnification condition the beam crosses the optical axis outside the lens region as shown in Fig.(2); however, under zero magnification condition the beam crosses the optical axis within the lens field at z/L = 0.8 and $U(z_i)/U(z_o) = 2$. As the refractive power of the lens increases with $U(z_i)/U(z_o)$, the ratio z/L at which the beam crosses the optical axis decreases.

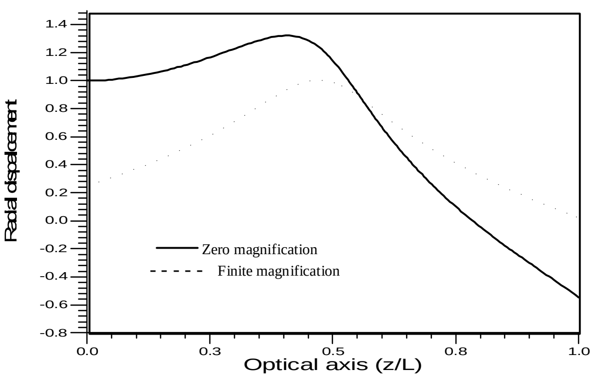

Fig. (2) : Ion beam trajectory under zero and finite magnification condition.

The relative image-side focal length fi/L as a function of the potential ratio $U(z_i)/U(z_o)$ is shown in Fig.(3) under zero magnification condition. The ratio fi/L has a minimum value of 0.4 at $U(z_i)/U(z_o) = 5.0$ where the charged particles beam crosses the optical axis at the terminal of the lens field. At this point the working distance is zero as shown in Fig.(4). At potential ratios $U(z_i)/U(z_o) > 5.0$ the beam crosses the optical axis within the lens field. The part of the field between the point of intersection and the point where the field terminates acts as a converging field. As the potential ratio increases the emerging beam trajectory converges further towards the optical axis until the beam emerges parallel to it where one would have a telescopic lens action. At this point the focal length becomes infinite which is shown clearly in Fig.(4) at $U(z_i)/U(z_o) = 6.5$ and the working distance W is also infinite. The relative working distance W/L decreases with increasing $U(z_i)/U(z_o)$ until it diminishes as shown in Fig.(4) since the refractive power of the lens increases with the increase of potential ratio. At values of $U(z_i)/U(z_o) > 7.0$ the second loop of the focal length appears and the working distance would have values greater than zero. It should be mentioned that the above variations of the working distance are in similar manner as those for the two-electrode immersion lens.

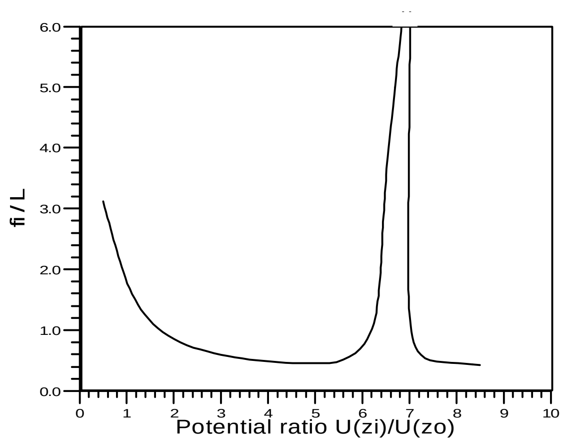

Fig.(3): The relative image-side focal length under zero magnification as a function of the potential ratio.

Fig.(5) shows the variation of the relative aberration coefficients for the three-electrode immersion lens as a function of the potential ratio under zero magnification condition. The minimum value of C_s/f_i is 12.0 at $U(z_i)/U(z_o)<3.0$. However it is quite acceptable to operate this lens at the potential ratios $1.5 < U(z_i)/U(z_o) < 5.0$ since the variation of Cs/fi is little within this range. The values of the relative chromatic aberration C_c/f_i are more favorable than those of Cs/fi within the above range of the potential ratio. At higher potential ratios both the relative aberration coefficients deteriorate where the lens is heading towards a telescopic action.

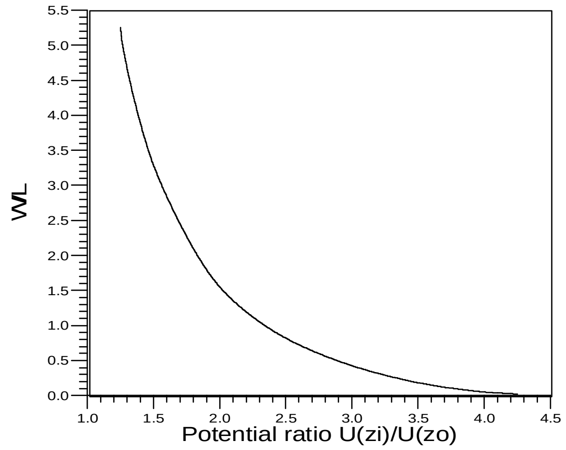

Fig.(4): The relative working distance under zero magnification condition as a function of the potential ratio.

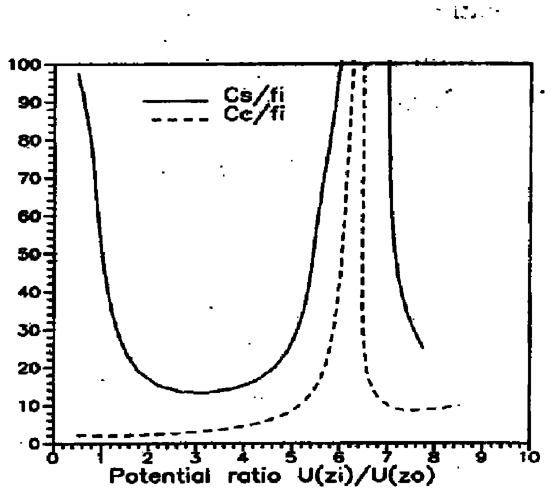

Fig.(5) : The relative spherical and chromatic aberration coefficients under zero magnification condition as a function of the potential ratio.

Fig.(6) shows the ion beam trajectory for different values of image-to-object potential ratios under initial conditions of infinite magnification. As the voltage ratio increases the beam crosses the optical axis at a shorter working distance until this distance diminishes. As the potential is increased further, the beam emerges from the lens field after crossing the optical axis once.

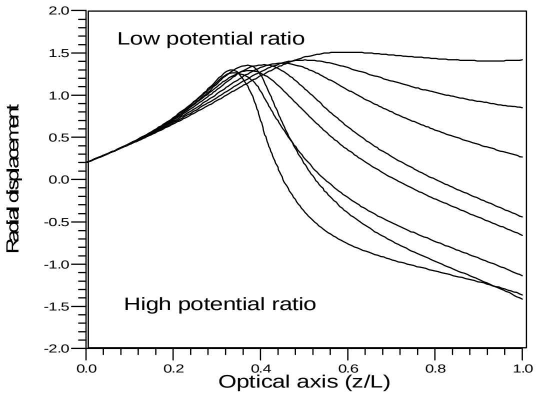

Fig.(6) : Ion beam trajectories at different potential ratios illustrating the change of magnification mode with the potential ratio.

Further details concerning the values of the various image parameters for the three-electrode immersion lens operated under zero magnification condition are given in Table (1). where the potential of the beam entrance electrode $U(z_o)$ is taken as a reference. At $0.0 < U(z_i)/U(z_o) < 1.0$ the lens operates in the deceleration mode; however at higher potentials the lens operates in the acceleration mode. By keeping $U(z_i)/U(z_o)$ constant a value for the ratio of the mid-electrode potential to that of the object side $[U(L/2)/U(z_o)]$ is determined at which the relative spherical aberration C_s/f_i is minimized; thus the corresponding relative chromatic aberration C_c/f_i, the relative image-side focal length f_i/L, the relative working distance W/L and the spot radius R_i are computed.

Fig.(7) is the three dimensional diagram of the two-interval three-electrode immersion lens obtained from the axial electrostatic potential of Fig(1). The potential applied on the first, second and third electrode is U, 0.2U and 2.0U respectively. Where the ion beam enters the lens from the left-hand side and emerges from the image side (right-hand side).

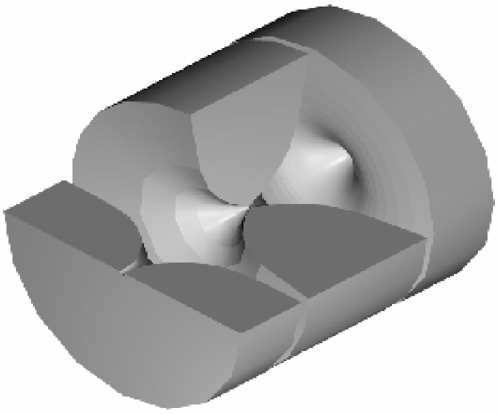

Fig.(7): A three-dimensional diagram of a three-electrode immersion lens obtained by two-interval spline lens model.

Table (1)
The optical properties and aberrations for two-interval three-electrode electrostatic immersion lens operated under zero magnification condition.

$U(z_i)/U(z_o)$	$U(L/2)/U(z_o)$	f_i/L	W/L	C_s/f_I	C_c/f_i	$R_i(\mu m)$
0.10	2.101	0.167	0.0405	0.1559	1.2217	4.082
0.12	2.124	0.195	0.0667	0.2025	1.1440	4.466
0.15	2.149	0.224	0.0829	0.2571	1.0761	4.823
0.25	2.250	0.338	0.1406	0.5443	0.8860	5.995
0.30	2.300	0.394	0.1651	0.7350	0.8224	6.478
0.40	2.400	0.499	0.2048	1.2306	0.7337	7.334
0.50	2.500	0.596	0.2325	1.8931	0.6800	8.123
0.75	2.750	0.790	0.2544	4.2318	0.6332	10.087
0.90	2.900	0.8726	0.2423	5.9250	0.6374	11.290
1.12	3.124	0.954	0.2022	8.4771	0.6656	13.065
1.25	3.249	0.983	0.1731	9.7717	0.6803	13.848
1.35	3.349	0.998	0.1479	10.709	0.6949	14.440
1.50	3.499	1.012	0.1086	11.935	0.7162	15.088
1.75	3.750	1.018	0.0429	12.506	0.7480	15.961
2.00	3.998	1.009	0.0195	13.568	0.7742	16.448
2.25	4.250	0.991	0.0000	14.247	0.7945	16.661

Conclusion

It appears from the present work that it is possible to design various types of electrostatic lenses with as small aberrations as possible operated under different potential ratios and magnification conditions by using the spline function. Although the spline lens model has not been used by many authors for investigating in detail the effect of the electrodes potential ratio on the focal properties, however, with the aid of this model, the present work has shown that the potential ratio is one of the most effective parameters in all types of electrostatic lenses. In addition the potential ratio is a crucial parameter since it governs the mode of operation and hence the path of the emerging beams. For instance, under zero magnification condition the beam may traverse a telescopic path.

It has been shown that the cubic spline function is an excellent tool for varying the basic parameters of the axial potential distribution in order to achieve the most favorable electrostatic lenses for a particular application. The total number of electrodes and their geometry depends on the axial distribution of the potential function. The inverse problem procedure has shown the possibility of determining electrostatic lenses for ion beam systems that are superior to conventional ones commonly used in present day systems. Although the full potentialities of these new lenses put forward in the present investigation have not yet been assessed in practice but it is already clear from the computations that they could bring about instrumental developments.

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